Advanced Reservoir Engineering

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Chapter 6 Principle and application of well test analysis

Section 1 Introduction

Section 2 Basic principles and concepts of transient well testing

Section 3 Conventional well test analysis methods (pressure drop test)

Section 4 Conventional well test analysis method (pressure build-up well test)

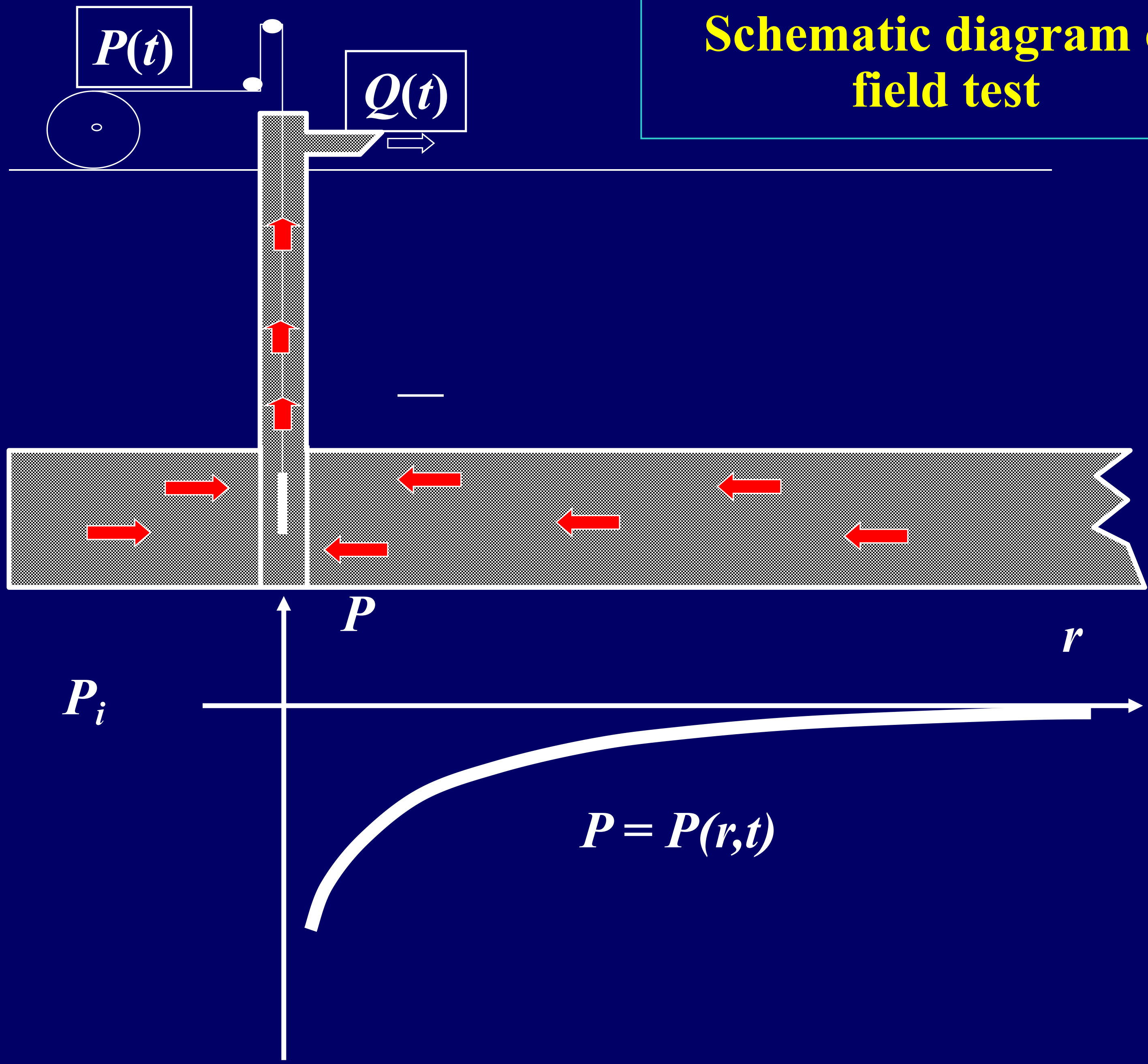
Section 5 Conventional well test analysis methods for dual-porosity reservoirs

Section 6 Conventional analysis method for fracturing wells in homogeneous reservoirs

Section 7 Conventional analysis method for horizontal wells in homogeneous reservoirs

- **〈Pressure Buildup and Flow Tests in Wells〉 1967**
- **〈Theory and Practice of Gas Well Tests〉 1975**
- **〈Advances in Well Test Analysis〉 1977**
- **〈Application of Pressure Recovery Curve in Oilfield Development〉 1977**
- **〈Theoretical Basis of Well Testing Analysis〉 1987**
- **〈Modern Well Test Analysis〉 1992**
- **〈Well Test Analysis〉 1993**
- **〈Pressure Transient Analysis 〉 1989**
- **〈Well Testing in Heterogeneous Formation〉**
- **Other**

Schematic diagram of field test



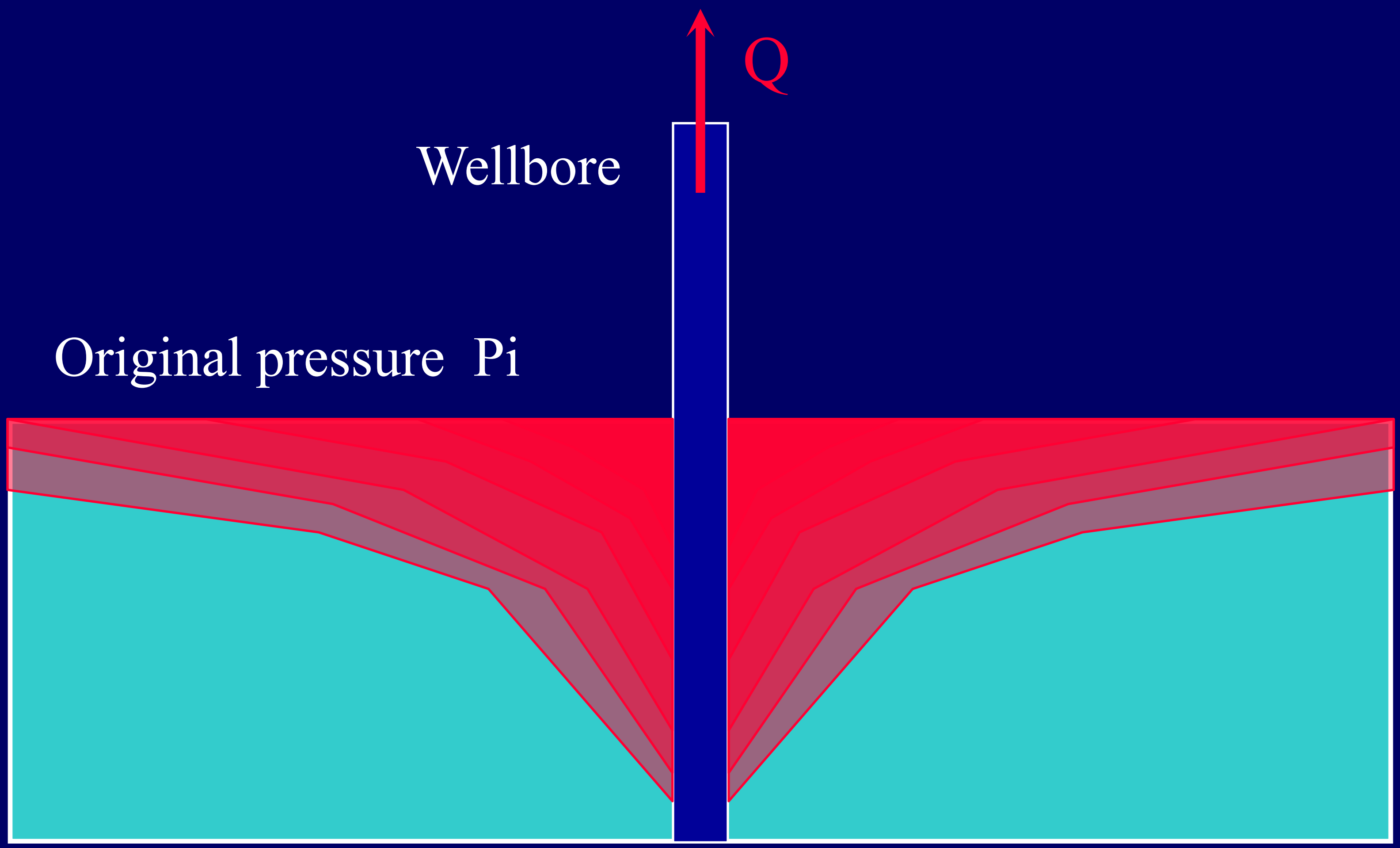
Section 1 Introduction

一、Well Testing and Its Classification

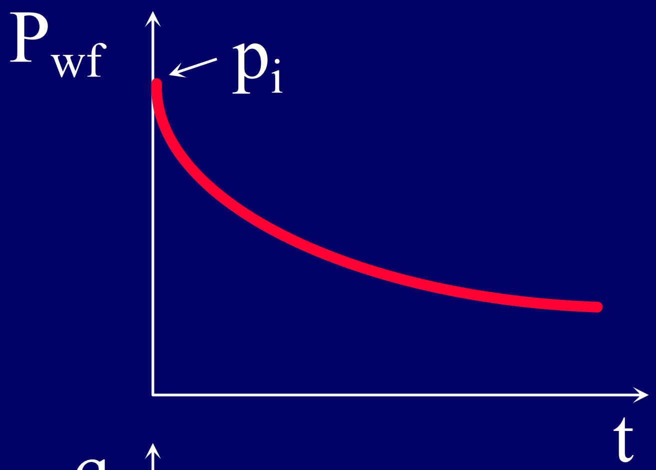
1、What is the well testing?

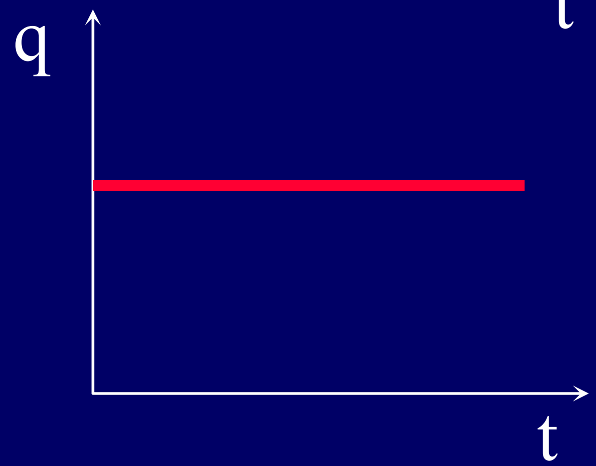
- Well testing is to test oil, gas or water wells. The test contents include output, pressure, temperature and sampling, etc.
- Well testing is a method based on the theory of percolation mechanics and by means of various testing instruments to study the physical parameters, production capacity of oil, gas, water layers and test wells, as well as the connection among oil, gas and water layers by testing the production performance of oil wells, gas wells or water wells.

Schematic Diagram of Pressure Wave Propagation in Oil (Gas) Well Production

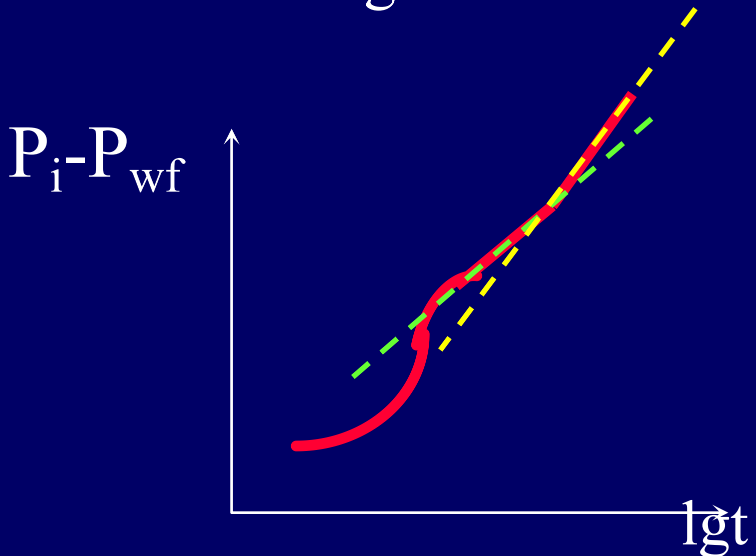


Measured Pressure-Production Curve





Pressure drop semilogarithmic curve



$$\Delta P = P_i - P_{wf}(t) = \frac{2.212\times10^{-3} q\mu B}{Kh}\lg t$$
$$+\frac{2.212\times10^{-3} q\mu B}{Kh}\times(\lg\frac{K}{\Phi\mu C_t r_w^{\ 2}} + 0.9077 + 0.8686S)$$

2 . Classification of Well Testing

（1） Productivity well test

Ÿ Productivity testing is to change the working system of several oil wells, gas wells or water wells, to measure the stable production and corresponding bottom hole pressure under different working systems, so as to determine the productivity equation and open flow rate of test wells (or test layers).

$$q = J \times \Delta P$$

$$q = c \times (P_R - P_{wf})^n$$

$$\Delta P = a \times q + b \times q^2$$

（2）Transient Well test

Transient well testing:

Change the production of the test well and measure the change of bottom hole pressure with time caused by it.

This pressure change is related to the production of the test process, and also to the characteristics of the test wells and test layers. Therefore, by using well test data, i.e. bottom hole pressure and production data in the process of testing, combined with other data, many characteristic parameters of test bed and test well can be calculated.

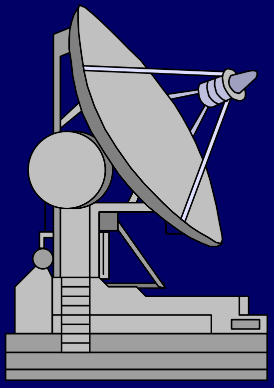
Usage of unstable well test analysis:

- Estimation of Completion Efficiency and Bottom-hole Pollution of Test Wells
- Judging whether stimulation measures (such as acidification and fracturing) are needed
- Analysis of the effect of measures to increase production
- Estimation of controlled reserves, formation parameters and formation pressure of test wells
- Detection of Oil (Gas) Layer Boundary and Interwell Connection near Test Well

二. History of Well Testing Interpretation

1 . Development history

- The method of bottom hole pressure measurement has lasted more than 50 years from static to dynamic. With the establishment of modern pressure testing theory, well testing has become an important means of understanding reservoirs since the 1950s. Since the 1960s, with the development of modern science and technology, especially the achievements in the 1980s, well test analysis has entered an important stage, greatly enriching the function of well test data and effectively improving the accuracy of analysis results.



2 . An Important Breakthrough in the 1980s

- **From the original:**
  - Interpretation results obtained by using uncorrelated methods are also different.

- **Develop to:**
  - Based on the single well test interpretation theory, a set of typical curve analysis methods are formed.

3 . Changes in the environment

The mechanical pressure gauge	1970
Electronic pressure gauge	1975
Ground measurements of production and pressure	1980
Simultaneous surface and bottom hole measurements	1983
Non-Frame Well Testing Interpretation Software	1983
Powerful personal computers	1986
Horizontal well	1990

In the past 20 years, the time difference between theoretical research and practical application of well testing is 5-10 years.

三、Well Testing Principle and Interpretation Method

1、Well Test Procedure

Direct measurement:

Core → Rock specimens of reservoirs

Sample → Fluid specimens of reservoirs

Direct measurement can only provide static and point data, which is not enough to describe the engineering characteristics of the system.

Well testing is an indirect measurement method.

Well testing is to change the original state of the testing system by some means, record the dynamic response by some means, and then interpret it according to a suitable theoretical model, so as to determine the parameters of the testing system.

Well testing is to disturb one (or several) wells, observe the response of disturbed wells or adjacent wells, and determine parameters by comparing with the theoretical characteristics of the reference system (model).

Well test program is a typical signal analysis problem in physics.

2、Well test analysis

- There are two types of problems in system analysis:
- Positive problem:

Given the structure of system S and input signal I, unknown output O is required.

- Inverse Problem:

The structure of the system S is inversely calculated from the known input signal I and output signal O.

$$I \times S \Rightarrow O$$

$$O / I \Rightarrow S$$

Positive and inverse problems

Positive problem: System $y = x + x^2$

Input	System	Output
x=2	$y = x + x^2$	y=6

The solution of the positive problem is unique.

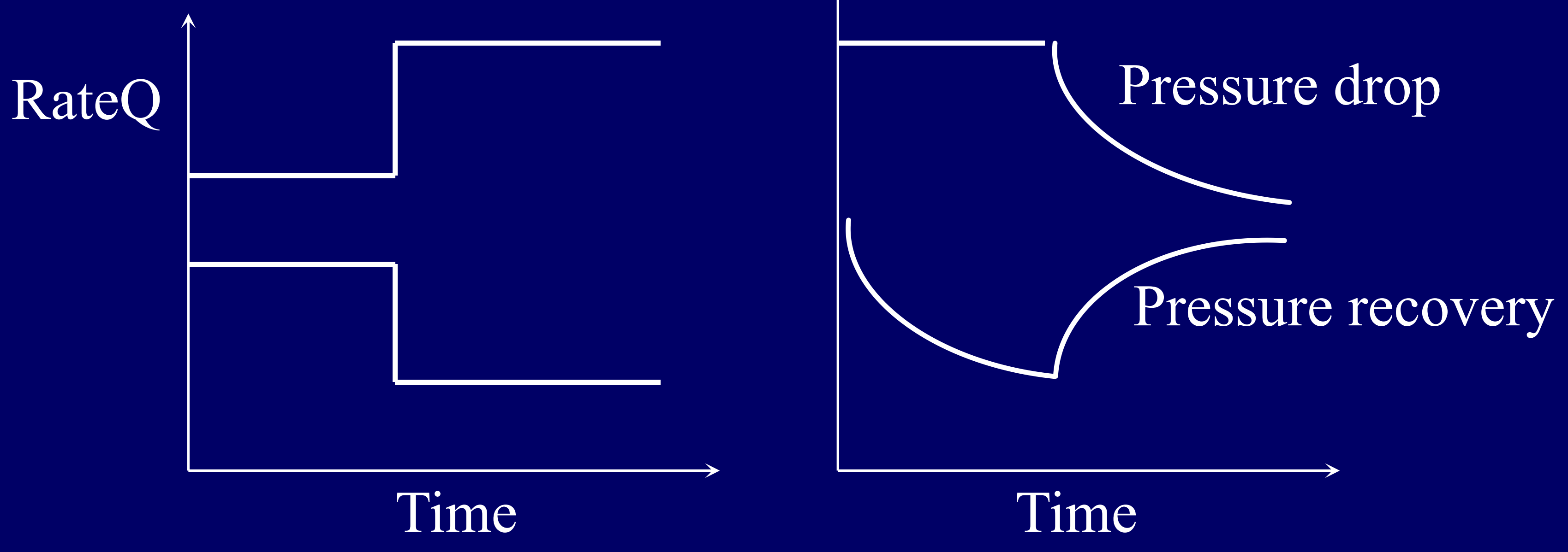
Inverse problem: Given the input and output signals, find the system.

Input	System	Output
x=2	?	y=6
x=3		y=12
x=4		y=20

The solution to the inverse problem is not unique.

The essence of well test research is:

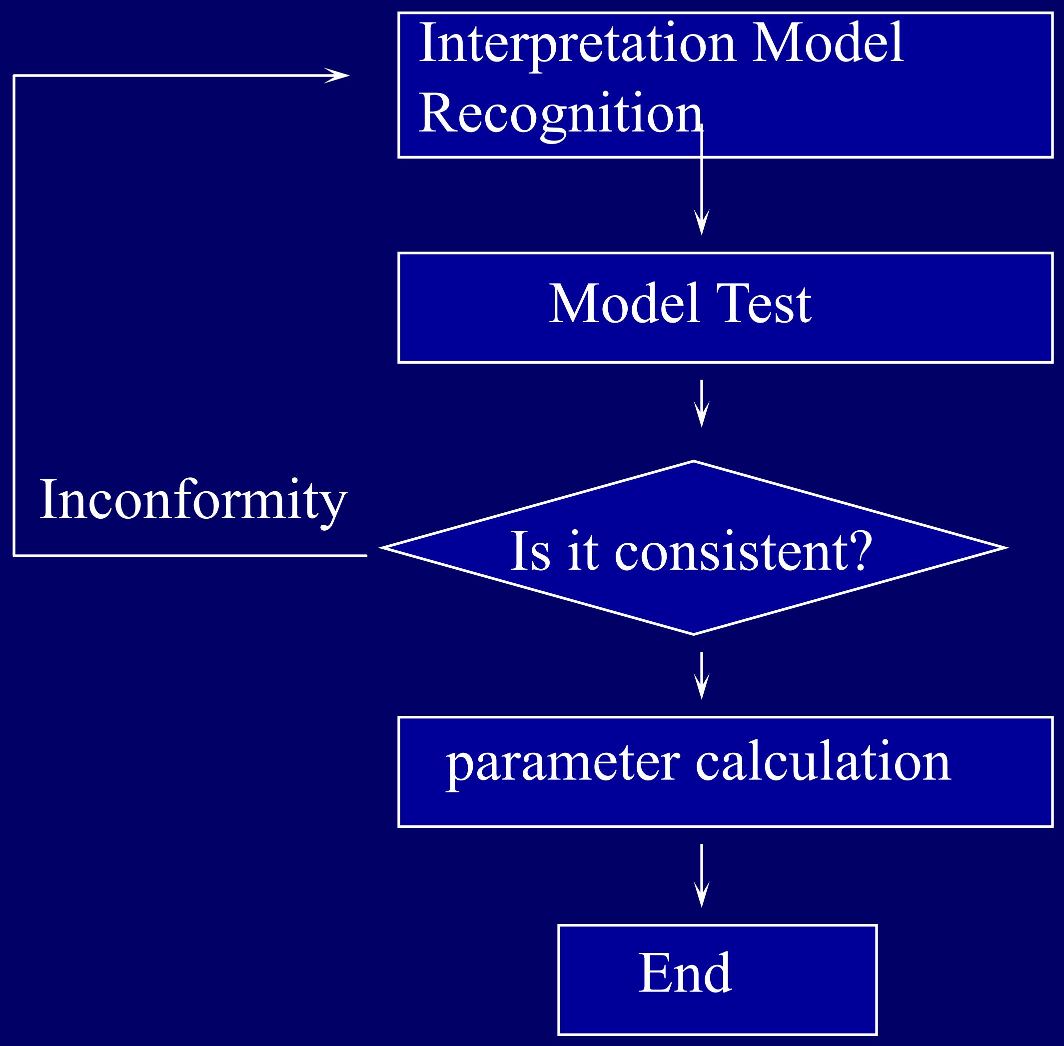
- Well testing is actually:
- Control rate Measuring pressure：



Objectives of well test analysis

- The non-uniqueness of the solution (or result) is overcome by collecting and measuring a large amount of reservoir information.
- Lots of test data： P,Q;
- Increasing reservoir information: geology, logging, drilling and production logging
- Inspection and identification.

