Advanced Reservoir Engineering

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Chapter 5 Steady-state productivity evaluation of oil wells

Section 1 Productivity evaluation method for vertical wells

Section 2 Productivity evaluation method for horizontal wells

Section 3 Productivity evaluation method for fractured wells

Mathematicalization of science is a main trend of contemporary scientific development. The essence of contemporary high-tech is mathematical technology. Mathematical science can not only quantitatively describe the object of scientific research, but also make theoretical analysis and scientific prediction of practical problems, thus pushing scientific research to a higher stage. **As Marx said, "Only when a science succeeds in applying mathematics can it reach a truly perfect level".** Therefore, in the development and exploitation of oil and gas fields, according to the theory of seepage mechanics, the establishment of mathematical models is the key to solve practical problems and improve development efficiency.

□ Dimensionless variables

Dimensionless is one of the important means to solve engineering problems. Theorems in dimensional analysis are the basis of dimensionless analysis. To transform a practical physical problem into a pure mathematical problem, first of all, we should transform the dimension (physical quantity) into the dimensionless (mathematical quantity), so that all analysis and calculation can be applied to all metric scales.

- **The dimensionless productivity index and production**

$$J_D = \frac{B\mu}{542.87kh}J\,,\quad q_D = \frac{qB\mu}{542.87kh(P_i - P_{wf})}$$

- Dimensionless Pressure of Oil Well

$$P_{jD} = \begin{cases} \dfrac{P_i - P_j}{P_i - P_{wf}} & , \quad P_{wf} = \text{Const.} \\ \dfrac{kh(P_i - P_j)}{1.842 \times 10^{-3} qB\mu} & , \quad q = \text{Const.} \end{cases} , \quad j = avg, wf$$

- Dimensionless Pressure of Gas Well

$$P_D(P) = \left\{ \begin{array}{ll} \dfrac{m(P_i) - m(P_j)}{m(P_i) - m(P_{wf})} & , \quad P_{wf} = \text{Const.} \\ \dfrac{78.55kh[m(P_i) - m(P_j)]}{Tq_{sc}} & , \quad q_g = \text{Const.} \end{array} \right\} , \quad j = avg, wf$$

- Dimensionless time

$$t_D = \frac{3.6kt}{\phi\mu c_t r_w^2} , \quad t_{DL} = \frac{3.6kt}{\phi\mu c_t L^2} , \quad t_{DA} = \frac{3.6kt}{\phi\mu c_t A}$$

- Dimensionless coordinates

$$r_{DL} = \frac{r}{L}, \quad r_{eDL} = \frac{r_e}{L}, \quad r_{wDL} = \frac{r_w}{L};$$

$$z_D = \frac{z}{h}, \quad z_w = \frac{z_w}{h}, \quad z_{rD} = z_{wD} + r_{wD} L_D$$

- Dimensionless Horizontal Well Half Length, Dimensionless Effective Thickness and Anisotropic Factor

$$L_D = \frac{L}{h}\sqrt{\frac{k_v}{k_h}}, \quad h_D = \frac{h}{r_w}\sqrt{\frac{k_h}{k_v}}, \quad \beta = \sqrt{\frac{k_h}{k_v}}$$

Dimensionless results make the mathematical and physical equations simple and neat, and easy to analyze and solve. Dimensionless results are actually helpful to the transformation of physical units and the inspection and verification of seepage differential equations.

□ Productivity index

Muskat (1949), a scholar, first defined the oil production index as the oil production volume per unit production pressure difference in $m^3/d/MPa$, which is expressed by formula as follows:

$$J = \frac{q}{P_{avg} - P_{wf}}$$

Productivity index is an important parameter to measure the production capacity of oil wells. Its more complete expression has been given by Economides (1994):

$$J = -\frac{dq}{dP_{wf}} = \frac{kh}{\alpha_p B\mu(P_{wD} + Skin)}$$

In the formula, α_p is the unit conversion constant. When the dimensionless bottom hole pressure is known, the productivity index of the well can be calculated directly by using the above formula.

