

## Exercise 2

2. The linear system in exercise-1 is under consideration for a waterflooding project, water-injection rate  $i_w = 1000$  bbl/day,  $u_0 = 10$  cp.

- (1) Calculate the fractional flow for the reservoir dip angles of **10°, 20°, and 30°**, assuming (a) **updip displacement** and (b) **downdip displacement**.
- (2) What conclusions can you draw?

$S_w$	$k_{ro}$	$k_{rw}$	$k_{ro}/k_{rw}$	$F_w$ , updip			$F_w$ , downdip		
				10°	20°	30°	10°	20°	30°
0.24	0.95	0							
0.3	0.89	0.01							
0.4	0.74	0.04							
0.5	0.45	0.09							
0.6	0.19	0.17							
0.65	0.12	0.22							
0.7	0.06	0.28							
0.75	0.03	0.36							
0.78	0	0.41							

(In field units, the fractional flow equation can be expressed as:

$$f_w = \frac{1 + \left( \frac{0.001127 k_o A}{\mu_o q_t} \right) \left[ \frac{\partial p_c}{\partial x} - 0.433 \Delta \rho \sin(\alpha) \right]}{1 + \frac{k_o \mu_w}{k_w \mu_o}}$$

- where  $f_w$  = fraction of water (water cut), bbl/bbl  
 $k_o$  = effective permeability of oil, md  
 $k_w$  = effective permeability of water, md  
 $\Delta \rho$  = water-oil density differences, g/cm<sup>3</sup>  
 $k_w$  = effective permeability of water, md  
 $q_t$  = total flow rate, bbl/day  
 $\mu_o$  = oil viscosity, cp  
 $\mu_w$  = water viscosity, cp  
 $A$  = cross-sectional area, ft<sup>2</sup>

Noting that the relative permeability ratios  $k_{ro}/k_{rw} = k_o/k_w$  and, for two-phase flow, the total flow rate  $q_t$  are essentially equal to the water-injection rate, i.e.,  $i_w = q_t$