Well test interpretation block diagram



四、Well test interpretation model

Well test interpretation model consists of the following three parts:

Basic model：
Basic Characteristics of Oil and Gas Reservoirs

Boundary condition：
Internal Boundary Conditions--Well bore and its vicinity
External Boundary Conditions--External Edge of Reservoir

Initial condition：
Pre-development Situation of Reservoirs

1、Basic model

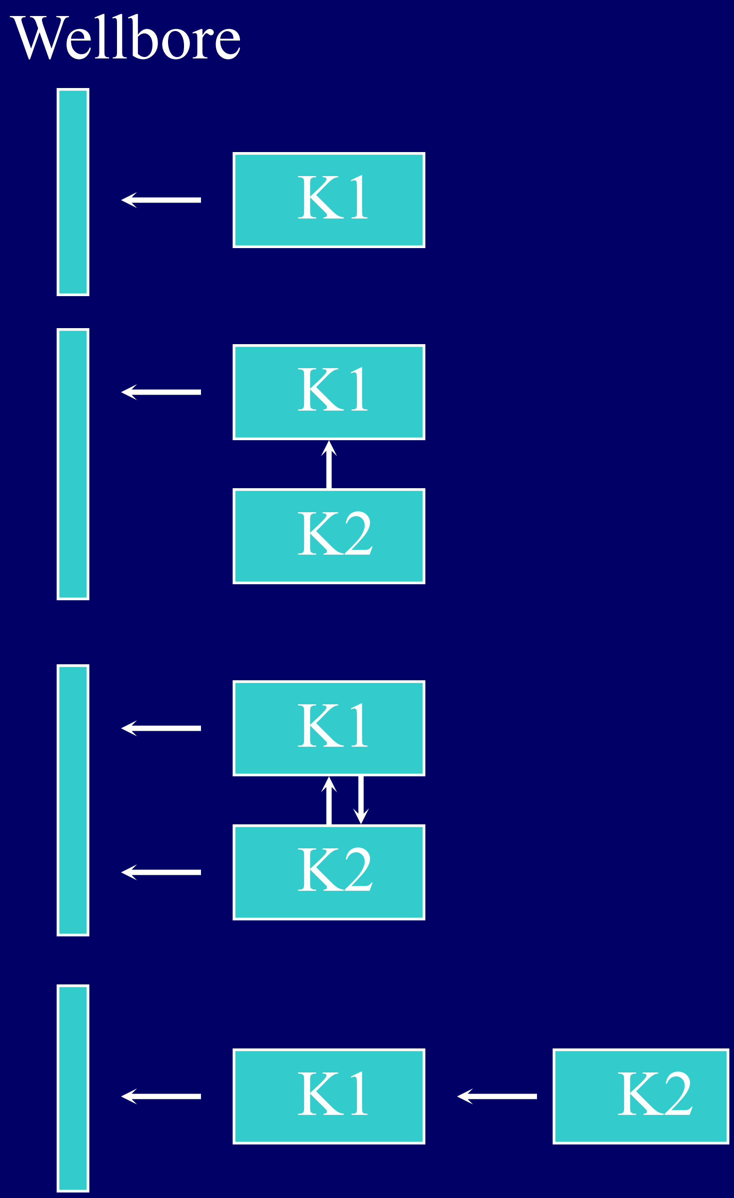
（1）Homogeneous reservoir model

（2）Heterogeneous reservoir model

Double-porosity

Dual-permeability

Composite reservoirs



2、Internal boundary conditions

（1）Wellbore storage effect

（2）Skin effect

（3）Fracture cutting wellbore

3、Outer boundary conditions

（1）Infinite stratum

（2）Impermeable boundary

（3）Constant pressure boundary

（4）Closed boundary

Example:

Basic model: Homogeneous reservoir

Internal boundary conditions： With wellbore storage and skin effect

External boundary conditions： The stratum is infinite and constant pressure is maintained at infinity.

$$\frac{\partial P_D}{\partial r_D} + \frac{1}{r_D} \cdot \frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial r_D}$$

$$P_d(r_{D,}0) = 0$$

$$P_D(\infty, t_D) = 0$$

$$C_D \frac{dP_D}{dt_D} - \left(\frac{\partial P_D}{\partial r_D}\right)_{r_d=1} = 1$$

$$P_{WD} = \left[P_D - S\left(\frac{\partial P_D}{\partial r_D}\right)\right]_{r_d=1}$$

五、Interpretation method for transient well testing

Since the Halner semi-logarithmic analysis method came out in the 1950s,Various unstable well test analysis methods have been developed successively, and have been widely used in production practice. Throughout all the analytical methods, they can be divided into two categories:

1、Conventional well test analysis（Semi-Log）

2、Modern well test analysis（Log-Log）

Modern well test analysis method, represented by Ramey and Agarwal methods

（1）Ramey, Agarwal Type Curves Analysis ；

（2）Earlougher Type Curves Analysis ；

（3）Gringarten Type Curves Analysis ；

（4）Mickinley Type Curves Analysis ；

（5）Bourder Type Curves Analysis

六、Unit System in Well Test Analysis

In petroleum engineering, a variety of unit systems are often used together. China now adopts the legal standard unit system.

The prototype of the equation for stable flow conditions is physical system.

$$\boxed{\Delta P = \frac{1}{2\pi} \cdot \frac{qB\mu}{Kh} \cdot \ln\left(\frac{r_e}{r_w}\right)}$$

Where：

P—— atm

q—— cm^3/S

μ —— cp

K—— d

h、r_e、r_w —— cm

Conversion to statutory unit system:

$$\Delta P = 1.842 \times 10^{-3} \frac{qB\mu}{Kh} \ln\left(\frac{r_e}{r_w}\right)$$

Where：

P—— MPa

q—— m^3/d

μ —— mPa.s

K—— μm^2

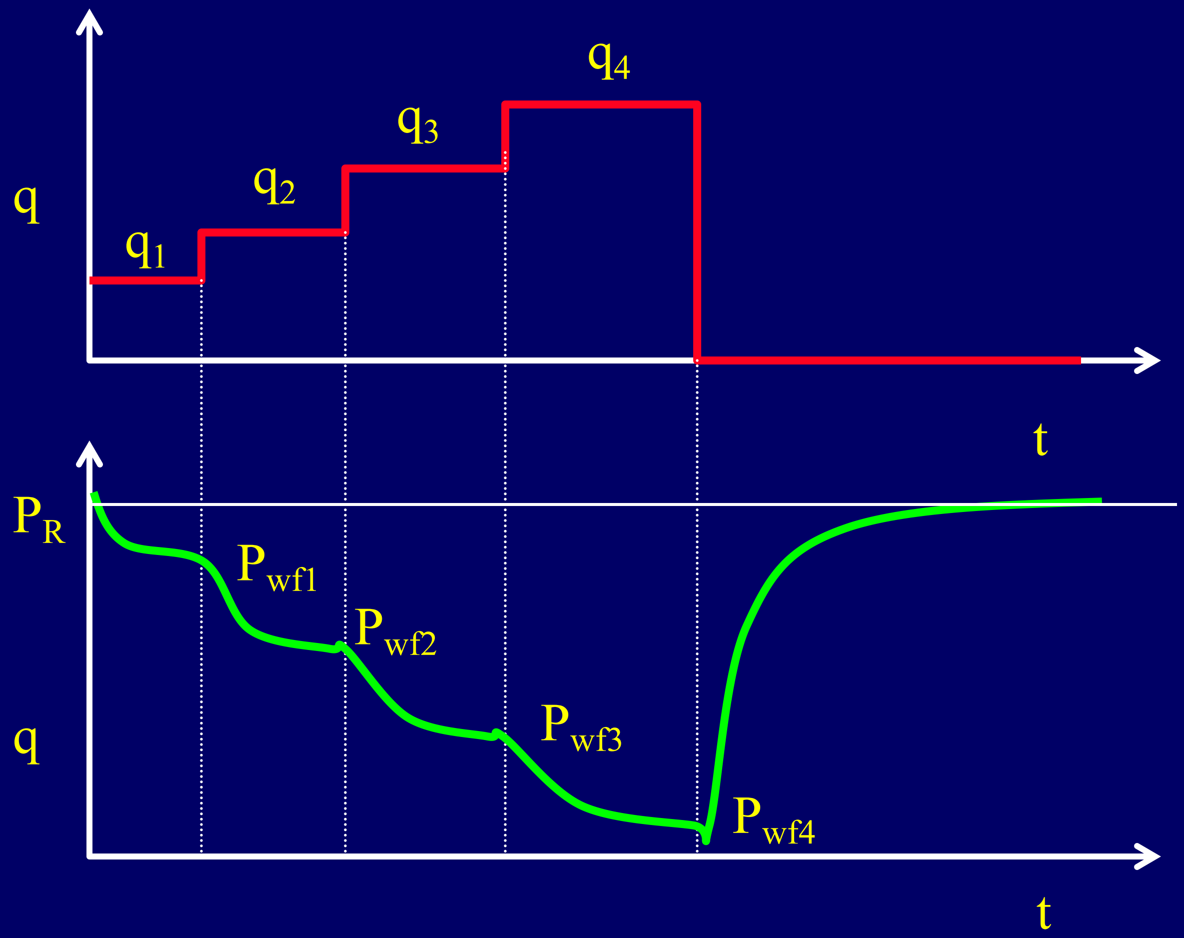
h、r_e、r_w —— m

Steady well test

Steady Well Testing (Systematic Well Testing, Back Pressure Well Testing)

Basic Principle: Darcy's Law

$$\boxed{q = \frac{Kh\Delta P}{1.842 \times 10^{-3} B\mu \ln(\frac{r_e}{r_w})}}$$



Backpressure well test schematic diagram

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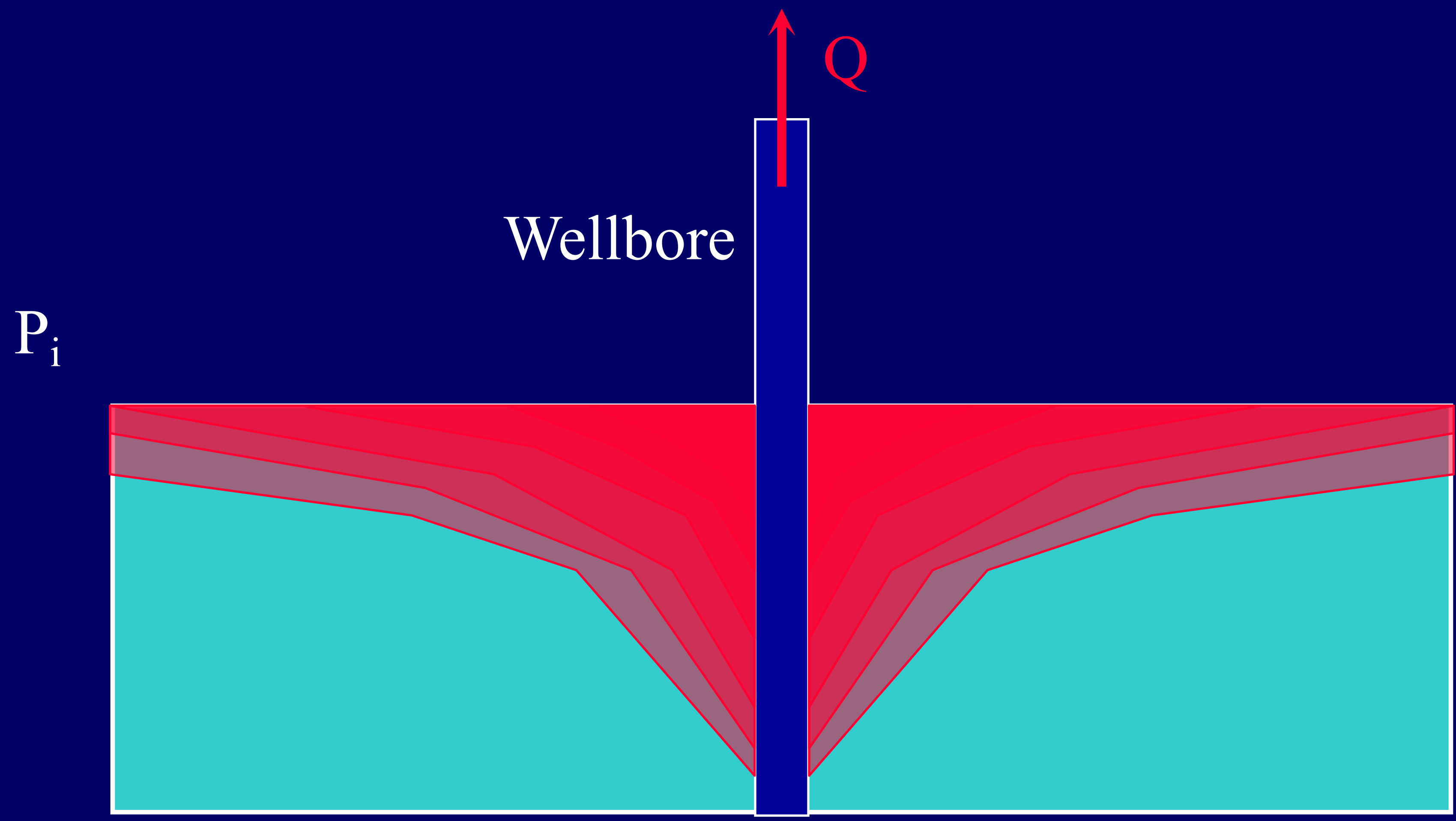
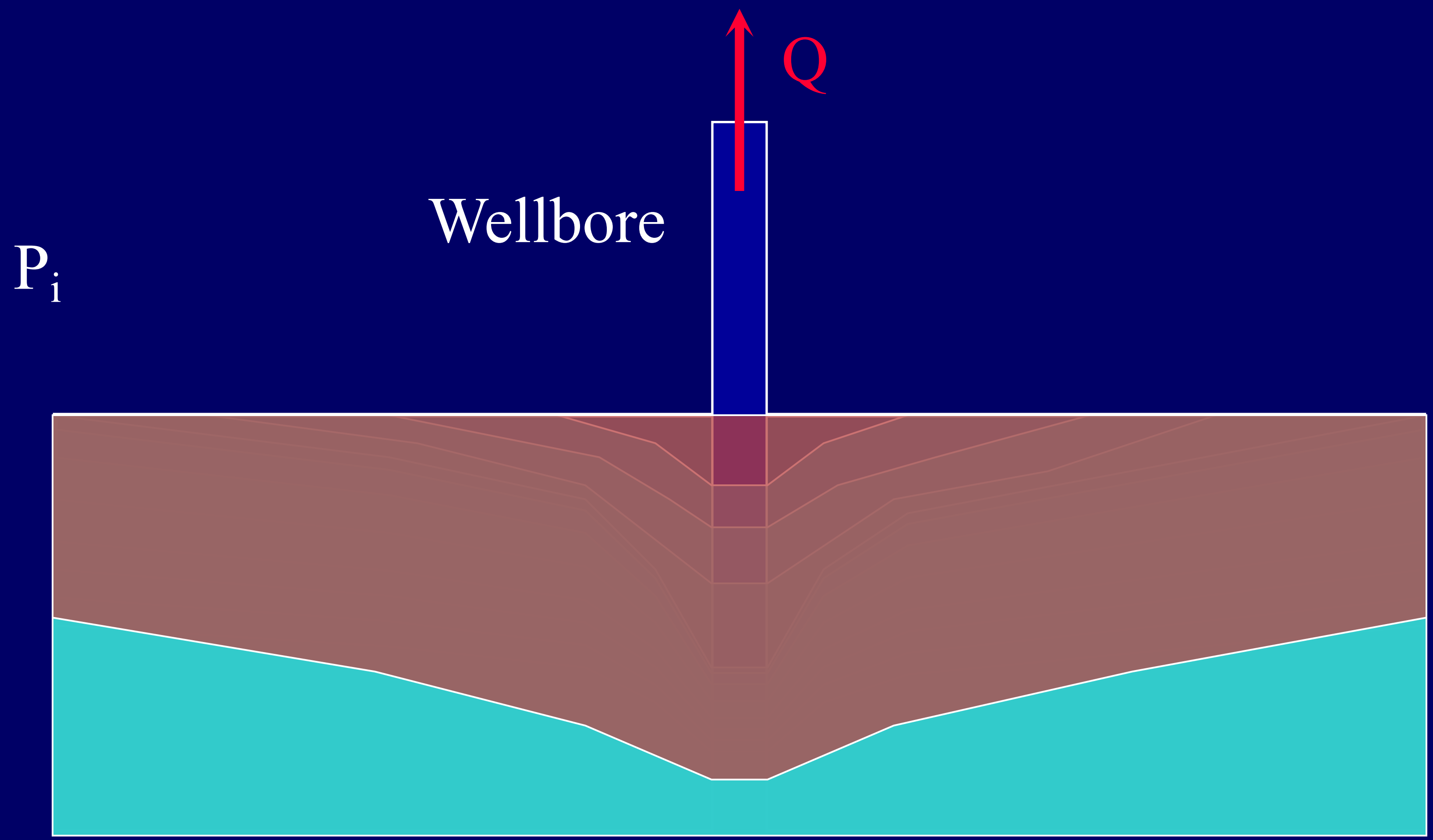
Schematic diagram of pressure wave transmission during well opening



Schematic diagram of pressure wave recovery at shut-in



一、Basic Differential Equations and Pressure Drop Formulas

The seepage of a single-phase weakly compressible liquid with constant compressibility in a horizontal, isotropic, infinitely isotropic homogeneous elastic porous medium obeys the following partial differential equation (diffusion equation):

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial p}{\partial r}=\frac{\varphi\mu C_t}{3.6K}\frac{\partial p}{\partial t}$$

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial p}{\partial r}=\frac{1}{3.6\eta}\frac{\partial p}{\partial t}$$

Initial and boundary conditions

$$p(r,0) = p_i$$

Outer boundary condition

$$p(\infty,t) = p_i$$

Inner boundary condition

$$\left(r\frac{\partial p}{\partial r}\right)_{r=r_w} = \frac{q\mu B}{172.8\pi Kh}$$

$$\left(r\frac{\partial p}{\partial r}\right)_{r=r_w} = \frac{1.842\times 10^{-3} q\mu B}{Kh}$$

P=P(r,t)－Pressure at t(h) moment at r(m) distance from well,MPa

Pi－Original formation pressure， MPa

r－Distance from well， m

t－Time from the time of well opening， h

K－Formation permeability， μm^2

h－Stratum thickness， m

μ－Fluid viscosity， mPa•s

φ－Formation porosity， 1

C_t－Comprehensive compression coefficient， MPa^{-1}

$$C_t = Cr + C_o S_o + C_w S_w + G_g S_g$$

r_w－Radius of well， m

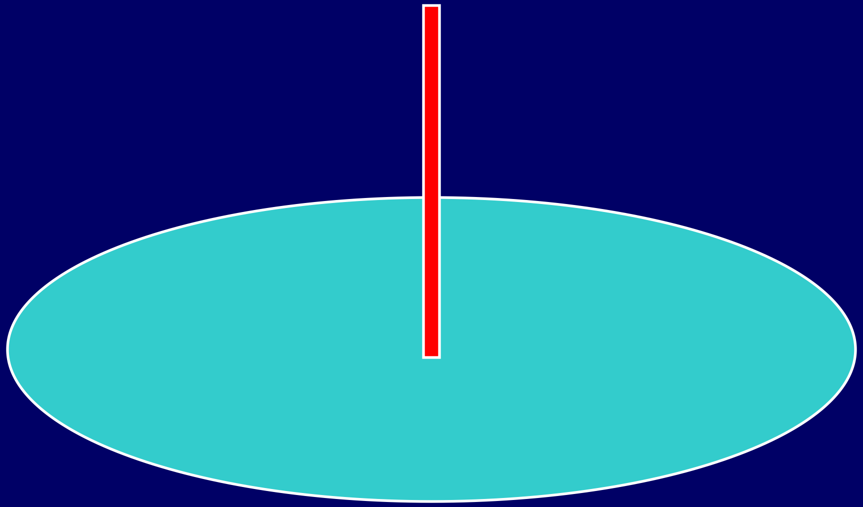
q－Ground production of wells， m^3/d

B－Volume Coefficient of Crude Oil， 1

$$\eta = \frac{K}{\varphi \mu C_t}$$ —Pressure coefficient， $\mu m^2 \cdot MPa/(mPa \cdot s)$

✦The coefficient of conductivity is a physical quantity that characterizes the difficulty of conduction pressure of formation and fluid.

Dimensions of the coefficient of conductivity： L^2 / t



The solution of equation (2-1) under definite conditions is as follows:

$$p = p(r,t) = p_i - \frac{q\mu B}{345.6\pi Kh}[-E_i(-\frac{r^2}{14.4\eta t})] \qquad (2\text{-}3)$$

Ei is a power integral.

$$E_i(-x) = -\int_x^{\infty} \frac{e^{-u}}{u} du$$

When x < 0.01, there are:

$$E_i(-x) \approx \ln x + 0.5772 \approx \ln(1.781x)$$

From (2-3), the bottom hole flow pressure P_{wf} (t) = P (r_w, t) is obtained as follows:

$$\boxed{p = p_{wf}(t) = p_i - \frac{q\mu B}{345.6\pi Kh}[-E_i(-\frac{r_w{}^2}{14.4\eta t}) + 2S]} \qquad (2\text{-}4)$$

An additional pressure drop due to borehole resistance is added to the equation.

$$\boxed{\frac{q\mu B}{345.6\pi Kh} \cdot 2S}$$

Write (2-4) in the form of pressure difference.

$$\boxed{\Delta p = p_i - p_{wf}(t) = \frac{q\mu B}{345.6\pi Kh}[-E_i(-\frac{r^2}{14.4\eta t})]} \qquad (2\text{-}5)$$

When $\frac{r_w^2}{14.4\eta t} < 0.01$, we have

$$\boxed{p_{wf}(t) = p_i - \frac{q\mu B}{345.6\pi Kh}[\ln\frac{8.085\eta t}{r_w^2} + 2S]} \qquad (2\text{-}6)$$

By changing to logarithm, we can get:

$$
\begin{aligned}
p_{wf}(t) &= p_i - \frac{2.121\times10^{-3} q\mu B}{Kh}\left[\lg\frac{Kt}{\varphi\mu C_t r_w^2} + 0.9077 + 0.8686S\right] \\
&= -\frac{2.121\times10^{-3} q\mu B}{Kh}\lg t + \left[p_i - \frac{2.121\times10^{-3} q\mu B}{Kh}\right. \\
&\quad \left.\times\left(\lg\frac{K}{\varphi\mu C_t r_w^2} + 0.9077 + 0.8686S\right)\right] \qquad (2\text{-}7)
\end{aligned}
$$

Write in the form of pressure difference:

$$\Delta p = p_i - p_{wf}(t) = \frac{2.121\times 10^{-3} q\mu B}{Kh}\lg t + \frac{2.121\times 10^{-3} q\mu B}{Kh} \times (\lg\frac{K}{\varphi\mu C_t r_w^2} + 0.9077 + 0.8686S) \qquad (2\text{-}8)$$

Formula (4) ~ (8) can be called "pressure drop formula".

二、Superposition principle

The so-called "superposition principle" is that if the definite solution conditions of a certain linear differential equation are linear, and they can be decomposed into several definite solution problems, and the corresponding linear combination of the differential equation and the definite solution conditions of these definite solution problems is exactly the original differential equation and the definite solution conditions, then the corresponding linear combination of the solutions of these definite solution problems is the original one. Solution of the problem.

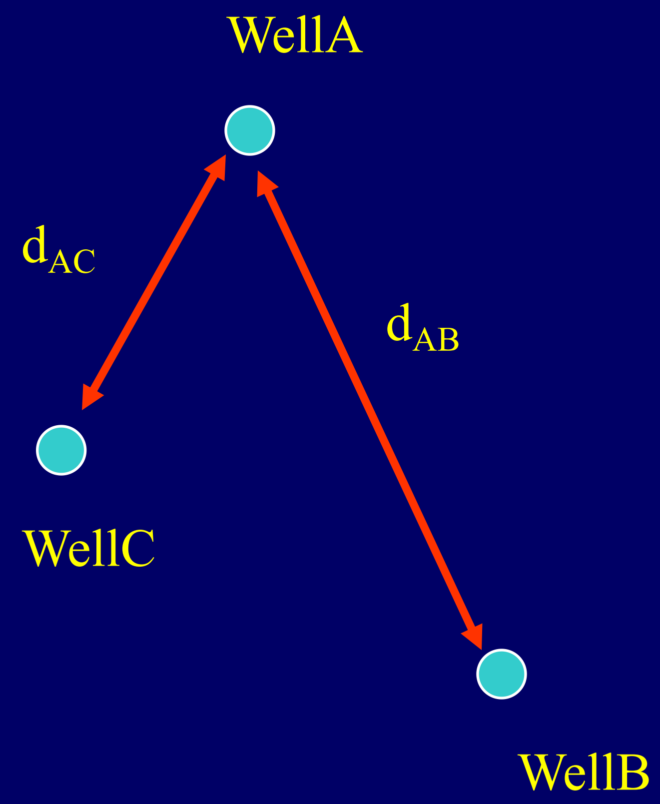
The application of superposition principle to well testing can be described as: The total pressure drop at any point in a reservoir is equal to the algebraic sum of the pressure drop generated at that point by each well in the reservoir.

When using the principle of superposition, attention should be paid to:

All wells should be in the same hydrodynamic system.

1、Application of multi-well system

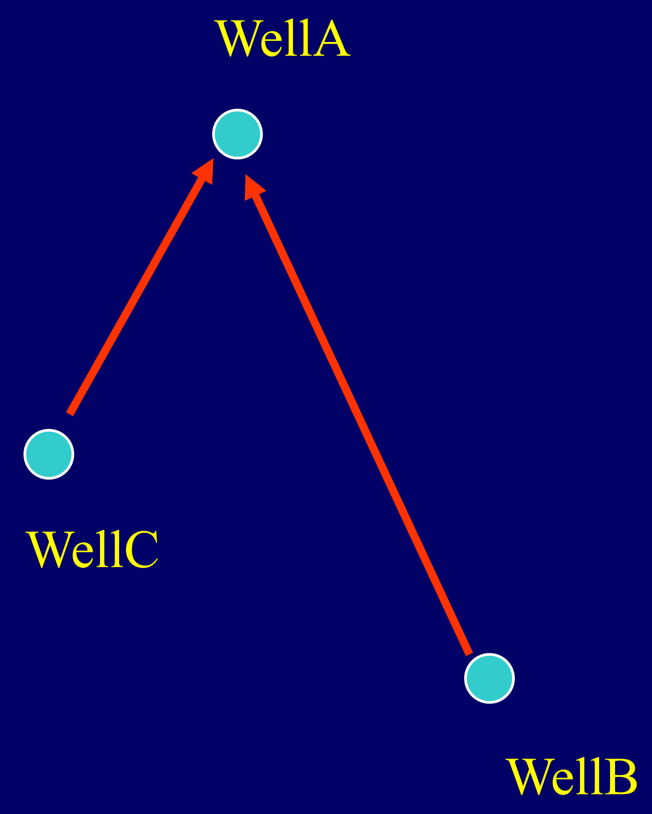
Assuming that there are three wells A, B and C in a reservoir, they start production at the same time with production q_A, q_B and q_C respectively. The known distances between B and C and A are d_{AB} and d_{AC} respectively. The pressure variation of well A should be calculated.



According to the superposition principle, the pressure change of well A is as follows:

$$\boxed{\Delta p = \Delta p_A + \Delta p_{B-A} + \Delta p_{C-A}}$$

In the upper formula, ΔP_A、ΔP_{B-A}、ΔP_{C-A} respectively indicate the pressure drop produced in well A when well A, B and C is produced with q_A, q_B and q_C.



If t is in the radial flow period, then

$$\boxed{\begin{aligned}\Delta p = \frac{9.21\times 10^{-4}\mu B}{Kh}[q_A(\ln\frac{Kt}{\varphi\mu C_t r_w^2} + 0.8091 + 2S) \\ - q_B E_i(-\frac{\varphi\mu C_t d_{BA}^2}{14.4Kt}) - q_C E_i(-\frac{\varphi\mu C_t d_{CA}^2}{14.4Kt})]\end{aligned}}$$

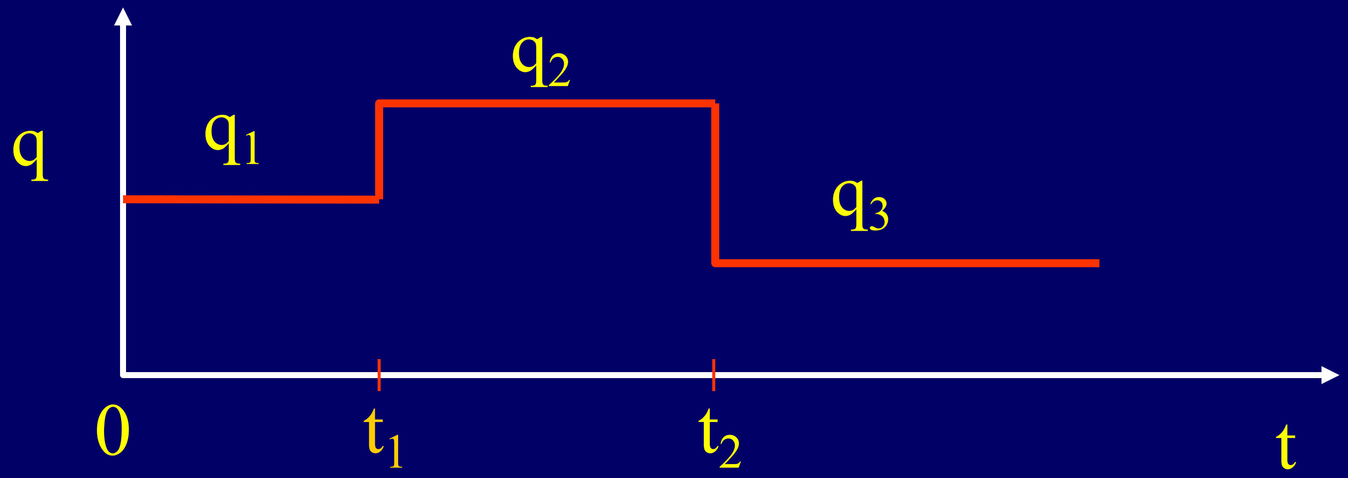
2 、Application of Variable Rate System

If wells are produced with several different yields, it can also be regarded as a problem of multi-well system, but the distance between wells is zero at this time.

Set up a well： q_1 production from 0 to t_1，

q_2 production from t_1 to t_2，

q_3 production from t_2，



It is envisaged that there are three wells in this well site:

Well 1 has been produced with q_1 since 0.

Well 2 starts production with production (q_2-q_1) only at t_2.

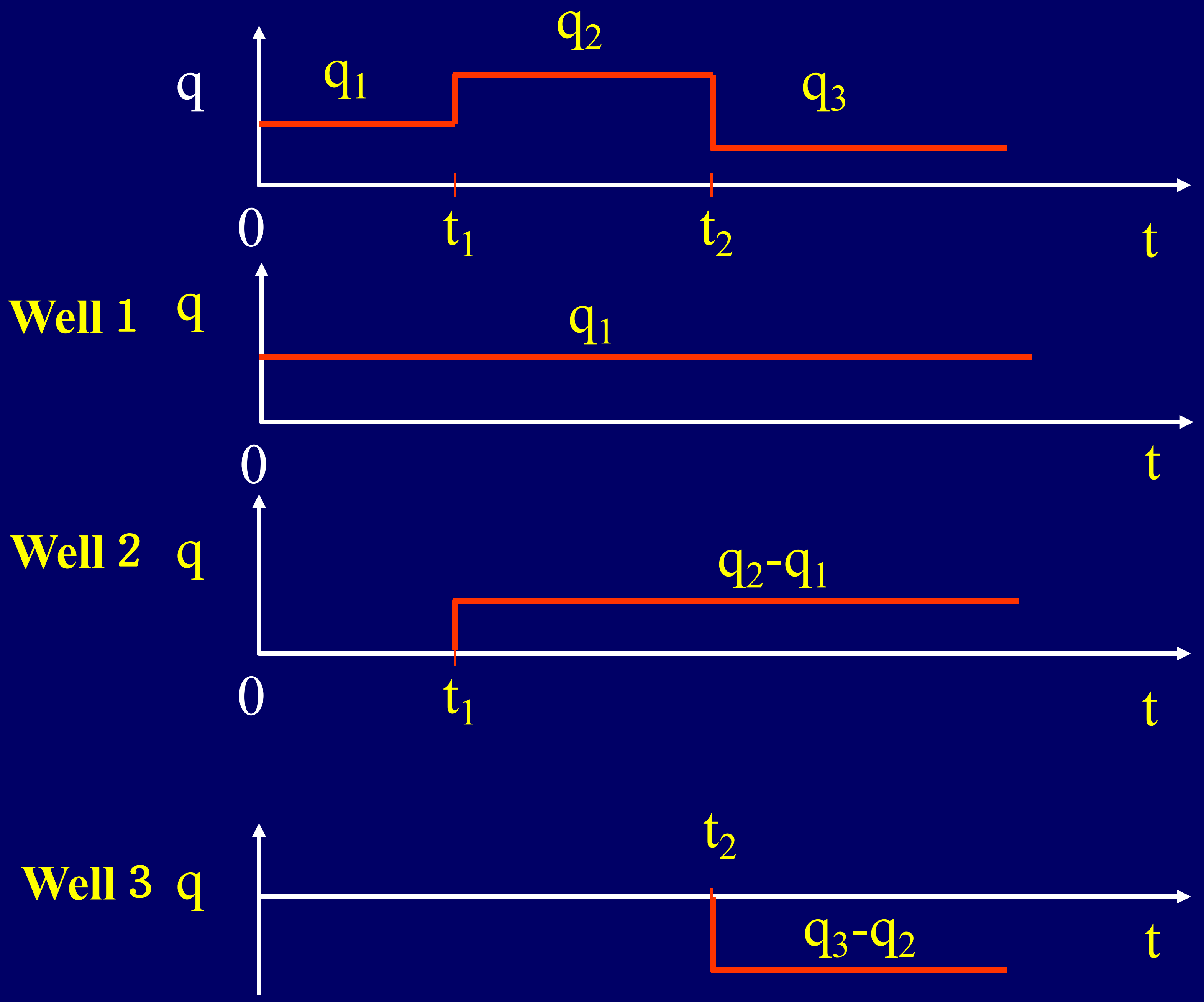
Well 3 has not been produced with production (q_3-q_2) since t_3.

The total effect of production in these three wells is the pressure drop caused by the production change of the wells.

Well 1： In $0 \sim t_1$， $q=q_1$;

Well 2： In $t_1 \sim t_2$， $q=q_1+(q_2-q_1)=q_2$

Well 3： After t_2， $q= q_1+(q_2-q_1)+(q_3-q_2)=q_3$



q
q1
q2
q3
0
t1
t2
t
Well 1
q
q1
0
t
Well 2
q
q2-q1
0
t1
t
Well 3
q
t2
q3-q2
t

The sum of the differential pressure caused by the "three wells" $\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3$ is the pressure change of the well.

If time t belongs to the radial flow section, then:

$$\boxed{\begin{aligned}\Delta p &= \Delta p_1 + \Delta p_2 + \Delta p_3 \\ &= -\frac{9.21\times 10^{-4} q_1 \mu B}{Kh}[E_i(-\frac{r_w^2}{14.4\eta t}) - 2S] \\ &\quad -\frac{9.21\times 10^{-4}(q_2 - q_1)\mu B}{Kh}[E_i(-\frac{r_w^2}{14.4\eta (t-t_1)}) - 2S] \\ &\quad -\frac{9.21\times 10^{-4}(q_3 - q_2)\mu B}{Kh}[E_i(-\frac{r_w^2}{14.4\eta (t-t_2)}) - 2S]\end{aligned}}$$

三、Dimensionless variables

General physical quantities have dimensions and can be expressed by basic dimensions.

Area: L^2

Rate: L^3/t

There are also some quantities that have no dimension. For example, the volume coefficient of crude oil, oil saturation, porosity and so on.

For certain purposes, some dimensionless physical quantities are often introduced, that is, new dimensionless quantities, or dimensionless quantities, are called dimensionless quantities. Use the subscript "D" to indicate "dimensionless".

Dimensionless pressure：

$$\boxed{p_D = \frac{Kh}{1.842\times 10^{-3} q\mu B}\Delta p} \qquad (2\text{-}9)$$

Dimensionless time

$$\boxed{t_D = \frac{3.6K}{\varphi \mu C_t r_w^2} t = \frac{3.6\eta}{r_w^2} t} \qquad (2\text{-}10)$$

Dimensionless wellbore storage constant：

$$\boxed{C_D = \frac{C}{2\pi\varphi C_t h r_w^2}} \qquad (2\text{-}11)$$

Dimensionless distance:

$$\boxed{r_D = \frac{r}{r_w}} \qquad (2\text{-}12)$$

Dimensionless method is not the only one.

Definition of dimensionless time t_D:

$$\boxed{t_D = \frac{3.6Kt}{\varphi \mu C_t r_w^2}}$$

Definition of radius of well

Definition by Converted Radius

Definition of Reservoir Area

Definition of fracture half-length

There are many advantages in discussing problems with dimensionless quantities:

1、Relations become simple, easy to deduce, memorize and apply.

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial p}{\partial r} = \frac{\varphi \mu C_t}{3.6K} \frac{\partial p}{\partial t}$$

$$\frac{\partial^2 p_D}{\partial {r_D}^2} + \frac{1}{r_D} \cdot \frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D}$$

$$p(r,0) = p_i$$

$$p(\infty,t) = p_i$$

$$(r\frac{\partial p}{\partial r})_{r=r_w} = \frac{q\mu B}{172.8\pi Kh}$$

$$p_D(r_D,0) = 0$$

$$p_D(\infty,t_D) = 0$$

$$(\frac{\partial p_D}{\partial r_D})_{r_D=1} = -1$$

$$p(r,t) = p_i - \frac{q\mu B}{345.6\pi Kh}[-E_i(-\frac{r^2}{14.4\eta t})]$$

$$p_D = \frac{1}{2}[-E_i(-\frac{r_D{}^2}{4t_D})]$$

$$p_{wf}(t) = p_i - \frac{q\mu B}{345.6\pi Kh}[-E_i(-\frac{r_w{}^2}{14.4\eta t}) + 2S]$$

$$p_D = \frac{1}{2}[-E_i(-\frac{1}{4t_D}) + 2S]$$

2、Because the dimensionless quantity is used, the derived formula is not affected and restricted by the unit system, so it is more convenient to use.

3、It can make the discussion under a certain premise of universal significance.

$$\boxed{\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial p}{\partial r} = \frac{\varphi \mu C_t}{3.6K} \frac{\partial p}{\partial t}}$$

Multiply both sides by r_w^2:

$$\boxed{\frac{\partial^2 p}{\partial (\frac{r}{r_w})^2} + \frac{\partial p}{\partial (\frac{r}{r_w})} = \frac{\varphi \mu C_t r_w^2}{3.6K} \frac{\partial p}{\partial t}}$$

$$\boxed{\frac{\partial^2 p}{\partial r_D{}^2} + \frac{\partial p}{\partial r_D} = \frac{\varphi \mu C_t r_w^2}{3.6K} \frac{\partial p}{\partial t}}$$

$$\boxed{\frac{\partial^2 p}{\partial r_D{}^2}+\frac{\partial p}{\partial r_D}=\frac{\partial p}{\partial\left(\frac{3.6K}{\varphi\mu C_t r_w^2}t\right)}}$$

$$\boxed{\frac{\partial^2 p}{\partial r_D{}^2}+\frac{\partial p}{\partial r_D}=\frac{\partial p}{\partial t_D}}$$

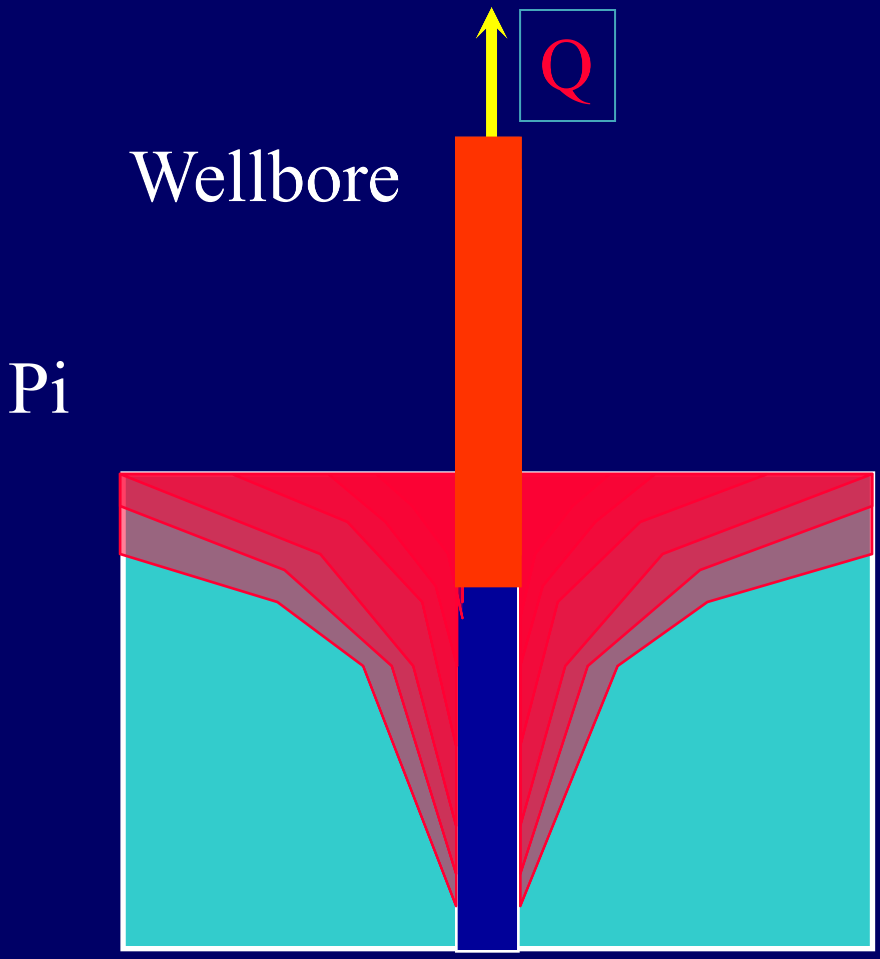
$$\boxed{\frac{\partial^2\left(\frac{Kh}{1.842\times 10^{-3}}\Delta p\right)}{\partial r_D{}^2}+\frac{\partial\left(\frac{Kh}{1.842\times 10^{-3}}\Delta p\right)}{\partial r_D}=\frac{\partial\left(\frac{Kh}{1.842\times 10^{-3}}\Delta p\right)}{\partial t_D}}$$

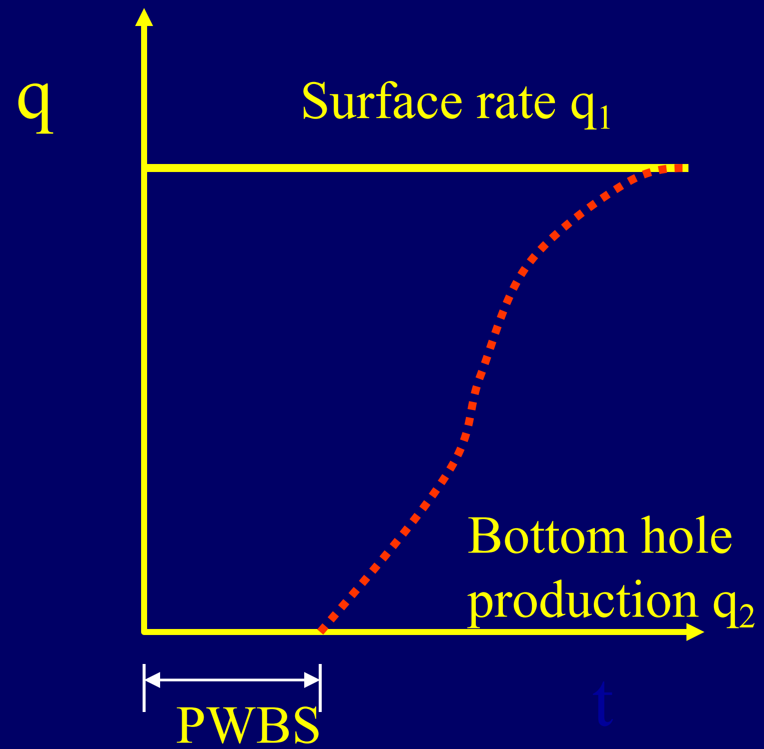
$$\boxed{\frac{\partial^2 p_D}{\partial r_D{}^2}+\frac{\partial p_D}{\partial r_D}=\frac{\partial p_D}{\partial t_D}}$$

四、Wellbore storage constant

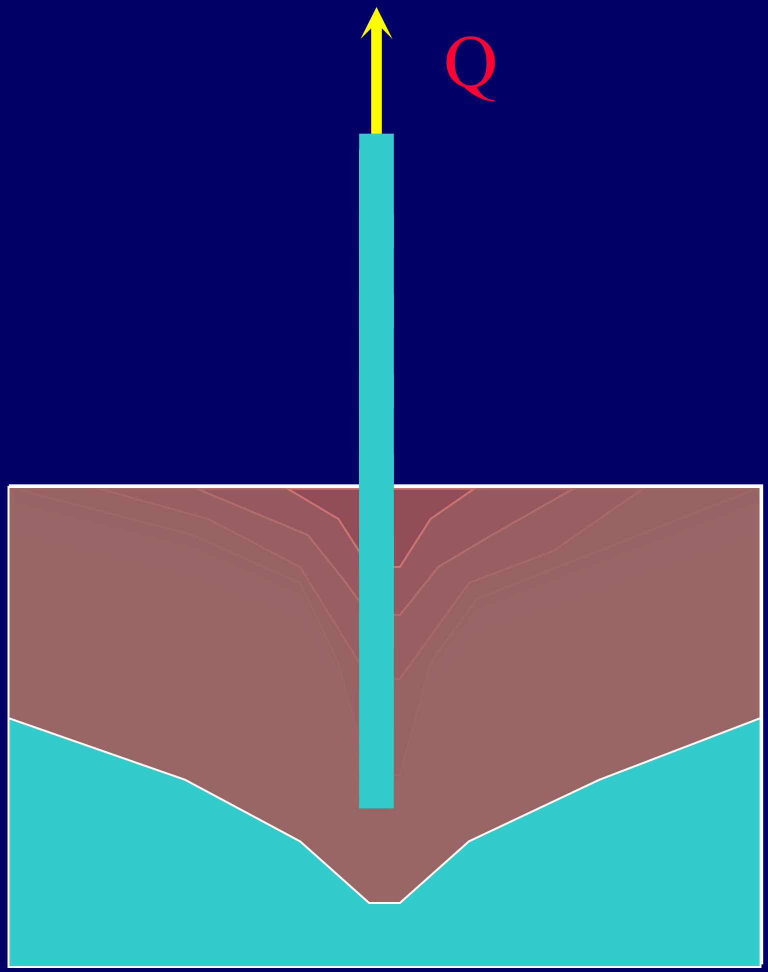
When oil wells are just opened or shut in, because crude oil has many reasons such as compressibility, surface production q_1 is not equal to bottom-hole production q_2.

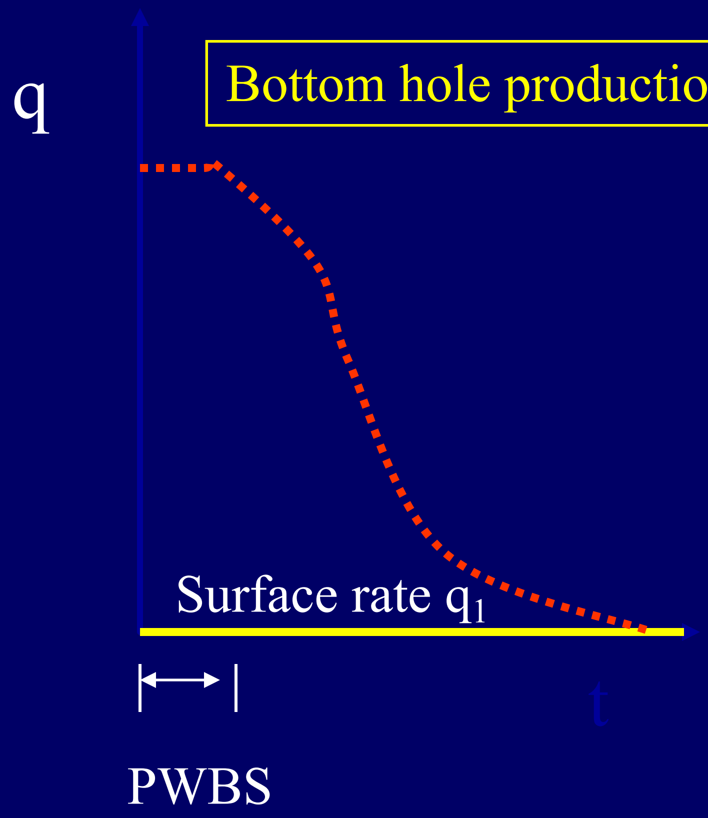
Wellbore unloading effect during well opening





Wellbore Continuous Flow Effect in Shut-in





Wellbore unloading effect and wellbore storage effect are collectively called wellbore storage effect (wellbore storage effect).

The period when $q_2 = 0$ (well opening) or $q_2 = q$ (well closing) is called "pure wellbore reservoir" stage，

PWBS（Pure Wellbore Storage）

The "wellbore storage constant" is used to describe the degree of wellbore storage effect, i.e. the ability of wellbore to store crude oil or release elastic energy of compressed crude oil in wellbore by compression of crude oil in wellbore, and C is used to represent the ability of wellbore to discharge crude oil.

$$\boxed{C = \frac{dV}{dP} \approx \frac{\Delta V}{\Delta P}} \qquad (2\text{-}13)$$

Where： C —Wellbore storage constant， m^3/MPa;

ΔV—Volume change of crude oil stored in wellbore,m^3；

ΔP—Change of wellbore pressure,MPa。

Wellbore storage constant C Physical meaning:

In the case of shut-in, in order to increase wellbore pressure by 1 MPa, crude oil from formation must flow into wellbore Cm3.When wellbore pressure decreases by 1 MPa, Cm3 crude oil can be discharged by the elastic energy of the crude oil in the wellbore.

１、Crude oil fills the whole wellbore.

Within t hours of opening or shutting-in, the volume of crude oil in the wellbore varies as follows

$$\boxed{\Delta V = \frac{|q_1 - q_2| t}{24} \quad (m^3)} \qquad (2\text{-}14)$$

Therefore

$$\boxed{C = \frac{\Delta V}{\Delta P} = \frac{|q_1 - q_2| t}{24 \Delta P}} \qquad (2\text{-}15)$$

In pure wellbore reservoir stage

Well opening $q_2=0$, $q_1=q$

Shut in situation $q_1=0$, $q_2=q$

Therefore

$$|q_1-q_2| = \begin{cases} q_1 = q \\ q_2 = q \end{cases}$$

Therefore, in the pure wellbore reservoir stage there are

$$\boxed{C = \frac{qt}{24\Delta P}} \quad (2\text{-}16)$$

$$\boxed{\Delta P = \frac{qt}{24C}} \quad (2\text{-}17)$$

If the crude oil is single phase, then

$$\boxed{\begin{aligned} \Delta V &= VC_o\Delta P \\ C = \frac{\Delta V}{\Delta P} &= \frac{VC_o\Delta P}{\Delta P} = VC_o \end{aligned}} \qquad (1\text{-}18)$$

Where： V－Wellbore volume， m^3;

Co－Compression coefficient of crude oil， 1/MPa.

The wellbore reservoir constants calculated from (1-18) are called "wellbore reservoir constants calculated from completion data".

2 、The liquid level does not reach the wellhead （Wellbore is not full）

$$\boxed{C = \frac{V_u}{9.80665 \times 10^{-3} \rho} \quad (m^3 / MPa)} \qquad (1\text{-}19)$$

Where： Vu — Cross-sectional area of tubing， m^2;

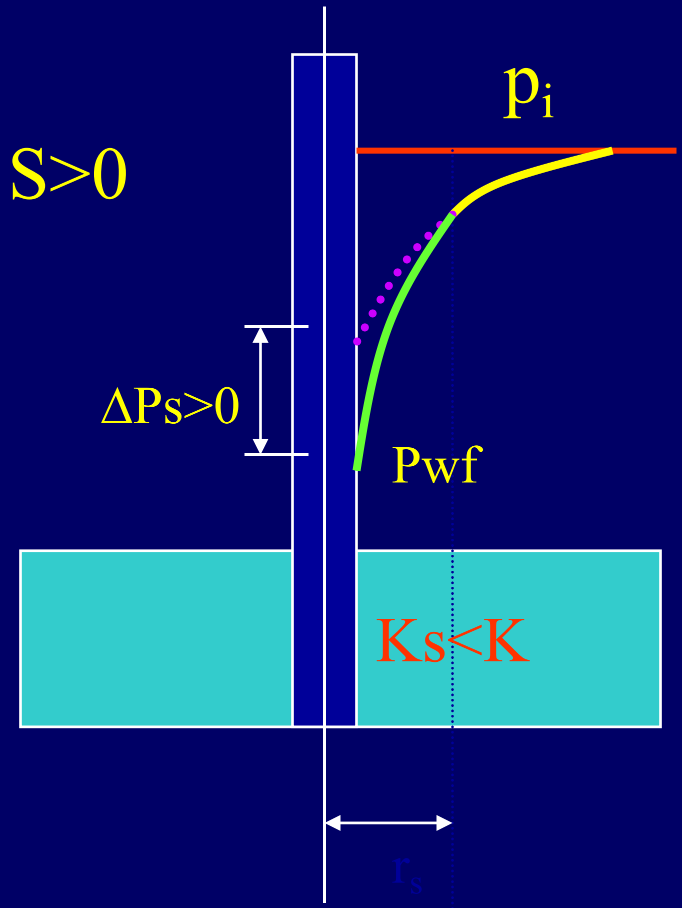
ρ — Density of crude oil in tubing， g/cm^3

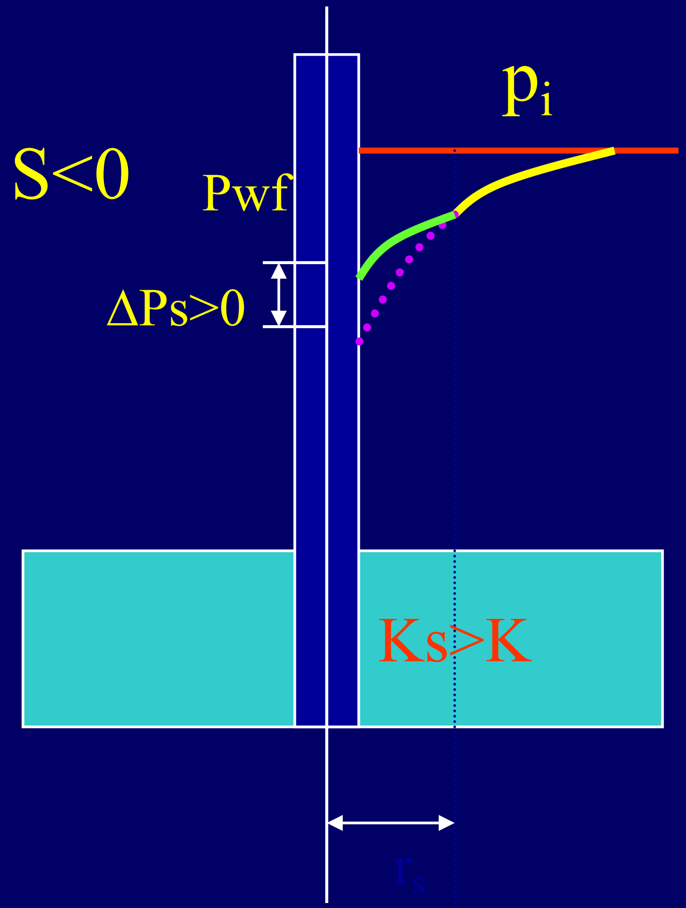
If the phase change occurs in the wellbore during the wellbore storage effect stage, the wellbore storage constant will also change.

五、Epidermal effect and epidermal coefficient

It is assumed that there is a small annular area around the wellbore. For various reasons, the permeability of this small ring area is different from that of the reservoir. Therefore, when crude oil flows from the reservoir into the wellbore, there is an additional pressure drop. This phenomenon is called the epidermal effect.

$$\boxed{S = \frac{Kh}{1.842 \times 10^{-3} q \mu B} \Delta P_S} \qquad (1\text{-}20)$$





Additional pressure drop：

$$\boxed{\Delta Ps = \frac{q\mu B}{345.6\pi Kh} \cdot 2S}$$

$$\boxed{p_{wf}(t) = p_i - \frac{q\mu B}{345.6\pi Kh}[\ln \frac{8.085\eta t}{r_w^2} + 2S]}$$

Can be rewritten into：

$$p_{wf}(t) = p_i - \frac{q\mu B}{345.6\pi Kh}[\ln\frac{8.085\eta t}{r_w^2} + \ln e^{2S}]$$

$$= p_i - \frac{q\mu B}{345.6\pi Kh}\ln\frac{8.085\eta t}{(r_w e^{-S})^2}$$

Let

$$\boxed{r_{we} = r_w e^{-S}} \qquad (1\text{-}21)$$

$$p_{wf}(t) = p_i - \frac{q\mu B}{345.6\pi Kh} \ln \frac{8.085\eta t}{r_{we}^{2}}$$

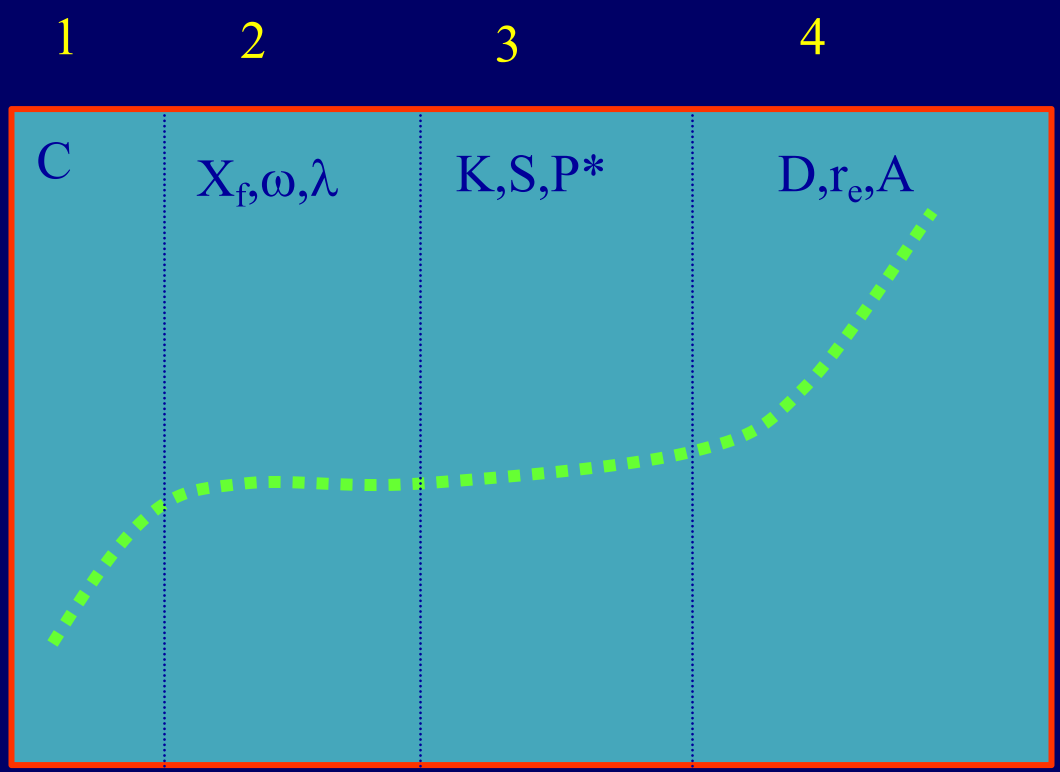
Where： r_{we} — Converted radius or effective radius。

$r_{we}<r_w$， $S>0$， The larger the value, the more serious the pollution is.；

$r_{we}=r_w$ ， $S=0$， Well not contaminated；

$r_{we}>r_w$, $S<0$， The greater the absolute value, the better the effect of increasing production.。

六、Information available at the flow stage and at each flow stage



Chapter 6 Principle and application of well test analysis

Section 1 Introduction

Section 2 Basic principles and concepts of transient well testing

Section 3 Conventional well test analysis methods (pressure drop test)

Section 4 Conventional well test analysis method (pressure build-up well test)

Section 5 Conventional well test analysis methods for dual-porosity reservoirs

Section 6 Conventional analysis method for fracturing wells in homogeneous reservoirs

Section 7 Conventional analysis method for horizontal wells in homogeneous reservoirs

Pressure drop test is to open wells for long-term closure, to measure the changes of production and bottom hole flowing pressure with time.

Pressure drop test includes equal production pressure drop test, variable production pressure drop test and exploratory edge test.

一、Equivalent Production Pressure Drop Well Testing

1 、Test program

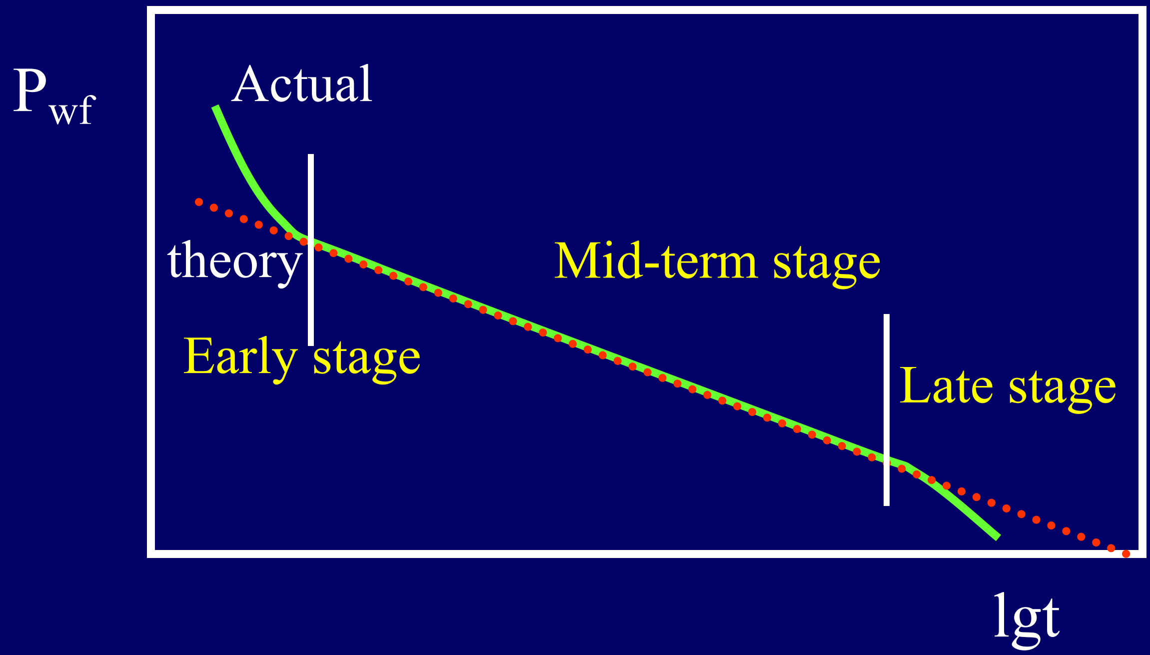
①Record the steady pressure (static pressure) by lowering the instrument to a predetermined position at the bottom of the well (as close as possible to the middle of the reservoir)；

②Opening production with constant production rate. At this time, the instrument records the change of bottom hole flowing pressure with time；

③Sampling for Physical Property when Necessary.

2 、Morphology of measured pressure drop curve

A complete pressure drop curve is generally composed of three flow stages: early, middle and late stages.



1）Characteristic of pressure curve in early stage

The early data are mainly controlled by wellbore storage effect.

2）Characteristic of pressure curve in mid-term

When the wellbore effect is no longer disturbed, the pressure curve enters the mid-term stage, which is called "infinite action radial flow stage". In this stage of flow, the measured pressure drop curve coincides with the theoretical pressure drop curve. In the semi-logarithmic coordinate system, it is often called the mid-term straight line segment.

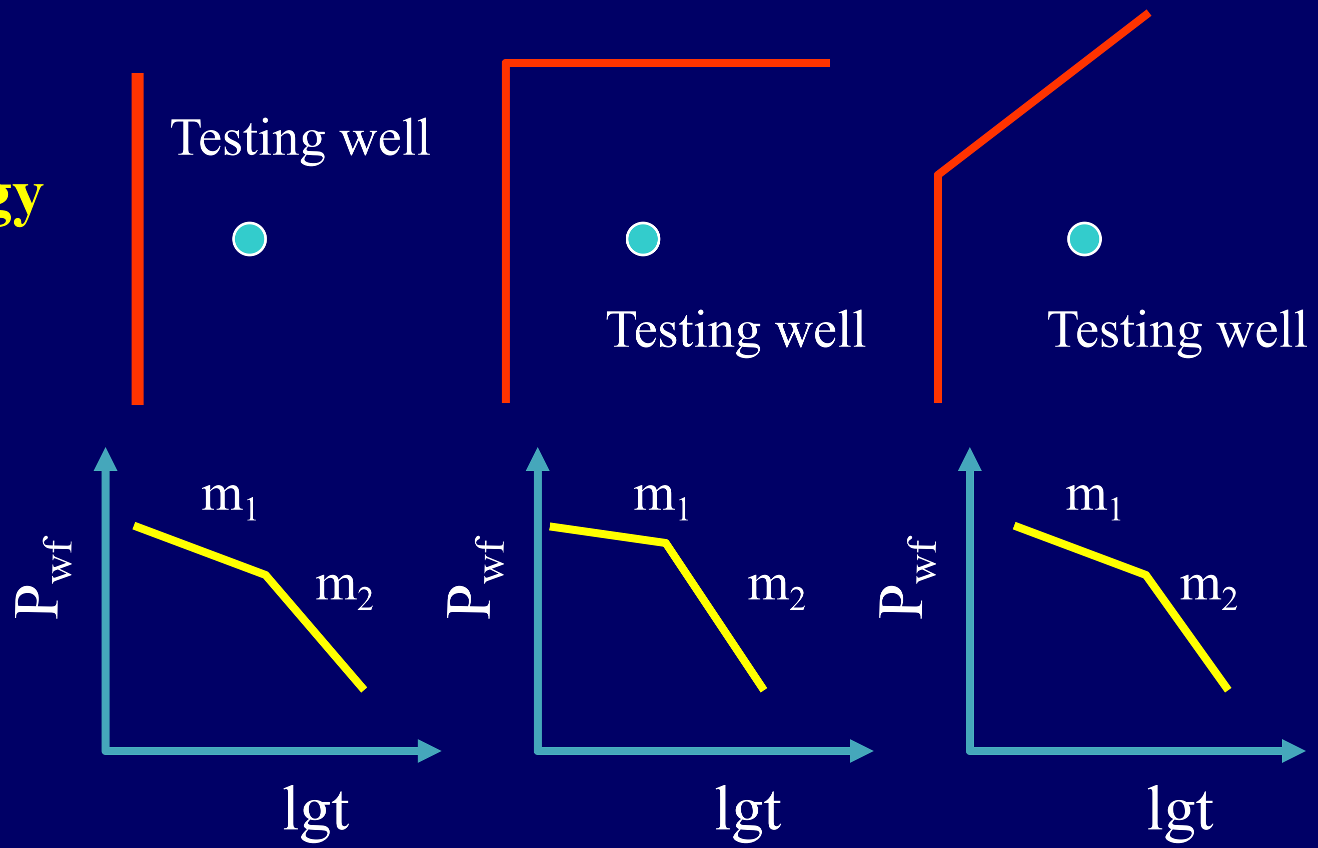
3）Characteristic of Pressure Curve in Late Stage

(1) The influence of linear impermeable boundary.

If there is a linear impermeable boundary near the test well, when the pressure propagates to this boundary, the pressure drop speed will be accelerated and the pressure drop curve will become steeper. In the semi-logarithmic coordinate system, another straight line segment is presented; the slope ratio of the straight line segment to the straight line segment (mid-term segment) $m_D = m_2/m_1$ varies with the geometrical shape of the impermeable boundary.

Fault morphology

Morphology of Middle and Late Stage of Pressure Drop Curve



2:1 4:1 3:1

$$\boxed{m_D = \frac{m_2}{m_1}}$$

(2) The influence of closed boundary

The so-called closed boundary is the entire boundary of a reservoir (also known as a closed system) surrounded by impermeable boundary.

When the pressure disturbance reaches the whole closed boundary, the flow in the reservoir enters the quasi-stable flow.

Thereafter, the relationship between PWF and t is linear, that is, the rate of change of flow pressure with time is constant:

$$\boxed{\frac{\partial P_{wf}}{\partial t} = \text{constant} \neq 0}$$

（3）Effect of Constant Pressure Boundary.

Large gas cap, wide and active edge water and edge water injection satisfying injection-production balance may form very similar constant pressure boundary.

When the pressure disturbance reaches the constant pressure boundary, a "steady flow" occurs in the reservoir. At this point, p_{wf} is independent of t：

$$\boxed{\frac{\partial P_{wf}}{\partial t} = 0}$$

The pressure drop curve becomes a horizontal line.

3 、Conventional analytical methods

Draw the pressure drop data (p_{wf} (t) - t) on the half (single) logarithmic coordinate paper and connect the data points of the radial flow section (i.e. the mid-term section) into a straight line (i.e. the semi-logarithmic straight line).

$$\boxed{\begin{aligned} P_{wf} &= -\frac{2.121\times 10^{-3} q\mu B}{Kh}\lg t + [p_i - \frac{2.121\times 10^{-3} q\mu B}{Kh} \\ &\times (\lg \frac{K}{\varphi \mu C_t r_w^2} + 0.9077 + 0.8686S)] \end{aligned}} \tag{2-24}$$

The absolute value of the slope of this line is:

$$\boxed{m = 2.121 \times 10^{-3} \frac{q\mu B}{Kh}} \qquad (2\text{-}25)$$

The intercept of 1 hour flow is:

$$\boxed{P_{1h} = P_i - m(\lg \frac{8.0854K}{\varphi \mu C_t r_w^2} + 0.8686S)} \qquad (2\text{-}26)$$

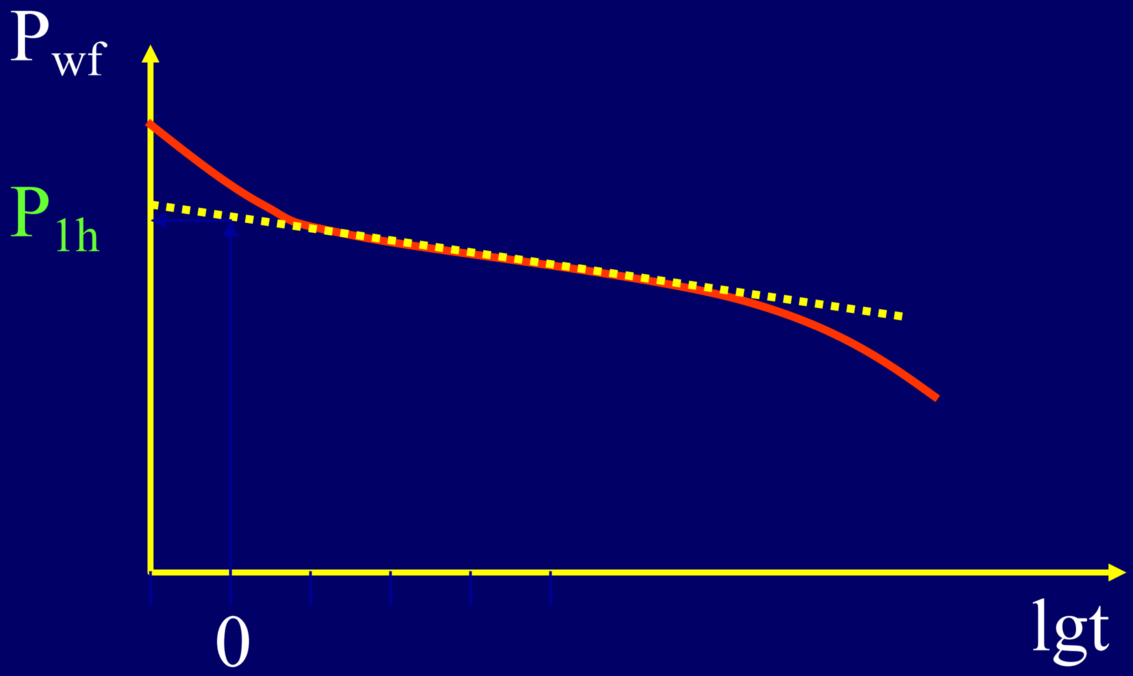
By measuring the slope m and P_{1h} of the semi-logarithmic straight line, the flow coefficient Kh/μ(or permeability K) and the skin coefficient S can be obtained.

$$\frac{Kh}{\mu} = \frac{2.121 \times 10^{-3} qB}{m} \qquad (2\text{-}27)$$

$$K = \frac{2.121 \times 10^{-3} q\mu B}{mh} \qquad (2\text{-}28)$$

$$S = 1.151 \times \left(\frac{P_i - P_{1h}}{m} - \lg \frac{K}{\varphi \mu C_t r_w^2} - 0.9077\right) \qquad (2\text{-}29)$$

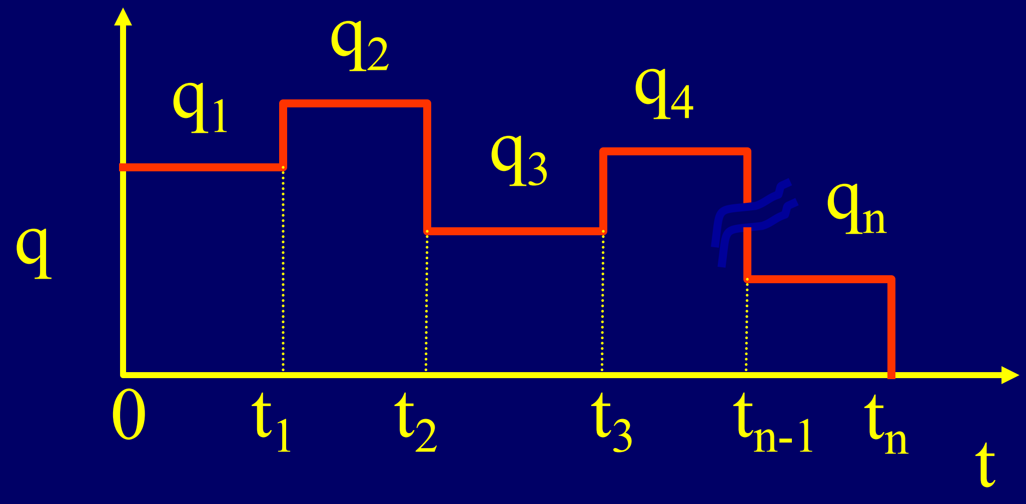
It must be noted that the P_{1h} must correspond to the pressure reading of T for 1h in the semi-logarithmic straight line segment (or its extension line) in equation (2-29).



二、Variable Production Pressure Drop Well Testing

Variable production pressure drop test is called variable production pressure drop test.

Let a well in an infinite reservoir change n times of production in the process of pressure drop.



In this case, when $t > t_{n-1}$, the：

$$P_{wf} = -m' \sum_{j=1}^{n} \frac{q_j - q_{j-1}}{q_n} \lg(t - t_{j-1}) + a \qquad (2\text{-}31)$$

$$m' = 2.121 \times 10^{-3} \frac{q_n \mu B}{Kh} \qquad (2\text{-}32)$$

$$a = P_i - m' \left(\lg \frac{K}{\varphi \mu C_t r_w^2} + 0.9077 + 0.8686 S \right) \qquad (2\text{-}33)$$

Where： q_i — Ith rate， m^3/d；

m’ — Slope of Linear Section in Variable Yield Data Analysis Map， $MPa \cdot (m^3/d)^{-1}$

t_{i-1} — the starting time for the Ith rate， （I=1,2,…,n)， h

If the nth time period is long enough, it can be drawn.

$$p_{wf} \sim \sum_{j=1}^{n} \frac{q_j - q_{j-1}}{q_n} \lg(t - t_{j-1})$$

$$\frac{Kh}{\mu} = \frac{2.121 \times 10^{-3} q_n B}{m'} \qquad (2\text{-}34)$$

$$Kh = \frac{2.121 \times 10^{-3} q_n \mu B}{m'} \qquad (2\text{-}35)$$

$$K = \frac{2.121 \times 10^{-3} q_n \mu B}{m' h} \qquad (2\text{-}36)$$

$$S = 1.151\left(\frac{P_i - a}{m'} - \lg \frac{K}{\varphi \mu C_t r_w^2} - 0.9077\right) \qquad (2\text{-}37)$$

If the original pressure Pi is unknown, it can be plotted

$$\boxed{\frac{p_{wf}}{q_n} \sim \sum_{j=1}^{n} \frac{q_j - q_{j-1}}{q_n} \lg(t - t_{j-1})}$$

The absolute value of slope is still m'. But the intercept becomes:

$$\boxed{a = \frac{P_i}{q_n} - m'(\lg \frac{K}{\varphi \mu C_t r_w^2} + 0.9077 + 0.8686S)}$$

(2-38)

三. Two-Production Well Testing

Well testing with two production rates has certain advantages：

Testing and analysis can be simplified；

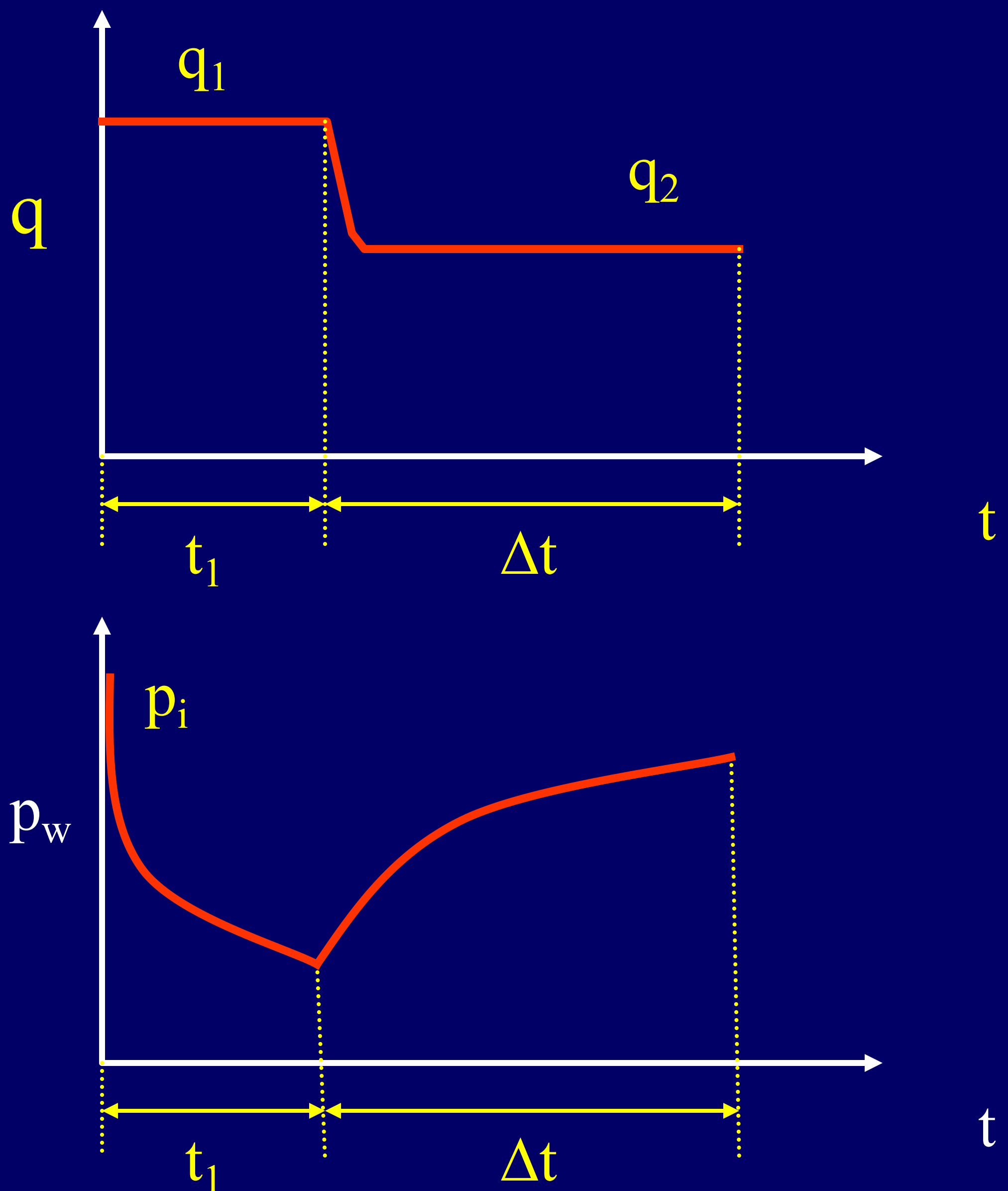
Wellbore storage effect can be reduced.

1.Test program

1 ） q_1 must be strictly kept constant before testing；

2 ） To stabilize q_2 production；

3 ） Accurately record pressure and output during q_2 production.


q
q_1
q_2
t_1
Δt
t
p_w
p_i
t_1
Δt
t

2 、Analysis method

Two-production well test is a special case of variable production well test.

$$P_{wf} = P_i - 2.121\times 10^{-3}\frac{q_1 \mu B}{Kh}[(\lg\frac{t_1+\Delta t}{\Delta t} + \frac{q_2}{q_1}\lg \Delta t) + \frac{q_2}{q_1}\sum S] \quad (2\text{-}41)$$

$$\sum S = \lg\frac{K}{\varphi \mu C_t r_w^2} + 0.9077 + 0.8686S$$

Make relation curve of test data in Cartesian coordinates

$$\boxed{p_{wf} - (\lg\frac{t_1 + \Delta t}{\Delta t} + \frac{q_2}{q_1}\lg\Delta t)}$$

The relationship curve should be straight line with slope m (1) and intercept B (1), respectively:

$$m_{(1)} = \frac{2.121 \times 10^{-3} q_1 \mu B}{Kh} \qquad (2\text{-}42)$$

$$b_{(1)} = P_i - m_{(1)}\frac{q_2}{q_1}\sum S \qquad (2\text{-}43)$$

From this, permeability K, S and Pi can be obtained.

$$\boxed{K = \frac{2.121\times 10^{-3} q_1 \mu B}{m_{(1)} h}} \qquad (2\text{-}44)$$

$$S = 1.151\left(\frac{P_i - b_{(1)}}{m_{(1)}}\frac{q_1}{q_2} - \lg\frac{K}{\varphi\mu C_t r_w^2} - 0.9077\right) \qquad (2\text{-}45)$$

If P_i is unknown, it can be obtained by the following formula:

$$\boxed{P_i = b_{(1)} - \frac{q_2}{q_1 - q_2}[P_{wf}(\Delta t = 0) - P_{wf}(\Delta t = 1)]} \qquad (2\text{-}46)$$

If the q_1 stability time before changing the flow rate is quite long, that is, $t_1 \gg \overline{\Delta} t$, the above analysis formula can be simplified as follows:

$$\boxed{P_{wf} = -m_{(2-1)} \lg \Delta t + b_{(2-1)}} \qquad (2\text{-}47)$$

Where：

$$m_{(2-1)} = -\frac{2.121 \times 10^{-3} (q_2 - q_1) \mu B}{Kh} \qquad (2\text{-}48)$$

$$b_{(2-1)} = P_i + m_{(2-1)} \frac{q_2}{q_2 - q_1} (\lg \frac{K}{\varphi \mu C_t r_w^2} + 0.9077 + 0.8686 S + \frac{q_1}{q_2} \lg t_1) \qquad (2\text{-}49)$$

The formulas for calculating the values of K and S are as follows:

$$\frac{Kh}{\mu} = -\frac{2.121\times10^{-3}(q_2 - q_1)B}{m_{(2-1)}} \tag{2-50}$$

$$K = -\frac{2.121\times10^{-3}(q_2 - q_1)\mu B}{m_{(2-1)}h} \tag{2-51}$$

$$S = 1.151[\frac{P_{1h} - P_{wf}(\Delta t = 0)}{m_{(2-1)}} - \lg\frac{K}{\varphi\mu C_t r_w^2} - 0.9077] \tag{2-52}$$

四、Boundary detection

1 、Test Method of Quasi-stable Flow Edge Detection

In the quasi-steady flow phase of pressure drop test, the flow pressure can be written as follows：

$$P_{wf} = P_i - \frac{9.210\times10^{-4} q\mu B}{Kh}\left(\ln\frac{2.246A}{r_w^2 C_A} + 2S\right) - \frac{qB}{24V_P C_t} t$$

(2-53)

It can be seen from the formula that in rectangular coordinates, P_{wf} and t are straight lines.

Slope m* and intercept b* are respectively：

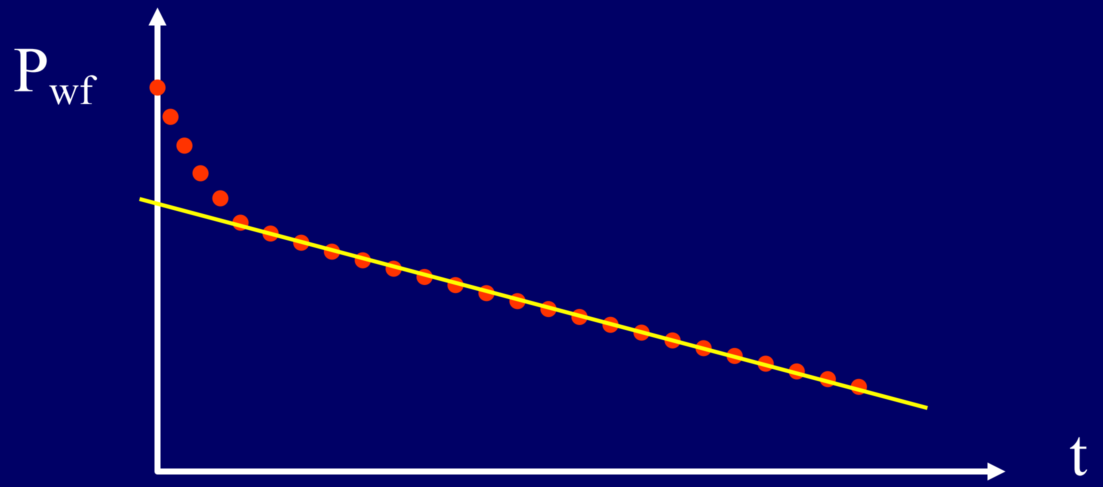
$$m^* = -\frac{qB}{24V_pC_t} \quad (2\text{-}54)$$

$$b^* = p_i - \frac{9.210\times10^{-4}q\mu B}{Kh}(\ln\frac{2.246A}{r_w^2C_A} + 2S) \quad (2\text{-}55)$$

Where：

V_P —Pore volume of closed reservoir, $V_P = Ah\varphi$， m^3

C_A —Shape Coefficient of Closed Reservoir， 1



Pore Volume Vp and Reserve N of Reservoir:

$$V_p = \frac{qB}{24C_t m^*} \qquad (2\text{-}56)$$

$$N = 10^{-4} \times V_p \times S_o = \frac{10^{-4} qBS_o}{24C_t m^*} \qquad (2\text{-}57)$$

Where:

N——Crude oil reserves in closed reservoirs， $10^4 m^3$；

S_o——Oil saturation， 1

2 、Y-function edge detection method

Park Jones combines the pressure dynamics of unstable and quasi-stable flows and proposes a calculation parameter which is useful for identifying boundary and estimating reserves. This method is called Y function edge detection method.

（１）Fundamental equation of Y function

In constant production pressure drop, the bottom hole pressure during radial flow is as follows:

$$\boxed{\begin{aligned} P_{wf} &= P_i - \frac{2.121\times 10^{-3} q\mu B}{Kh} \\ &\quad \times (\lg \frac{Kt}{\varphi \mu C_t r_w^2} + 0.9077 + 0.8686S) \end{aligned}}$$

If we derive the equation from t, we can get it：

$$\boxed{Y=\frac{D'}{2t}} \tag{2-58}$$

Where： Y——Rate of pressure change per unit time output under unstable flow conditions［$MPa\cdot h^{-1}/(m^3\cdot d^{-1})$］

$$\boxed{Y=\frac{1}{qB}\cdot\frac{d(p_i-p_{wf})}{dt}} \tag{2-59}$$

D’——Darcy constant（$mPa\cdot s/\mu m^2\cdot m$)， Defined as：

$$\boxed{D'=1.842\times10^{-3}\frac{\mu}{Kh}} \tag{2-60}$$

By taking logarithms on both sides of equation (2-28), the following results are obtained:

$$\boxed{\lg Y = \lg \frac{D'}{2} - \lg t} \qquad (2\text{-}61)$$

As can be seen from the above formula, a straight line with a slope of -1 can be obtained by plotting Y and T in the double logarithmic coordinate system.。

When t=1h， Its Y value is：

$$\boxed{Y_{1h} = \frac{D'}{2} = 9.21 \times 10^{-4} \frac{\mu}{Kh}} \qquad (2\text{-}62)$$

By (2-53) derivation of t, we can get that:

$$\boxed{Y_{ps} = \frac{S_o}{24 \times 10^{-4} N C_t}} \qquad (2\text{-}63)$$

Double logarithm

$$\boxed{\lg Y_{ps} = \lg \frac{S_o}{24 \times 10^{-4} N C_t}} \qquad (2\text{-}64)$$

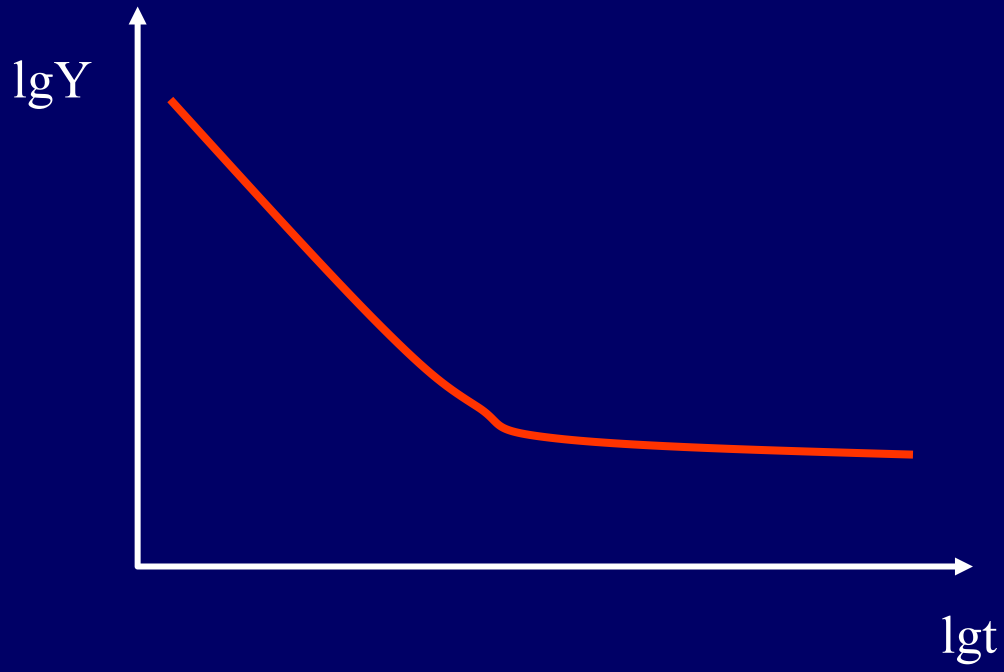
Where：

Yps——Pressure Drop Rate of Unit Time Output under Quasi-stable Flow Conditions，［$MPa \cdot h^{-1}/(m^3 \cdot d^{-1})$］

(2) The shape of Y-function curve

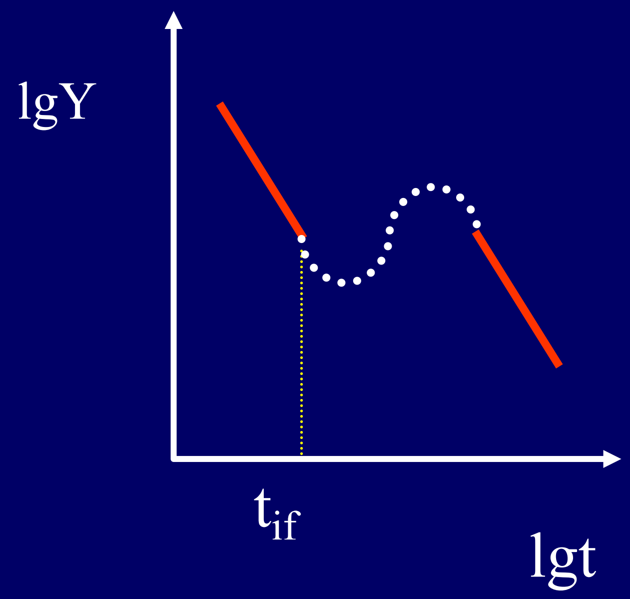
１）Y-function curve of closed reservoir

It starts with unstable flow and Y function decreases along a straight line with slope of -1. Then it transits to quasi-stable flow and Y function curve becomes a horizontal straight line.



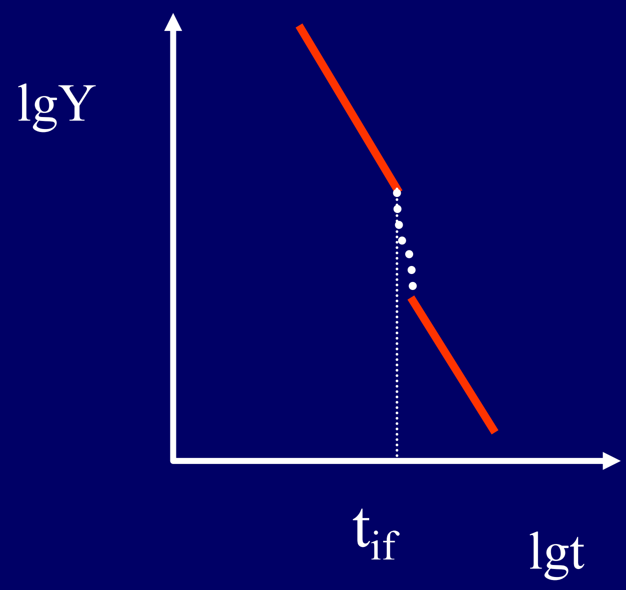
2）Y-function curve disturbed by viscous fluid

When pressure disturbance encounters high resistance fluids (e.g. gas to oil or water, thin oil to heavy oil, etc.), the Y function moves to the right. And the distance from well to fluid interface can be estimated by time t_{if} of interference point.



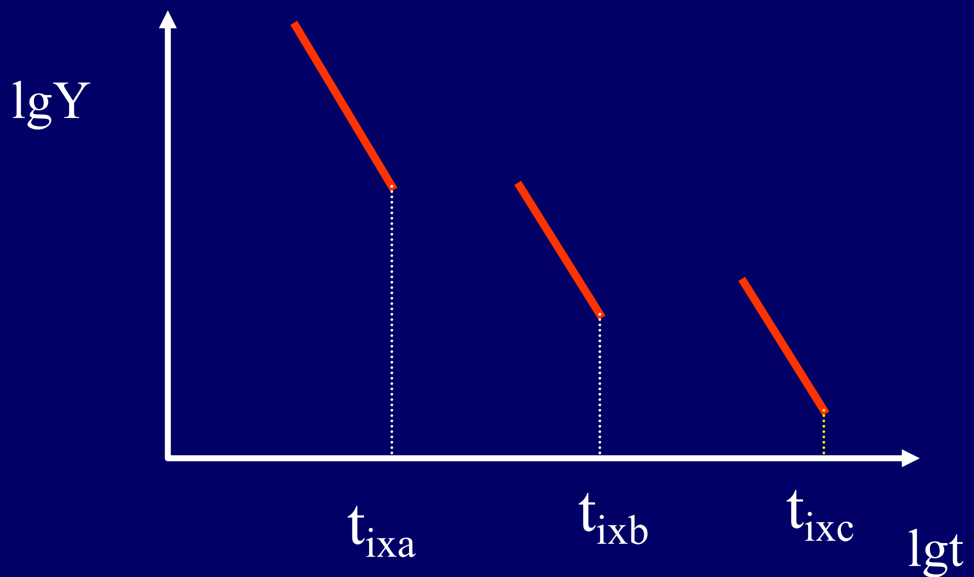
3） Y-function curve disturbed by low viscous fluid

When pressure disturbance encounters low resistance fluid, the Y function moves to the left.



4） Y-function curve affected by faults

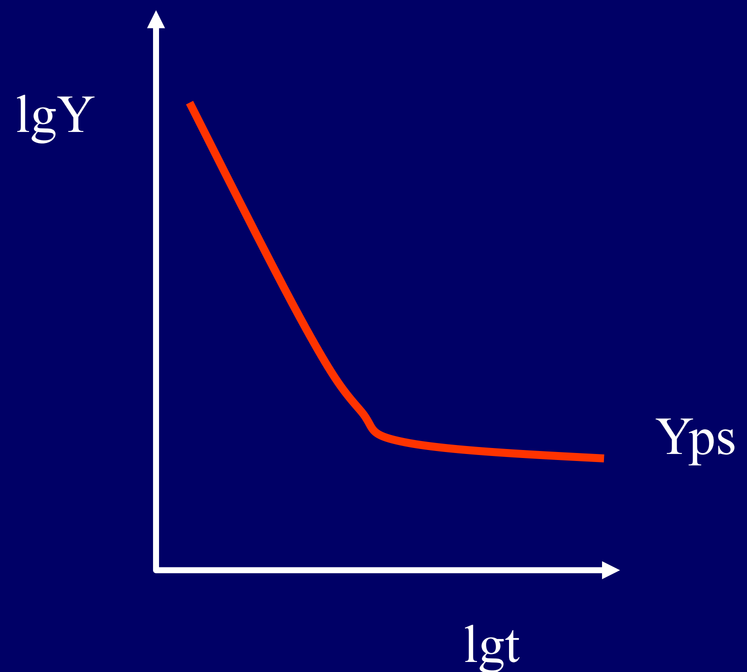
When the pressure disturbance meets a fault, the Y function moves to the right.



（3）Calculation of parameters

1）Calculating Geological Reserves of Closed Reservoirs

$$\boxed{N = \frac{S_o}{24 \times 10^{-4} Y_{ps} C_t}}$$



2 ） Calculating Kh/μ Or K:

$$\boxed{\begin{aligned} \frac{Kh}{\mu} &= 9.21\times10^{-4}\frac{1}{Y_{1h}} \\ K &= 9.21\times10^{-4}\frac{\mu}{Y_{1h}h} \end{aligned}}$$

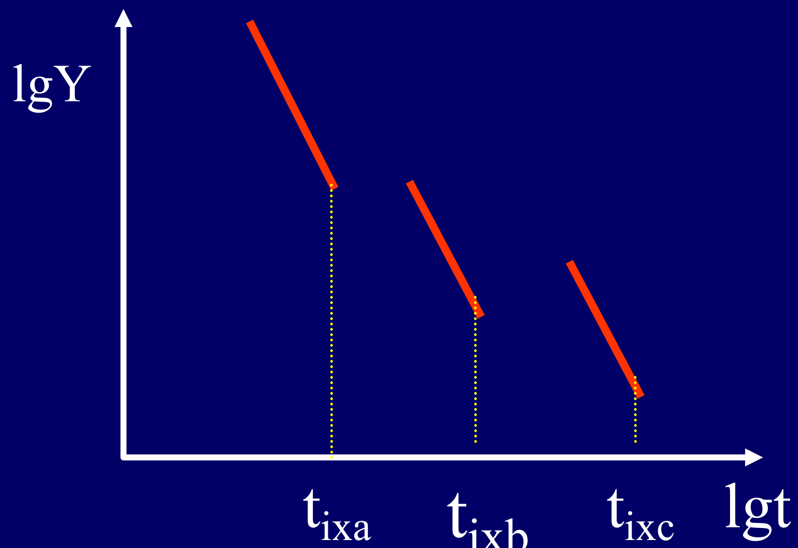
3）Calculate the distance from well to fault

$$d = 1.422\sqrt{\frac{Kt_{ix}}{\varphi \mu C_t}}$$

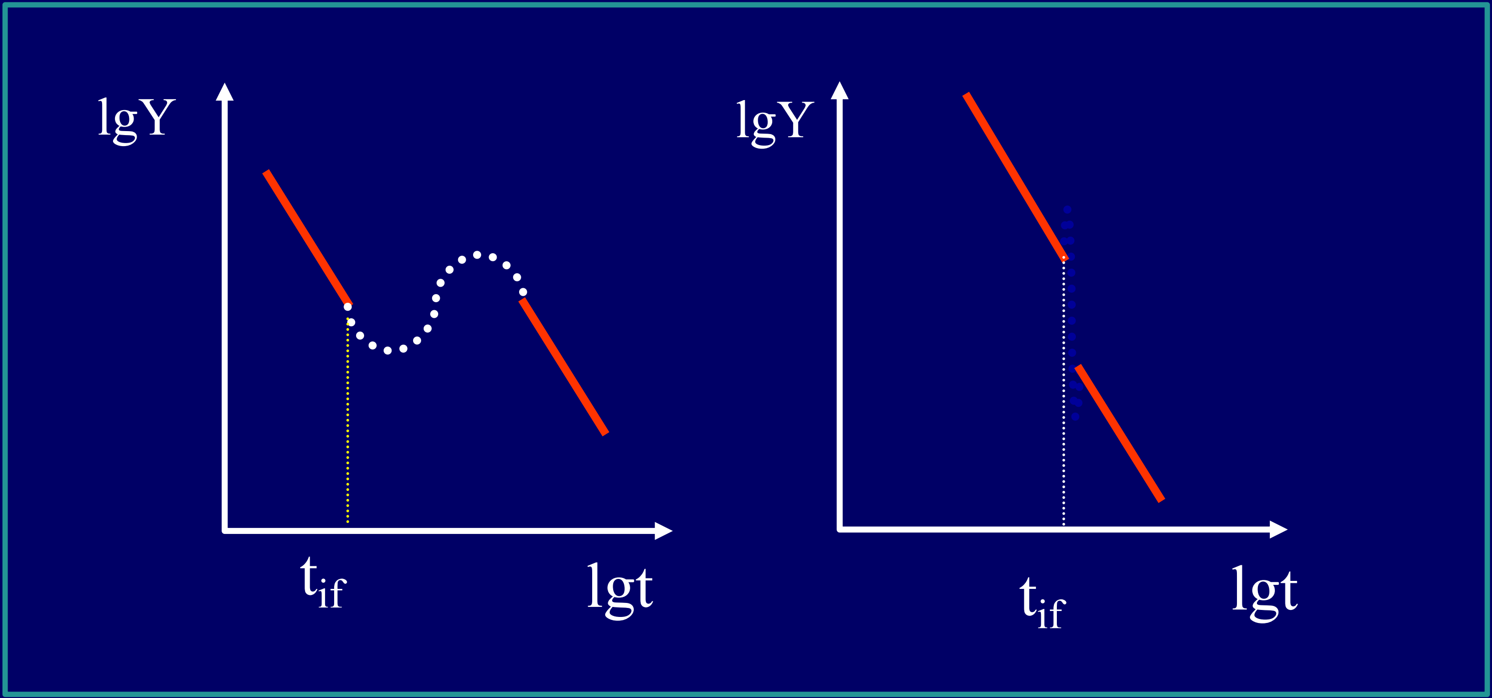
Where：

d——Distance from well to fault， m

t_{ix}——Time Corresponding to Fault Interference Points



4）Calculating the distance from the test well to the fluid boundary



$$r_{if} = 3.795\sqrt{\frac{Kt_{if}}{\varphi \mu C_t}}$$

Where：

r_{if}—Distance from well to fluid boundary， m

t_{if}—Corresponding time of interference point， h

Chapter 6　Principle and application of well test analysis

Pressure build-up well testing is referred to as recovery well testing, which is a method of transferring a well from a stable production state to a shut-in state and measuring the pressure rise after shut-in. It includes two methods: equal production well test and variable production well test.

一、Equivalent production well testing

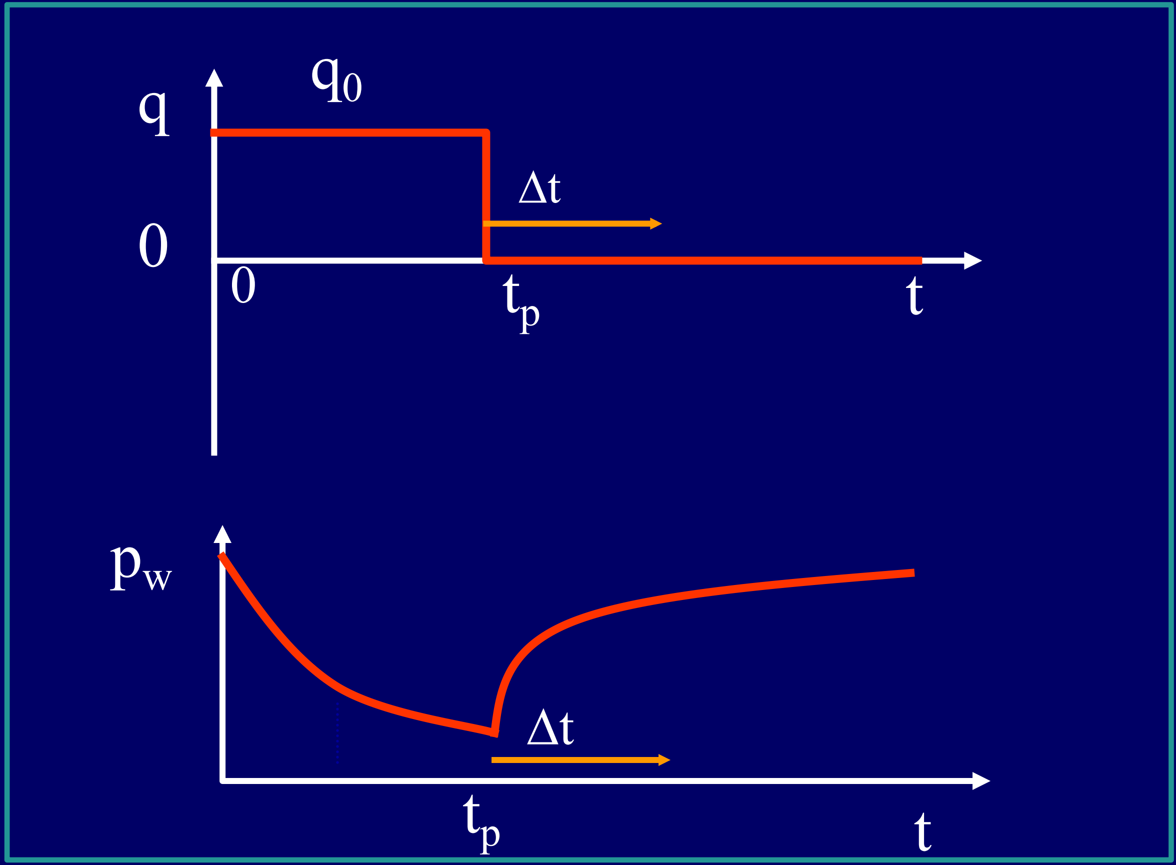
Equivalent production test is to measure the change of bottom hole flowing pressure with time when the fluid in reservoir is stable (usually the bottom hole flowing pressure is required to be stable).

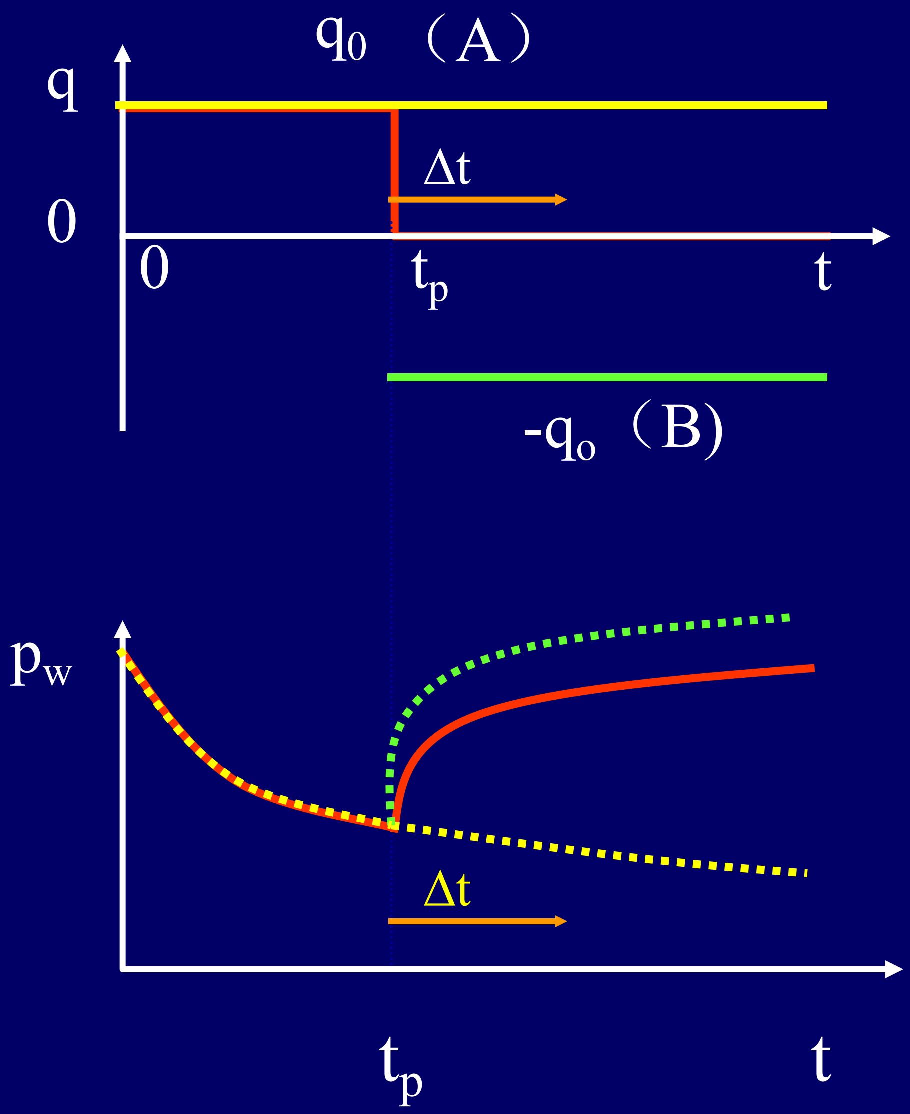
1、Horner method
2、MDH method
3、Muskat method
4、MBH method

1、Horner method

1）Analysis of Infinite Strata

Testing wells are produced from zero to constant production q and shut down at t_p.





It can be analogous to the flow system of two wells (well spacing is 0):

Well A has been producing with constant output q since zero hour. Well B produces (or injects) at a constant negative production rate (-q) starting at t_p time (shut-in time).

According to the superposition principle, if both TP and (tp+Δt) are in the radial flow stage, the variation of the actual bottom hole recovery pressure with time is Horner's formula.

$$\boxed{p_{ws} = p_i + \frac{2.121 \times 10^{-3} q \mu B}{Kh} \lg \frac{\Delta t}{t_p + \Delta t}} \qquad (2\text{-}71)$$

Or

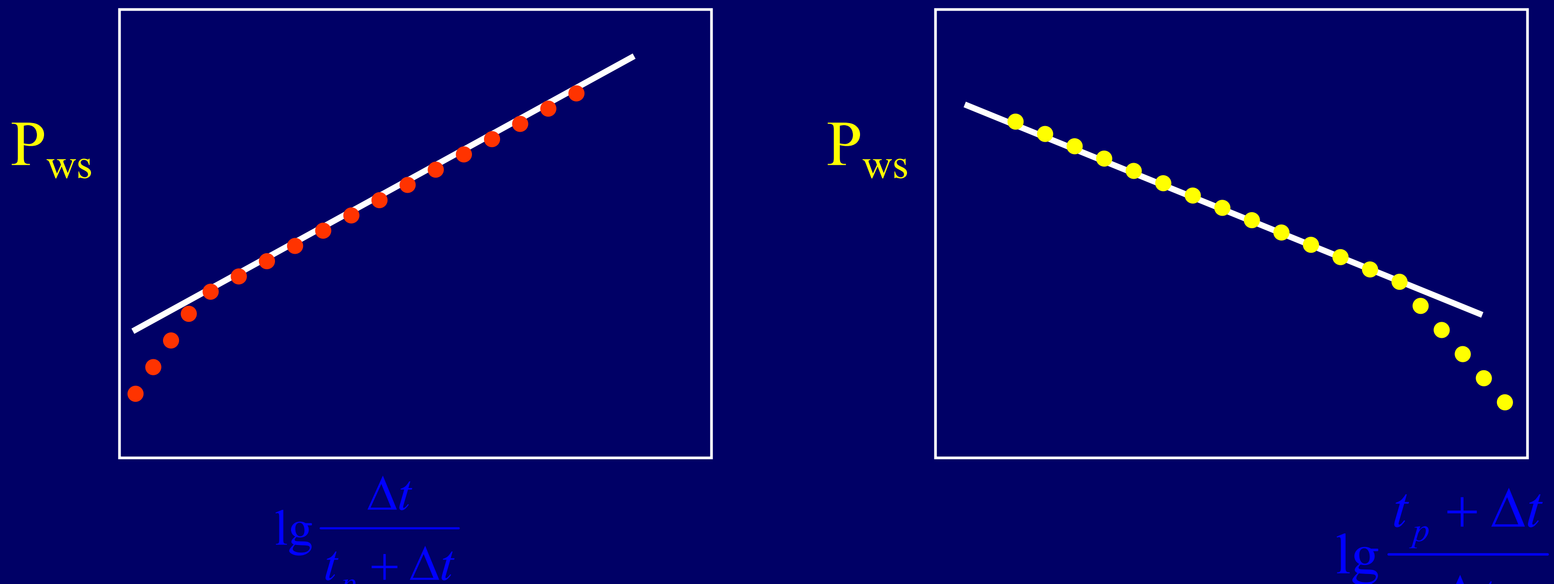
$$\boxed{p_{ws} = p_i - \frac{2.121 \times 10^{-3} q \mu B}{Kh} \lg \frac{t_p + \Delta t}{\Delta t}} \qquad (2\text{-}72)$$

Where： P_{ws} —Shut-in recovery pressure， MPa；

Δt —Shut in time， h；

t_p —Production time before shutdown， h。

From the formulas (2-71) and (2-72), it is known that in the semi-logarithmic coordinate system, the relation curve (Horner curve) between Pws and $(t_p+\Delta t) / \Delta t$ (or $\Delta t / (t_p+\Delta t)$) is a straight line (Horner line).



Horner curve sketch

$$\boxed{p_{ws} = p_i + \frac{2.121 \times 10^{-3} q\mu B}{Kh} \lg \frac{\Delta t}{t_p + \Delta t}}$$

Absolute value of slope：

$$\boxed{m = \frac{2.121 \times 10^{-3} q\mu B}{Kh}} \qquad (2\text{-}73)$$

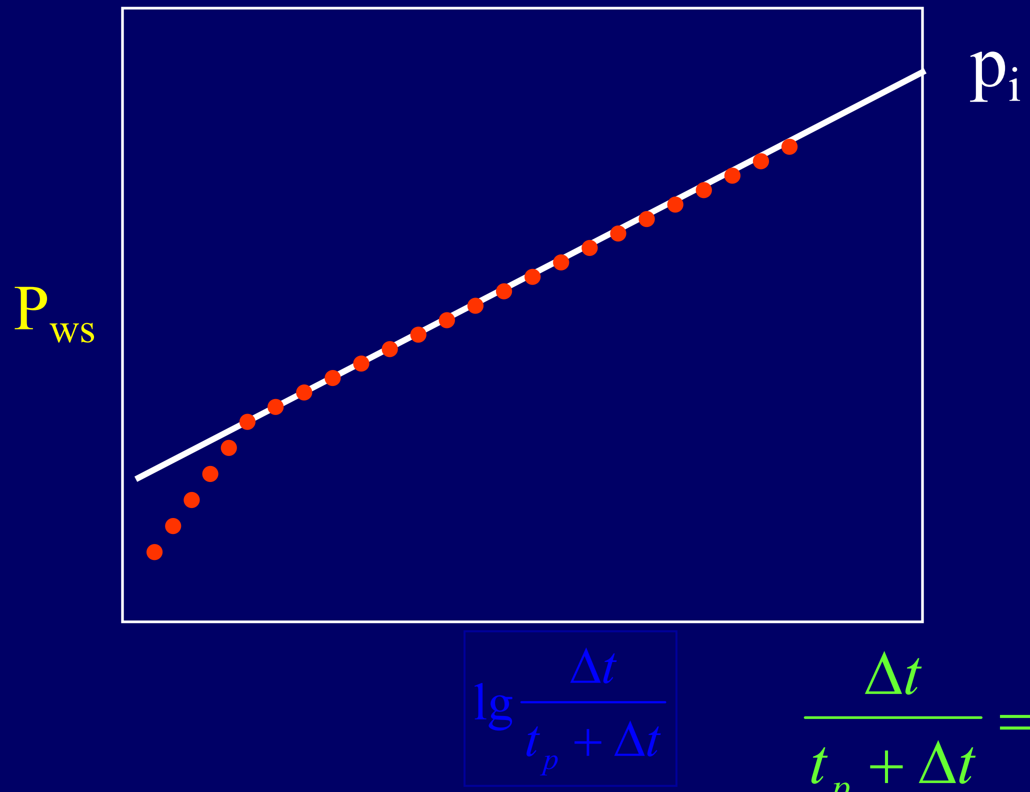
From this formula, the flow coefficient and permeability can be calculated.

$$\boxed{p_{ws} = p_i + \frac{2.121 \times 10^{-3} q\mu B}{Kh} \lg \frac{\Delta t}{t_p + \Delta t}}$$

When the shut-in time is infinite, the shut-in pressure should reach the reservoir static pressure (or original pressure):

$$\boxed{\lim_{\Delta t \to \infty} p_{ws} = p_i} \qquad (2\text{-}74)$$

The Horner line is extrapolated to the abscissa Δt / (t_p+ Δt) =1 by graph method. When t_p is small, the corresponding pressure is the original pressure.



$$\frac{\Delta t}{t_p + \Delta t} = 1$$

Production is not stable before shut-in， t_p . It can be determined by the following formula， Also known as converted production time：

$$\boxed{t_p = \frac{24 N_p}{q}} \qquad (2\text{-}75)$$

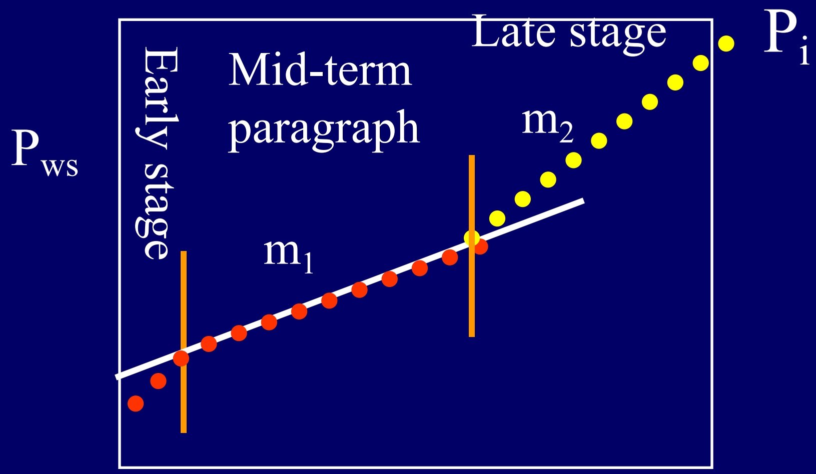
Where：

Np——Accumulated production during this production period， m^3；

q——Stable production before shutdown， m^3/d。

2 ）Extrapolation Pressure of Bounded Reservoirs

For bounded reservoirs, due to the influence of boundary, the pressure extrapolated to lg $[\Delta t / (t_p + \Delta t)] = 0$ in mid-term straight line segment is not equal to Pi, which is usually called extrapolated pressure, expressed as P *. P* Conversion Pressure for Calculating Average Reservoir Pressure



3 ） Conventional analytical methods

（ 1 ） Computing test data

$t_p =$			
	Δt	P_{ws}	$\dfrac{\Delta t}{t_p + \Delta t}$ 或 $\dfrac{t_p + \Delta t}{\Delta t}$
0 1 ⋮ ⋮			

(2) Drawing on semilogarithmic coordinate paper；

$$\boxed{p_{ws} - \lg \frac{t_p + \Delta t}{\Delta t}\left(\text{或} \lg \frac{\Delta t}{t_p + \Delta t}\right)}$$

3) Determine the start time of the mid-term period；

(4) Calculate the slope m of the mid-term straight line segment and read it out $\Delta t = 1h$ P_{ws} ；

(5) Calculating formation parameters；

$$\boxed{\frac{Kh}{\mu}=\frac{2.121\times10^{-3}qB}{m}} \qquad (2\text{-}76)$$

$$\boxed{K=\frac{2.121\times10^{-3}q\mu B}{mh}} \qquad (2\text{-}77)$$

$$\boxed{S=1.151[\frac{P_{ws1h}-P_{wf}}{m}-\lg\frac{K}{\varphi\mu C_t r_w^2}+\lg\frac{t_p+1}{t_p}-0.9077]} \qquad (2\text{-}78)$$

Where：

m——Horner straight line slope， MPa/cycle；

P_{wf}——Flow pressure before shut-in， MPa；

P_{ws1h}——The Horner straight line or its extension line corresponds to the pressure of t=1h， MPa.

If $t_p > 1$, it can be simplified to

$$\boxed{S = 1.151\left[\frac{P_{ws1h} - P_{wf}}{m} - \lg\frac{K}{\varphi\mu C_t r_w^2} - 0.9077\right]} \quad (2\text{-}79)$$

(6) Determining the Primary Reservoir Pressure by Graphic Method P_i

The mid-term straight line segment is extrapolated to $\Delta t / (t_p + \Delta t) = 1$, and the corresponding pressure is P^*.

$$\boxed{\ln\left(\frac{t_p+\Delta t}{\Delta t}\right)_x = -E_i\left(\frac{\varphi\mu C_t d^2}{3.6Kt_p}\right)} \qquad (2\text{-}80)$$

From the value of the inverse Ei function, we can get that:

$$\boxed{d=\sqrt{\frac{3.6Kt_p}{\varphi\mu C_t}E_i^{-1}[x]}} \qquad (2\text{-}81)$$

$Ei^{-1}(x)$ is the inverse Ei function value of X.

$$\boxed{x=\frac{1}{2}\ln\left(\frac{t_p+\Delta t}{\Delta t}\right)_x} \qquad (2\text{-}82)$$

2 、MDH method

$$\boxed{p_{ws} = p_i - \frac{2.121\times 10^{-3} q\mu B}{Kh}\lg\frac{t_p + \Delta t}{\Delta t}}$$

If the production time t p before testing is much larger than the test time t_p, i.e. $t_p >> \Delta t$, then $t_p \approx (t_p + \Delta t)$ can be approximated as follows:

$$\boxed{p_{ws} = p_i - \frac{2.121\times 10^{-3} q\mu B}{Kh}[\lg t_p - \lg \Delta t]}$$

(2-83)

$$p_{ws} = p_{wf} + \frac{2.121\times10^{-3} q\mu B}{Kh}[\lg\frac{K}{\varphi\mu C_t r_w^2} + 0.9077 + \lg\Delta t + 0.8686S] \qquad (2\text{-}84)$$

Formula (2-83) (2-84) shows that P_{ws}~lgΔt has a linear relationship.

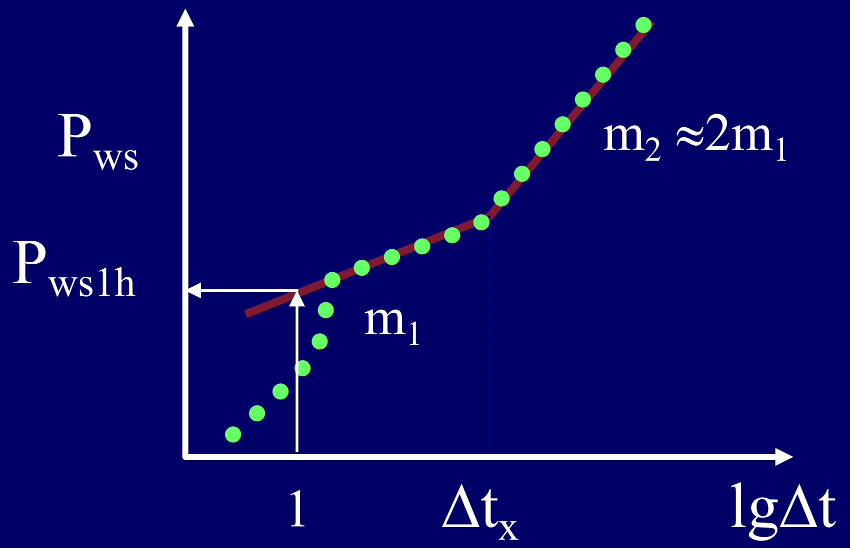
This formula is basically the same as the pressure drop formula (2-24), so when $t_p >> \Delta t$, the pressure recovery analysis can be carried out by a method similar to the pressure drop analysis.

This method is called MDH method.（Miller-Dyes-Hutchinson）.

The steps of pressure recovery analysis by MDH method are as follows:

(1) Making P_{ws}—lgΔt Curve on Semi-logarithmic Coordinate Paper;

(2) Determining the Starting Time of Mid-term Linear Segment;



(3) Calculate the slope m of the mid-term straight line segment and the P_{WS} value corresponding to Δt =1h in the straight line segment (or its extension line).

(4) Calculating formation parameters:

$$\boxed{\frac{Kh}{\mu} = \frac{2.121\times10^{-3}qB}{m}} \qquad (2\text{-}85)$$

$$\boxed{K = \frac{2.121\times10^{-3}q\mu B}{mh}} \qquad (2\text{-}86)$$

$$\boxed{S = 1.151\left[\frac{P_{ws1h} - P_{wf}}{m} - \lg\frac{K}{\varphi\mu C_t r_w^2} - 0.9077\right]} \qquad (2\text{-}87)$$

(5) If there are linear faults and the slope ratio is 2:1, the distance from the test well to the fault can be calculated by the Δt_x corresponding to the intersection of two straight lines.

$$\boxed{d = 1.422\sqrt{\frac{K\Delta t_x}{\varphi \mu C_t}}} \qquad (2\text{-}87)$$

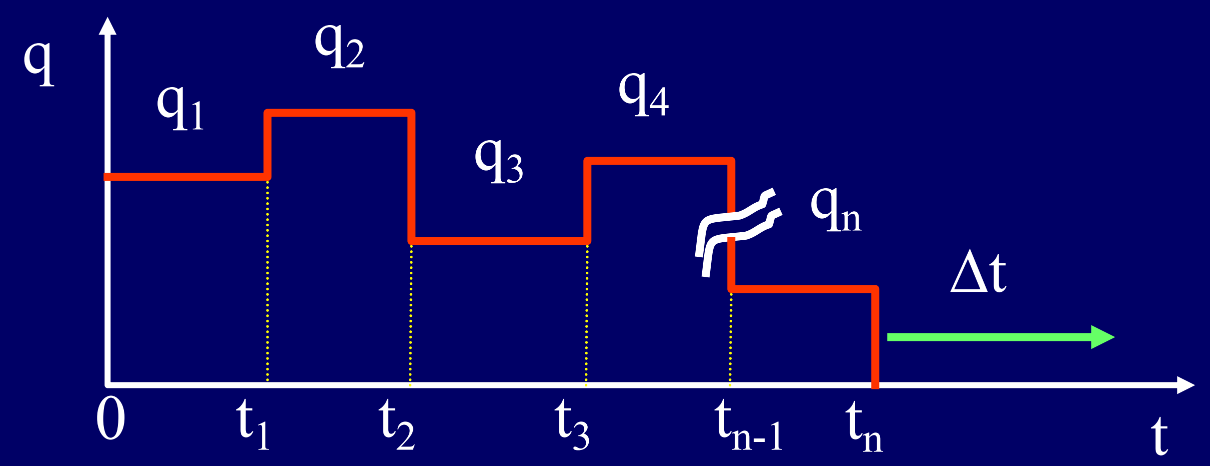
Note：

MDH method is an approximate method based on $t_p >> \Delta t$.

When $t_p >> \Delta t$ is not satisfied, Horner method can only be used.

二、Variable Production Recovery Well Testing

A well in an infinite reservoir is set up to continuously produce with q_1, q_2,... And q_N before shutting in for recovery testing. The production end time of each production is t_1, t_2,... And t_N respectively, and the well is shut down at t_N time.



Using superposition principle, the basic equation of variable production recovery well test can be obtained.

$$P_{ws} = P_i - m_n T \qquad (2\text{-}94)$$

$$T = \sum_{i=1}^{n} \frac{q_i}{q_n} \lg \frac{t_n + \Delta t - t_{i-1}}{t_n + \Delta t - t_i} \qquad (2\text{-}95)$$

Where：

m_n —Slope of Pws-T Relational Curve， MPa;

q_n —Production before shutdown， m^3/d

After making the P_{ws}~T curve and measuring the slope m, we can calculate:

$$\boxed{m_n = \frac{2.121 \times 10^{-3} q_n \mu B}{Kh}}$$

$$\boxed{\frac{Kh}{\mu} = \frac{2.121 \times 10^{-3} q_n B}{m_n}}$$

$$\boxed{K = \frac{2.121 \times 10^{-3} q_n \mu B}{m_n h}}$$

$(t_n-t_{n-1})>>1h$， Then the application formula (2-79) calculates S:

$$S = 1.151[\frac{P_{ws1h} - P_{wf}}{m} - \lg\frac{K}{\varphi\mu C_t r_w^2} - 0.9077]$$

If this condition is not met, then S is calculated by t_p in (t_n-t_{n-1}) substitution formula (2-78) :

$$S = 1.151[\frac{P_{ws1h} - P_{wf}}{m} - \lg\frac{K}{\varphi\mu C_t r_w^2} + \lg\frac{t_p + 1}{t_p} - 0.9077]$$

When the extended line intersects t = 0, the pressure Pws corresponding to the intersection point is P_i (or P*):

$$\boxed{\lim_{T \to 0} P_{ws} = P_i}$$

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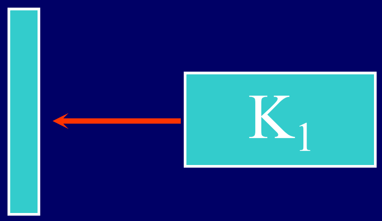
Section 5 Conventional well test analysis methods for dual-porosity reservoirs

Section 6 Conventional analysis method for fracturing wells in homogeneous reservoirs

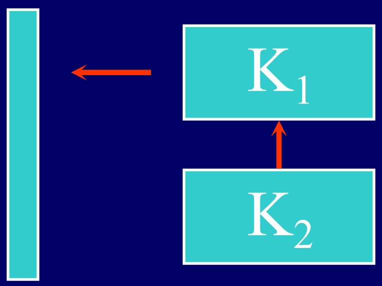
Section 7 Conventional analysis method for horizontal wells in homogeneous reservoirs

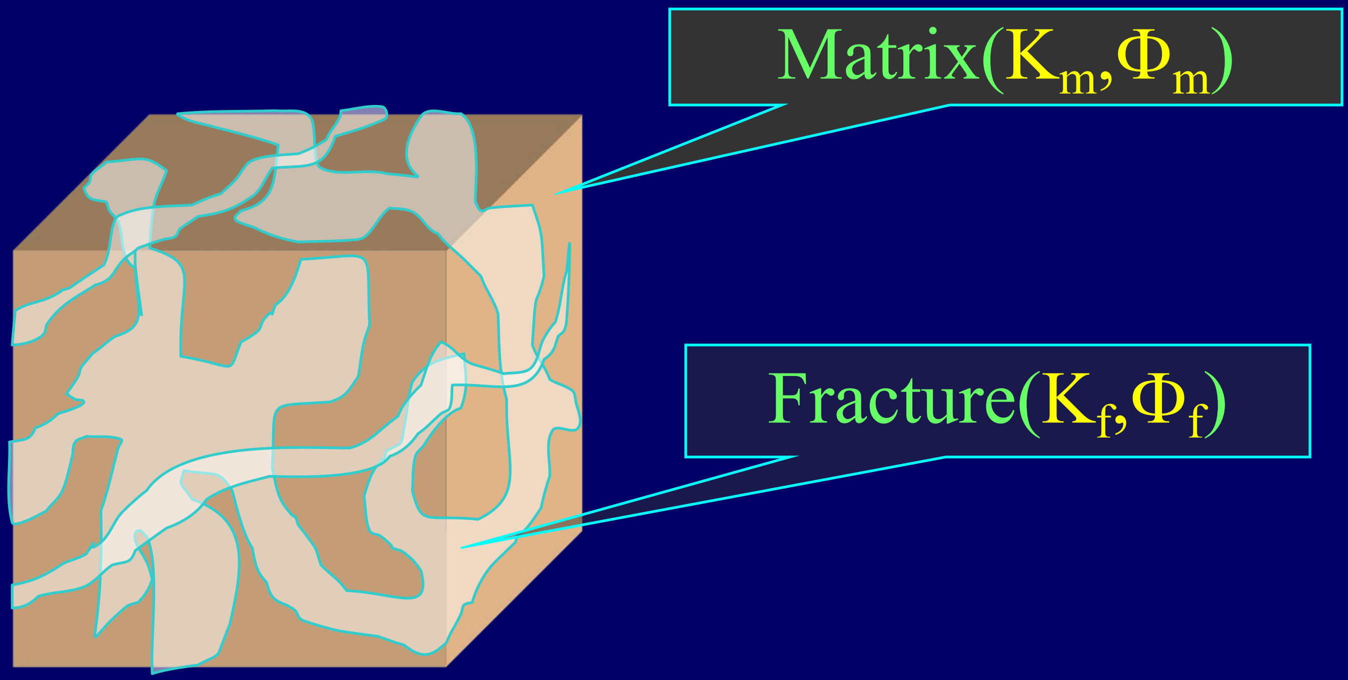
一、Concept and Physical Model of Dual-porous Media Reservoir

Homogeneous reservoir model



Dual-porous reservoir model





$K_f > K_m$, Fracture is the main flow passage

$\Phi_m > \Phi_f$, Bedrock is the main reservoir space

There are three stages in the dynamic change of pressure of fluid seepage in double-porous media formation:

- **Stage one:**
  - Crude oil flowing into oil wells in fracture system;
  - The bedrock system remains stationary.
- **Stage two:**
  - The pressure difference between the bedrock and the fracture forms, and the fluid in the bedrock begins to flow to the fracture (transition zone).
- **Stage three:**
  - Fluid from bedrock flow to fracture system;
  - Crude oil flows into wellbore from fracture system.

Classification of Channel Flow in Transition Zone:

Pseudo-steady state

The pressure in bedrock is the same everywhere, and the channeling flow is only related to the pressure difference between bedrock and fracture.

Transient

The pressure of each point in the bedrock is different, and there is unstable seepage in the bedrock itself.

二、Conventional well test analysis of infinite double-hole reservoir

A well in the center of a horizontally equal-thickness infinite double-porous medium formation is quasi-steady state with constant production Q and cross-flow between matrix and fracture.

$$\frac{k_f}{\mu}\left(\frac{\partial^2 P_f}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial P_f}{\partial r}\right)-\frac{\alpha}{\mu}(P_m-P_f)=\beta_f\frac{\partial P_f}{\partial t}$$

$$\beta_m\frac{\partial P_m}{\partial t}+\frac{\alpha}{\mu}(P_m-P_f)=0$$

$$P_f(r,0)=P_m(r,0)=P_i$$

$$r\cdot\left.\frac{\partial P_f}{\partial r}\right|_{r=r_w}=\frac{\mu q}{2\pi k_f h}$$

$$P_f(\infty,t)=P_m(\infty,t)=P_i$$

Warren and Root give approximate analytical solutions to the above models:

$$P_f(r_w,t) = P_i - \frac{q\mu}{4\pi k_f h}\left\{\ln\frac{\vartheta t}{r_w^2} + E_i(-at) - E_i(-a\omega t) + 0.80907\right\}$$

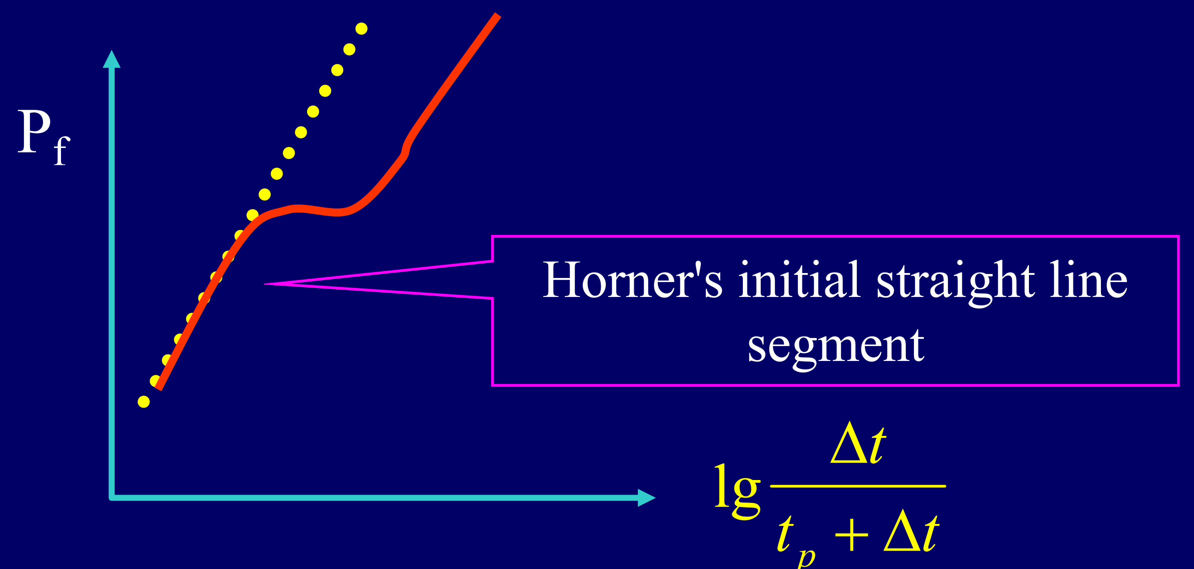
The bottom hole pressure recovery formula is as follows:

$$P_f(r_w,t) = \frac{q\mu}{4\pi k_f h}\left\{\ln\frac{\Delta t}{t_p+\Delta t} + E_i[\frac{-\lambda\theta\Delta t}{\omega(1-\omega)}] - E_i[\frac{-\lambda\theta\Delta t}{1-\omega}]\right\}$$

When $x \to 0$， $Ei(-x) = \gamma + \ln(x)$

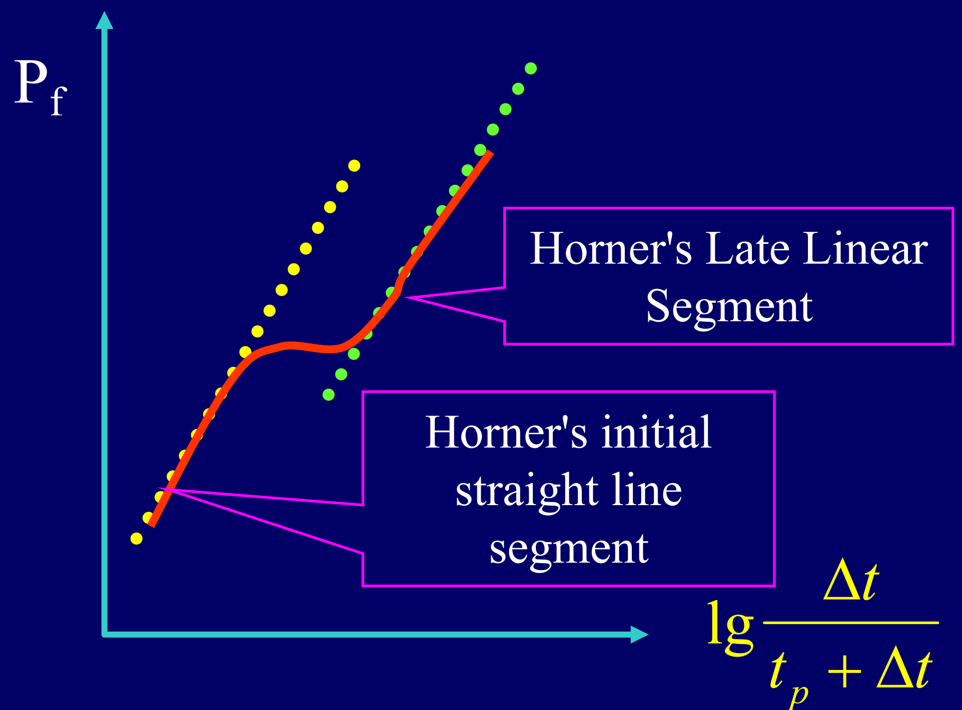
When the shut-in time is not too long, if Δt is not very large, then:

$$\boxed{P_f(r_w, t) = P_i + \frac{0.183 q\mu}{k_f h}\left\{\lg\frac{\Delta t}{t_p + \Delta t} + \lg\frac{1}{\omega}\right\}}$$



When shut-in time is long, Δt is large, then:

$$P_f(r_w,t) = P_i + \frac{0.183 q\mu}{k_f h} \lg \frac{\Delta t}{t_p + \Delta t}$$



$$m_1 = m_2 = \frac{0.183 q\mu}{k_f h}$$

Intercept difference is

$$D_p = m \lg \frac{1}{\omega}$$

Calculating formation parameters:

$$\boxed{\begin{aligned}\frac{k_f h}{\mu} &= \frac{0.183q}{m}\\ \omega &= \exp(-2.3D_p / m)\end{aligned}}$$

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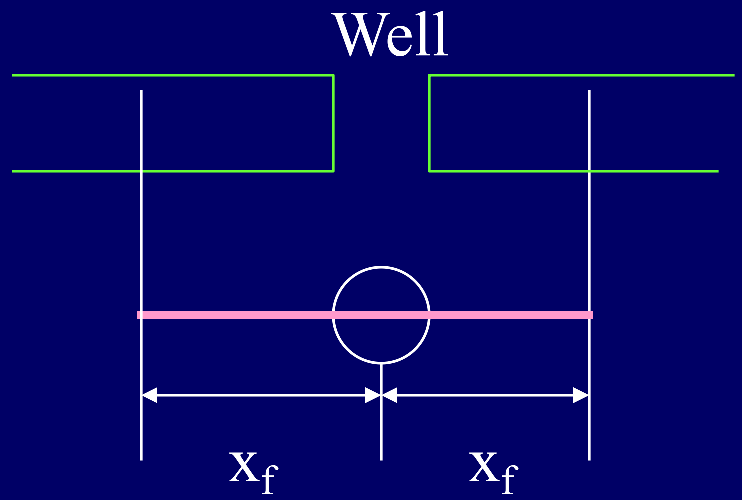

Well
x_f
x_f

Basic Mathematical Model for Well Test Analysis of Vertical Fractured Wells

- Since the existence of vertical fractured wells, there are six kinds of theoretical models used to analyze vertical fractured wells：（Point Source Flow Model），（Efficiency Well Radii Flow Model），（Linear Flow Finite-Conductivity Model），（Bilinear Infinite-Conductivity Flow Model），（Trilinear Linear Finite-Conductivity Flow Model），（Elliptical Flow Model）. The development of these models is gradually from simple to complex, from linearization to non-linearization.

- **一、Wellbore storage-based flow**
- Pump shutdown is the closing of wellbore on the ground, when the wellbore contains compressible liquid, the stopping of pump will cause obvious response to wellbore storage effect. The duration of wellbore storage effect mainly depends on the volume of wellbore from the ground to the reservoir and the compressibility of fluid in the wellbore. The unit slope of the pressure and pressure derivative in the log coordinate chart represents the flow of wellbore storage, and the system is positive. The skin factor and the logarithm of pressure and pressure stored in a constant wellbore reflect a straight line of unit slope.

✦ 二、Fracture linear flow

- The fluid storage of the fracture itself can control the initial pressure instability of the vertical fracture with limited conductivity. Under the condition of small wellbore storage effect, the decrease of steady flow in the vertical fracture with limited conductivity is mainly caused by the diffusion of fluid in the fracture. **The range of this linear flow can be represented by the slope of half of the logarithmic plot of pressure change P relative to time change t.**

- ## 三、Bilinear flow
- Bilinear flow may occur in a finite vertical fracture because there are two linear flow structures in the system at the same time and the effect of fracture end does not affect the instability of the well. **The range of the bilinear flow can be represented by the slope of 1/4 of the logarithmic plot of pressure change P versus time change t.**

✦ 四、Formation linear flow

- For a finite conductive fracture with dimensionless fracture conductivity over 80, the second linear flow may occur because the fracture conductivity is high enough. In this case, the pressure instability of the well is determined by the compressible linear flow of the reservoir perpendicular to the fracture plane, which is usually called the linear flow of the formation. The range of this linear flow can be represented by the slope of 1/2 of the logarithmic plot of pressure change P versus time change t.

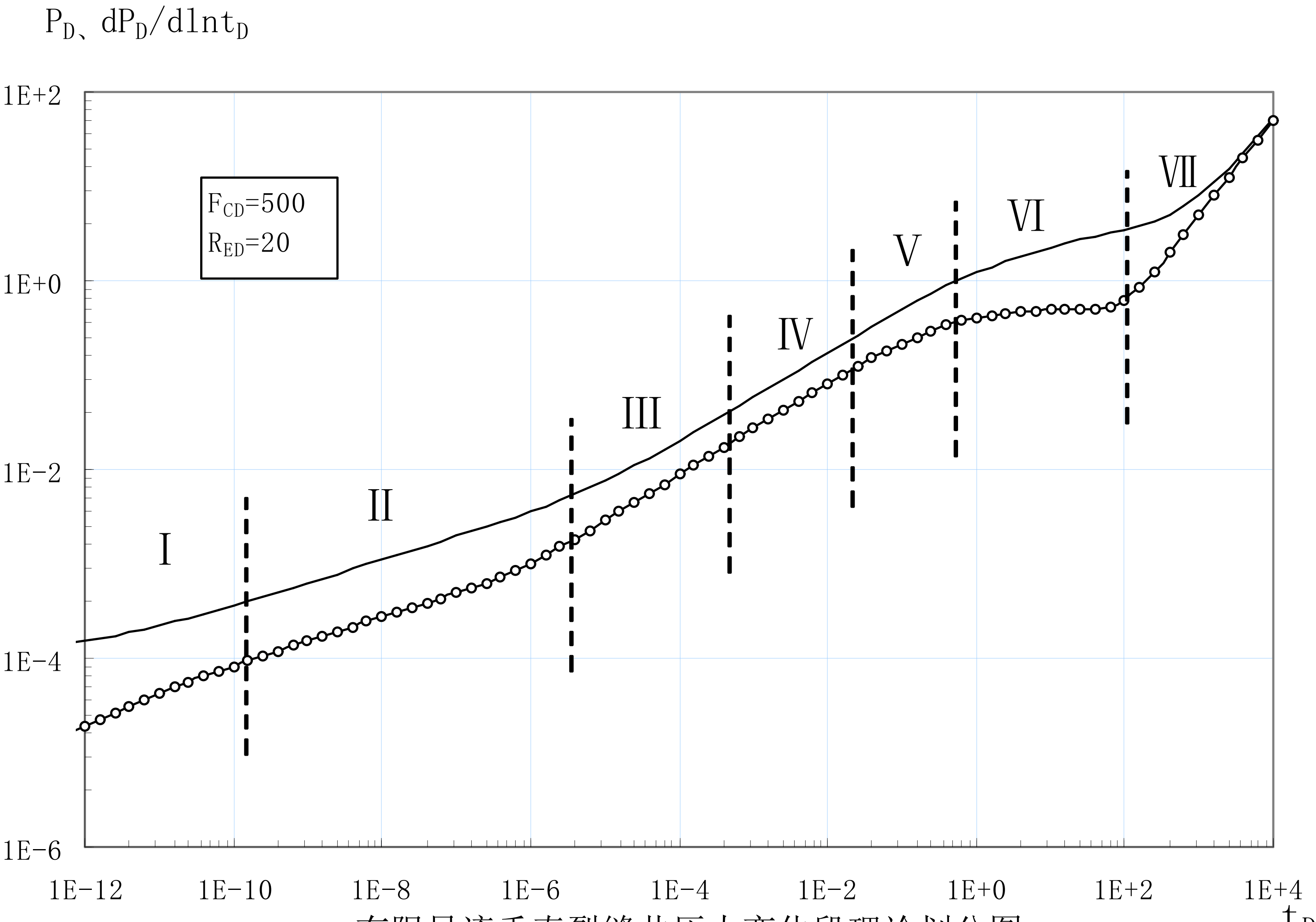
✦ 五、Quasi radial flow

Before the boundary effect occurs, the pseudo-radial flow characteristics may occur in all vertical fractured wells at the later stage. During the quasi-radial flow period under infinite boundary action, the flow rate in the fracture is stable. At this time, the instability of the well is equivalent to that of an uncracked well whose effective radius r'w is enlarged. The skin factor of the radial flow caused by this flow range is only a function of C_{fD}. Before quasi-radial flow, the steady-state skin factor of radial flow caused by vertical cracks should be considered. In the early stage of instability of vertical fracture, the flow rate in the fracture is unstable, and the steady skin factor of radial flow is a function of C_{fD} and time.

$$\boxed{p_{wD} = \frac{1}{2}\left[\ln t_{Dr_w'} + 0.8091\right]}$$

✦ 六、Quasi steady state flow

✦ In the later stage of finite reservoir, the actual completion method has not completely controlled the pressure instability of wells. The dimensionless pressure in the closed system varies with the area and shape of reservoir oil release, the location of wells, formation characteristics and time. The single slope straight line on the log plot of pressure and pressure derivative is used to characterize the quasi-steady-state instability of well development. Reservoir geometry is related to the shape of reservoir drainage area, well location, well type and drainage area as well as the ratio xe/xf of fracture half-length. It can refer to the geometric shape coefficient of vertical fractured wells in reservoir engineering code.（John Wiley and Sons,2002）。

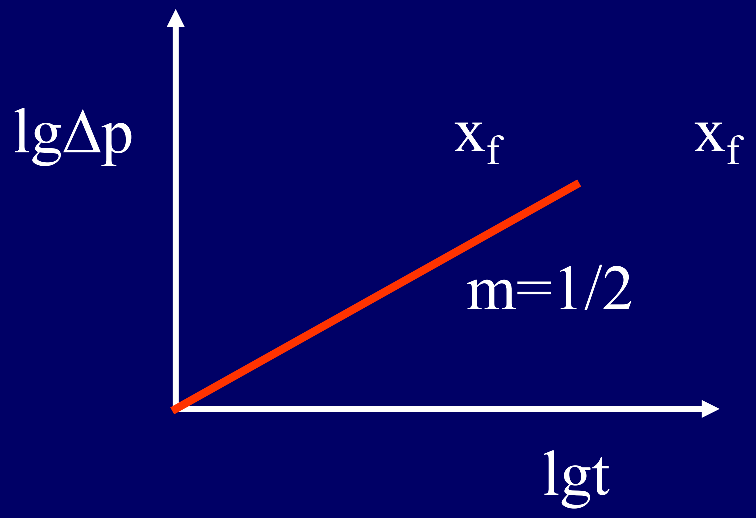


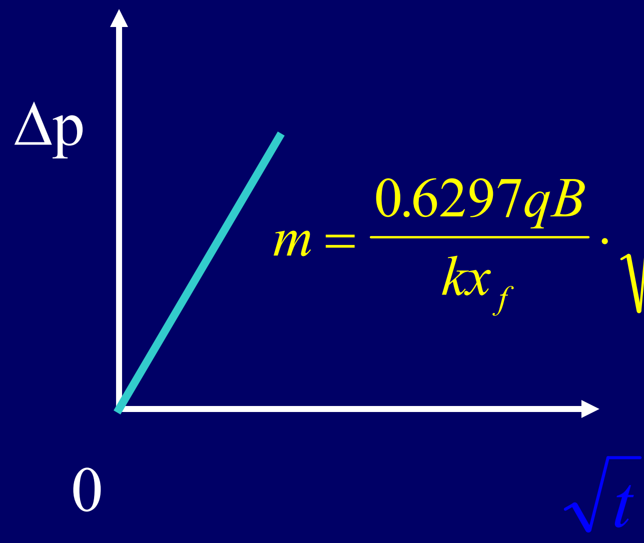
有限导流垂直裂缝井压力变化段理论划分图

一、Vertical Fracture Model of Infinite Conductivity and Pressure Analysis

Early Segment Characteristics

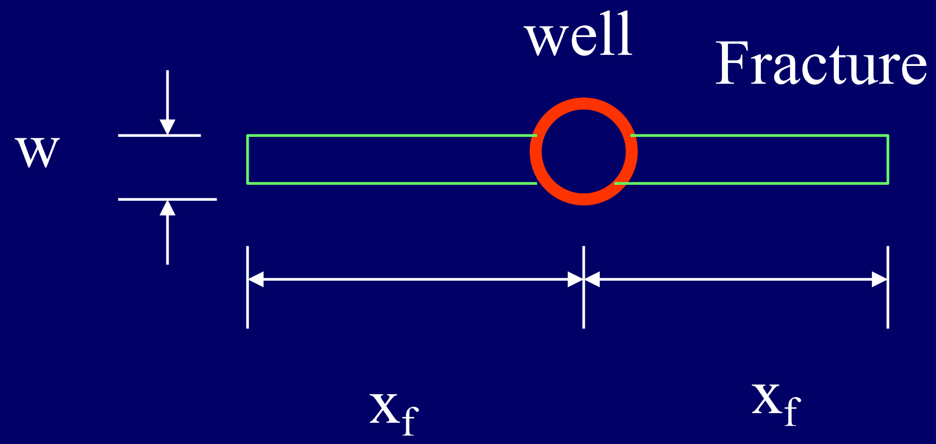
$$\boxed{\lg \Delta p = \frac{1}{2}\lg \Delta t + \lg(\frac{0.6297qB}{kx_f}\cdot\sqrt{\frac{\mu}{\phi C_t k}})}$$





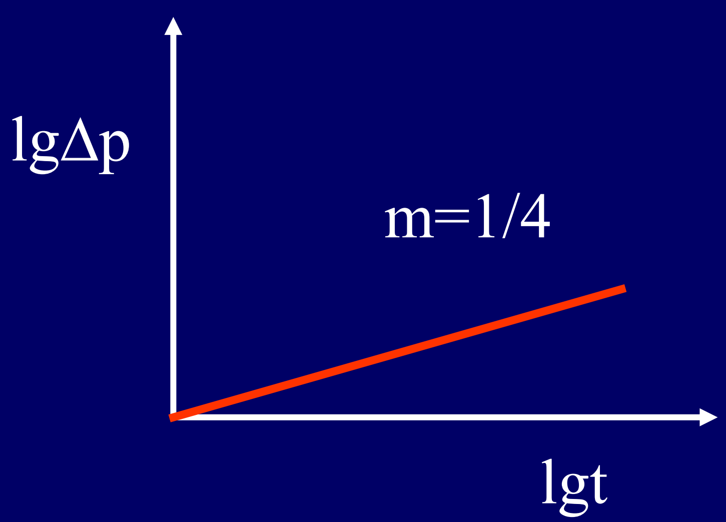
$$m = \frac{0.6297qB}{kx_f}\cdot\sqrt{\frac{\mu}{\phi C_t k}}$$

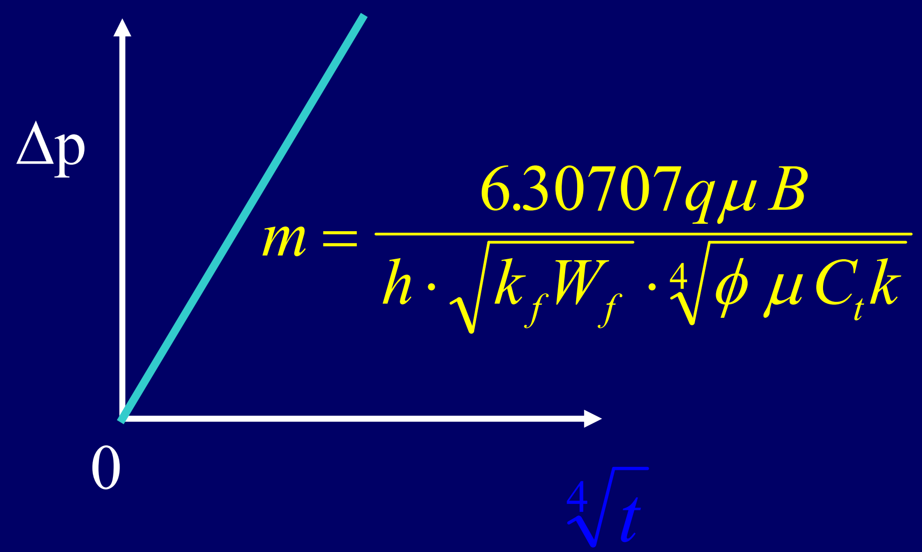
二、Finite Conductivity Vertical Fracture Model and Pressure Distribution



Early Pressure Characteristics of Vertical Fractures with Finite Conductivity

$$\boxed{\lg \Delta p = \frac{1}{4} \lg \Delta t + \lg\left(\frac{6.30707 q \mu B}{h \cdot \sqrt{k_f W_f} \cdot \sqrt[4]{\phi \mu C_t k}}\right)}$$





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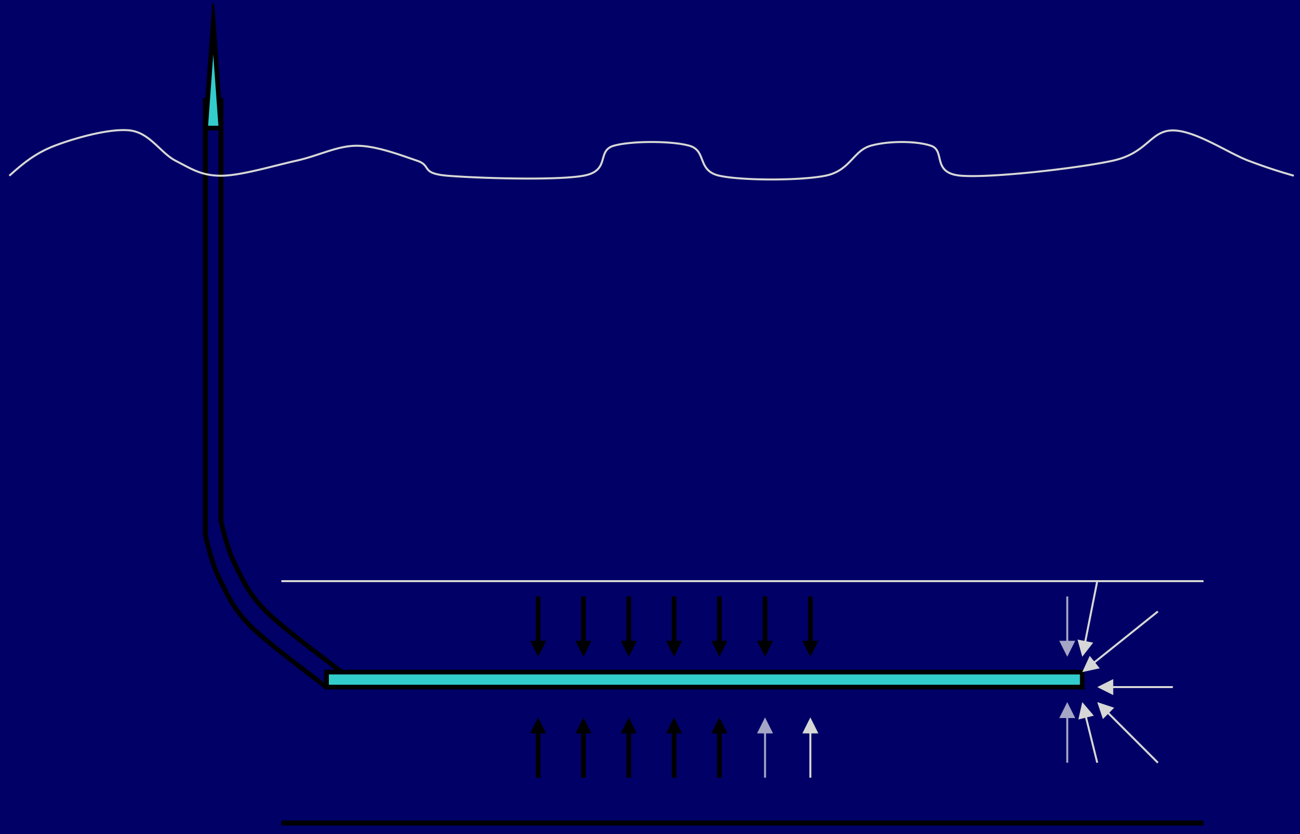
Section 7 Conventional analysis method for horizontal wells in homogeneous reservoirs

- The seepage problem of horizontal wells is more complex than that of vertical fractured wells. The main reason is that the seepage of horizontal wells is greatly affected by length, outer boundary, especially upper and lower boundary. It is a three-dimensional problem. For horizontal wells, it is very important to correctly diagnose the shape of well test curve.

Horizontal well model

- There are radial flow, linear flow and spherical flow in reservoirs near horizontal wells.
- There is a variable mass flow in the wellbore, and the pressure will change.



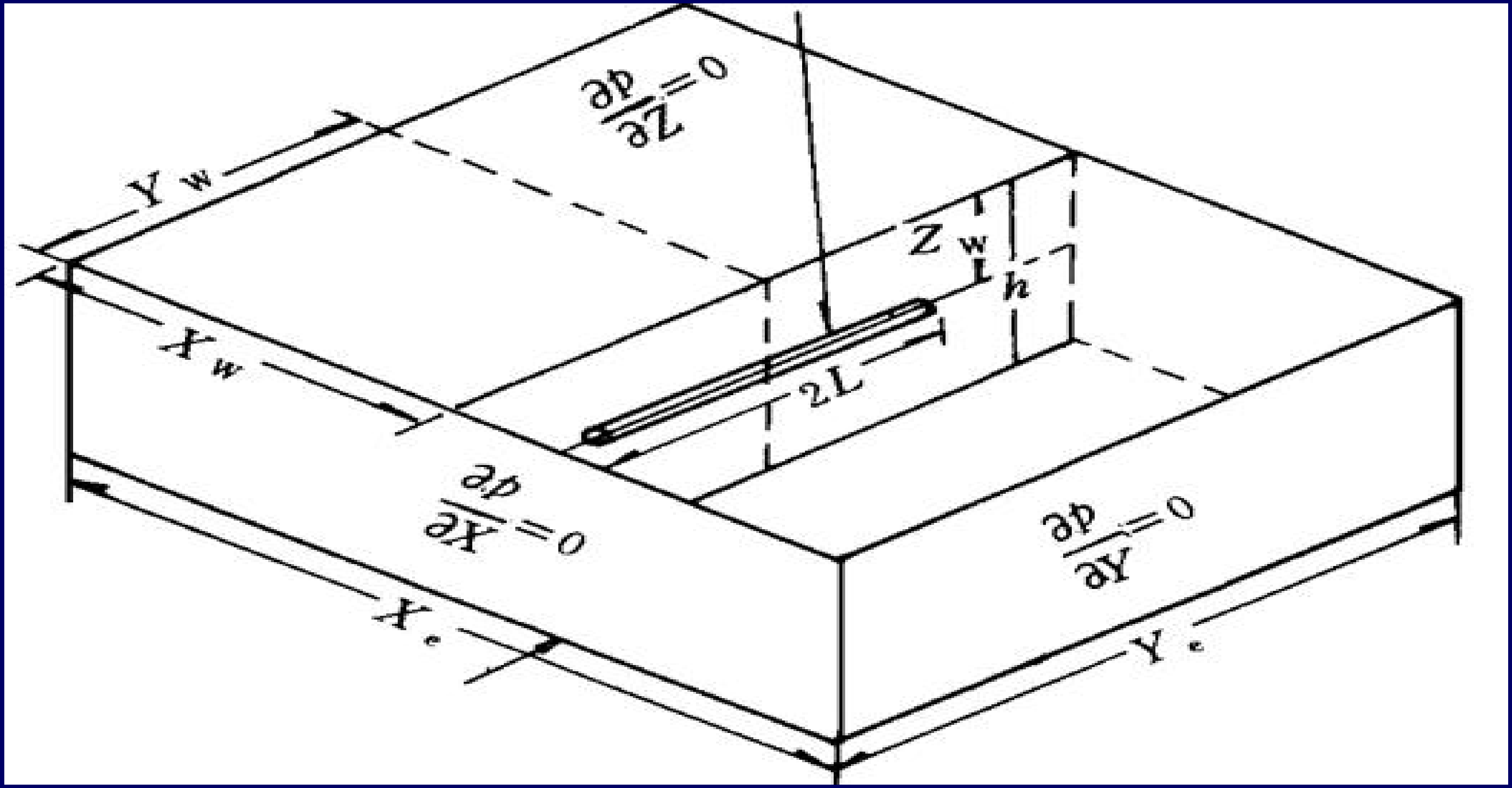


Fig. 1 A sketch map showing horizontal well in box-shaped reservoir

$$\frac{\partial p_D}{\partial t_D} - \left(\frac{\partial^2 p_D}{\partial x_D^2} + \frac{\partial^2 p_D}{\partial y_D^2} + \frac{\partial^2 p_D}{\partial z_D^2} \right) = 2\pi z_{eD} \delta\left(x_D - x_{wD}\right) \delta\left(y_D - y_{wD}\right) \delta\left(z_D - z_{wD}\right)$$

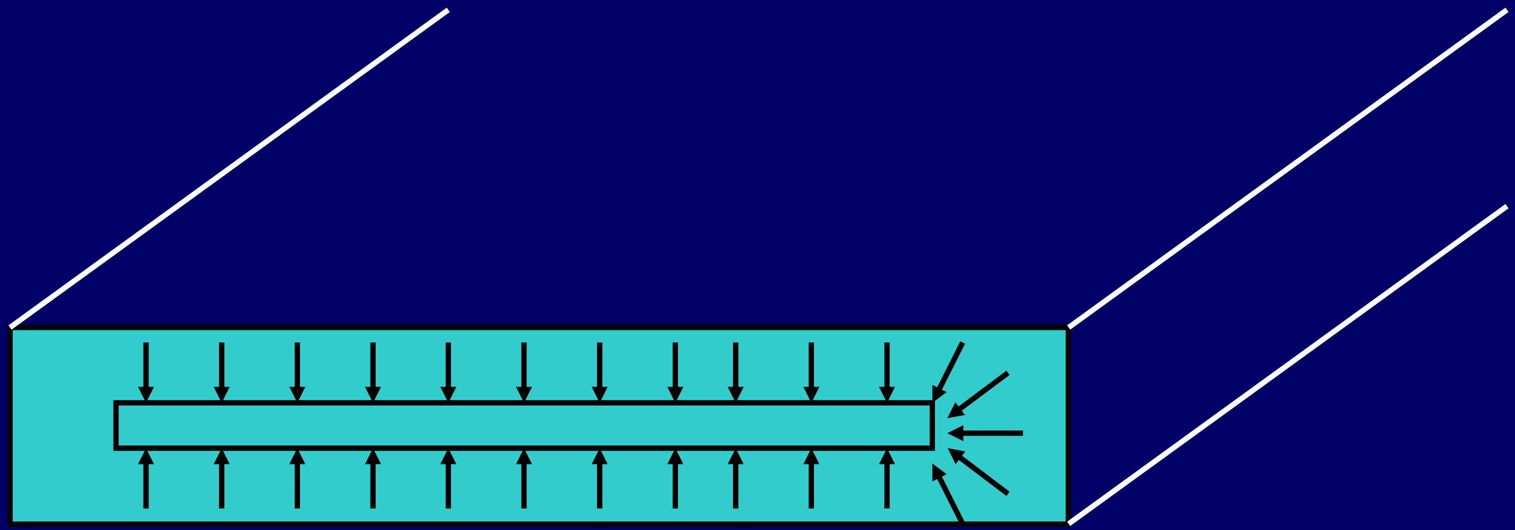
$$\left.\frac{\partial p_D}{\partial x_D}\right|_{x_D=0, x_{eD}} = 0 \text{；} \quad \left.\frac{\partial p_D}{\partial y_D}\right|_{y_D=0, y_{eD}} = 0$$

$$\left.\frac{\partial p_D}{\partial z_D}\right|_{z_D=0, z_{eD}} = 0 \text{；} \quad p_D\left(x_D, y_D, z_D, 0\right) = 0$$

Flow characteristics

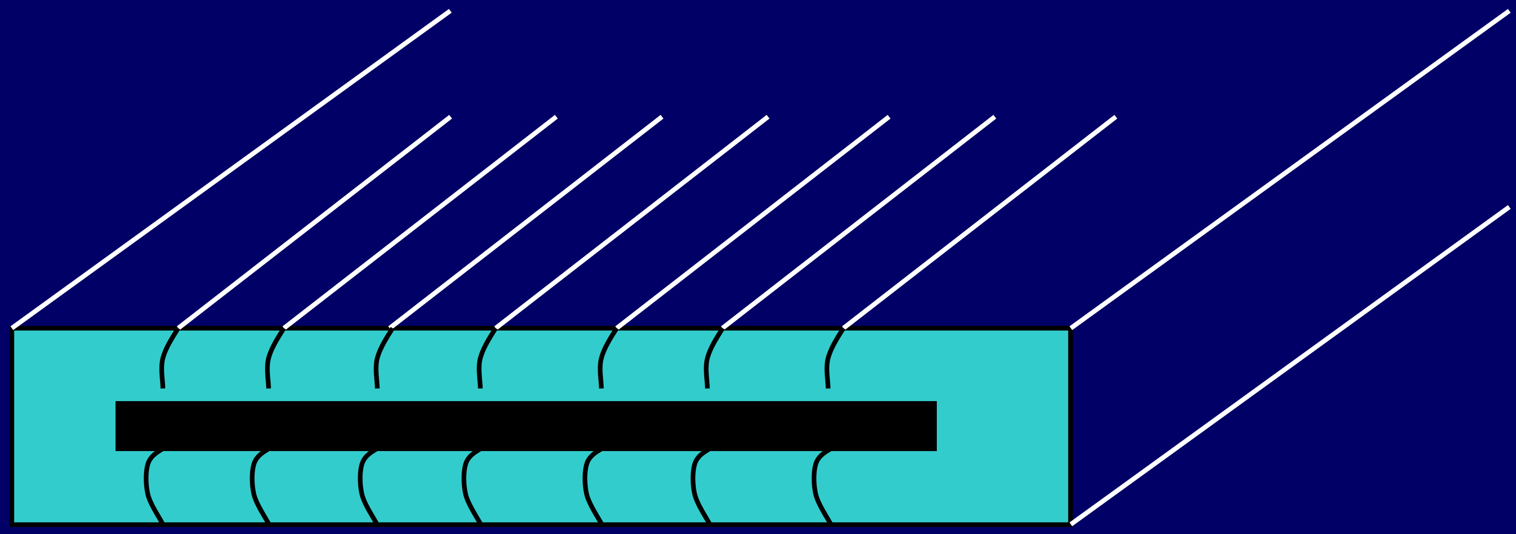
一、Initial radial flow stage

Horizontal wells are just beginning to produce, and the pressure in the wellbore drops suddenly. The fluid around the wellbore first flows into the wellbore, forming an early radial flow in the plane. (in the vertical plane)



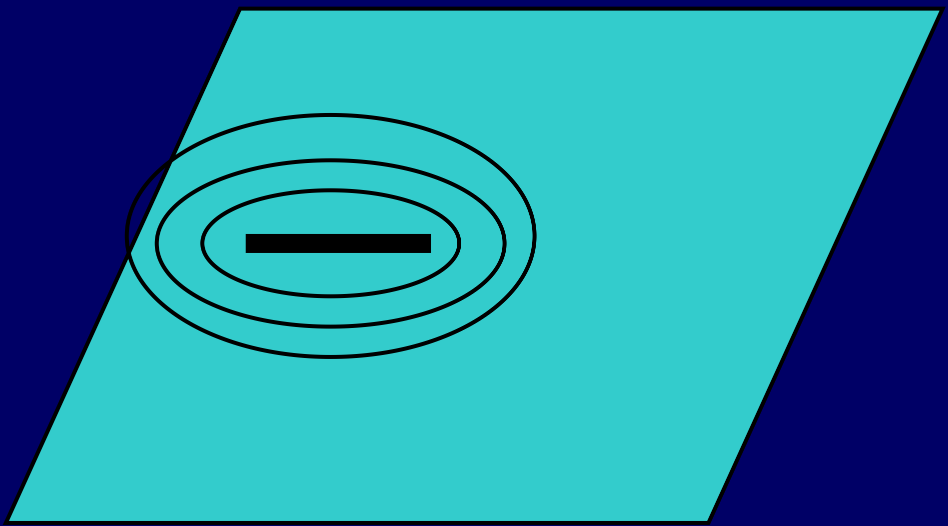
二、Medium-term linear flow stage

As time goes on, the pressure wave reaches the upper and lower boundaries, the radial flow phase disappears, the vertical flow reaches the quasi-steady state, and the horizontal flow plays a major role in the formation of a medium-term linear flow.



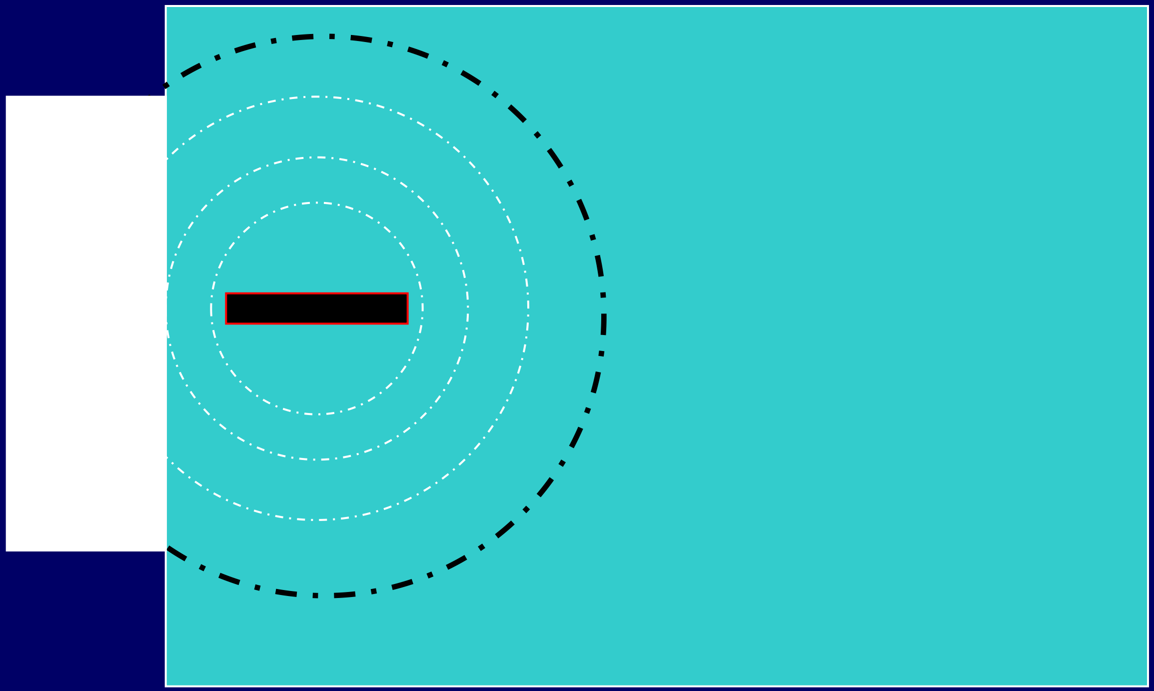
三、Medium-term radial flow stage

As time goes on, the flow range becomes wider and wider. The fluid in the distance is approximately considered to flow radially to the bottom of the well and linear flow appears again in the formation. Called mid-term quasi-radial flow.



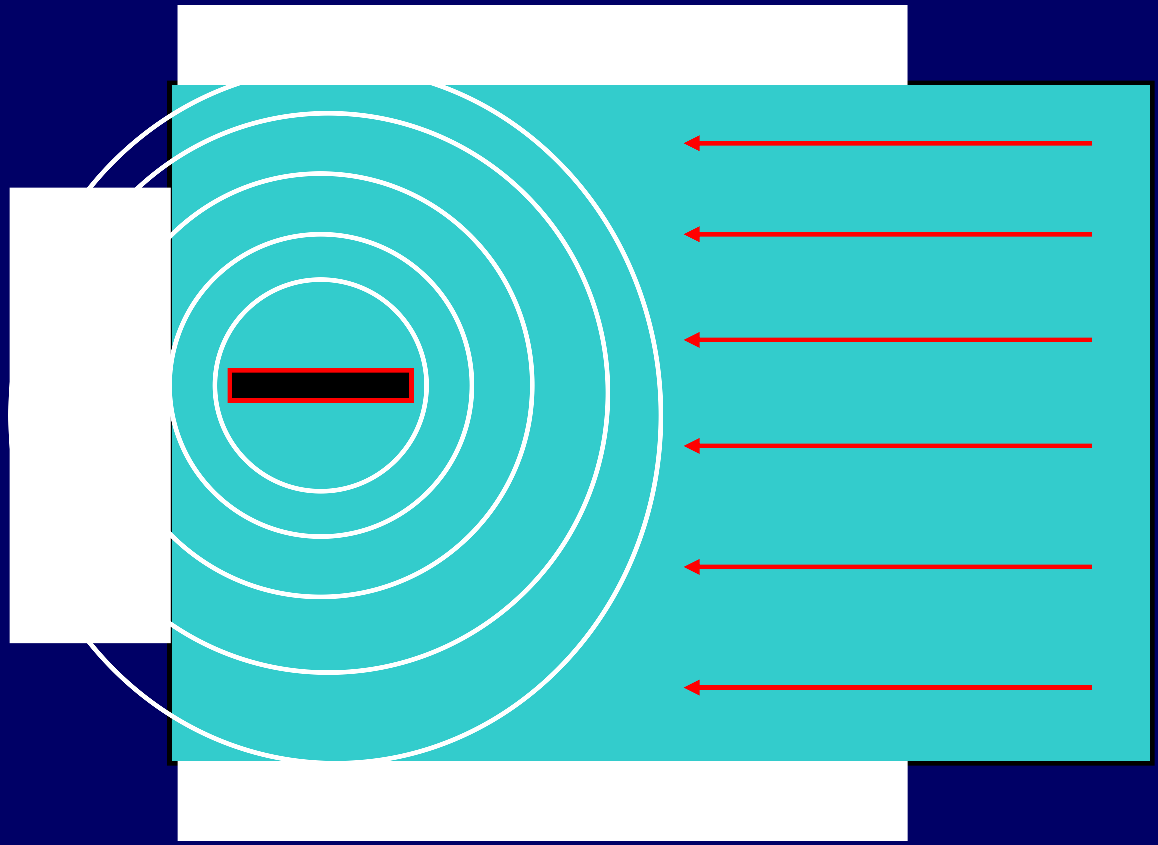
四、Radius-to-flow stage

As time goes on, due to the eccentricity of horizontal wells, when the flow range encounters the nearest vertical boundary, it is constrained and the radius flow appears in the formation.



五、Late linear flow stage

As time goes on, when the range of flow encounters the nearest horizontal boundary, it is restrained and late linearity appears in the formation.



六、Quasi-steady state stage

When all six directions encounter the boundary, they are constrained, and there is a quasi-steady state stage in the formation, with the slope of the straight line being 1.