Specific productivity index

For the convenience of analysis and calculation, the specific productivity index Jsp is defined as

$$J_{sp} = \frac{q\mu B}{542.87kh(P_e - P_w)}$$

Equivalent radius

Equivalent diameter of ordinary perfect vertical wells is equal to original diameter, while that of ordinary imperfect vertical wells (for example, in case of formation damage near wells) can be expressed by the following formula:

$$r_{equ} = r_w \exp(-S)$$

The specific production value can be calculated by the corresponding production formula of vertical wells.

Average formation pressure

For any type of well, if the volume of wellbore is V_w in the closed volume V_R range, the integral average of pressure distribution is the current average formation pressure, expressed by P_{avg}, and the formula is as follows:

$$P_{avg} = \frac{1}{V_R}\int_{V_w}^{V_R} P(x,y,z)\,dV$$

If the plane radial seepage occurs in the formation, the above formula can be written as follows：

$$P_{avg} = \frac{2}{r_e^2 - r_w^2}\int_{r_w}^{r_e} rP(r)\,dr$$

According to the principle of material balance, for flat-plate formation, if ρ_{avg} is the spatial average of fluid density, the average thickness of medium is h, and the area of formation drainage is A, there are:

$$V\frac{d}{dt}\left(\phi\rho_{avg}\right)=-(qB\rho)_{w}$$

$$\rho_{avg}=\rho_{w}\exp[c_{l}(P_{avg}-P_{w})]$$

After launching, the following conclusions can be obtained:

$$V\rho_{avg}\phi\left(\frac{1}{\rho_{avg}}\frac{d\rho_{avg}}{dP_{avg}}+\frac{1}{\phi}\frac{d\phi}{dP_{avg}}\right)\frac{dP_{avg}}{dt}=V\phi\rho_{avg}c_t\frac{dP_{avg}}{dt}=-(qB\rho)_w$$

$$\frac{dP_{avg}}{dt}=-\frac{(qB\rho)_w}{V\phi\rho_{avg}c_t}=-\frac{(qB\rho)_w}{Ah\rho_{avg}c_t}$$

We have

$$\frac{dP_{avg}}{dt}=-\frac{(qB\rho)_w}{V\phi\rho_{avg}c_t}=-\frac{qB\exp[-c_t(P_{avg}-P_w)]}{Ah\phi c_t}\approx-\frac{qB}{Ah\phi c_t}$$

$$\frac{2\pi kh(P_i-P_{avg})}{qB\mu}=\frac{2\pi kt}{A\phi\mu c_t}$$

According to the relevant definition of dimensionless, the dimensionless expression of average formation pressure is as follows:

$$P_{avgD}=2\pi t_D\frac{r_w^2}{A}=2\pi t_{DA}$$

一、Classification of productivity analysis methods

(1) Statistical analysis method

(2) Empirical Analysis Method

(3) Analysis Method of Theoretical Model

一、Classification of productivity analysis methods

(1) Statistical analysis method

Statistical analysis method is based on the actual production data of production wells, using some reference distribution rules, or using neural network method to establish models, and then analysis and prediction.

(2) Empirical Analysis Method

Empirical analysis method is to extend the productivity formulas in some simple cases to a certain extent and apply them to complex cases, so as to obtain some regression formulas.

(3) Analysis Method of Theoretical Model

The method of theoretical model analysis is based on seepage theory, which establishes a mathematical model describing fluid seepage.

一、Production Calculation of Ordinary Vertical Well in Circular Homogeneous Reservoir

(1) Steady and quasi-steady seepage

According to the theory of unstable seepage, when the pressure drop funnel reaches the boundary at a certain time t_B, the boundary will produce supply. At this time, the production of oil wells will be composed of two parts, Q_1 is the fluid flowing into the formation from the boundary, and Q_2 is the fluid expanding elastically within the boundary, and gradually decreases. When $q = q_1$, steady seepage will occur in the formation.

If the pressure drop funnel reaches the boundary at a certain time t_B, the formation will fail because the closed boundary can not produce fluid supply ($q_1 = 0$), and the pressure drop velocities at all points in the formation are equal in the process of failure, which indicates that the energy dissipation velocity is uniform, and then the formation will undergo quasi-steady flow. After quasi-steady flow occurs, dissolved gas flooding may occur in oil wells, while dry gas wells can continue to produce until the energy is exhausted.

(2) Steady-state productivity equation

Considering a perfect vertical well, fixed bottom hole production, constant boundary pressure, and plane radial steady Darcy seepage, the seepage mathematical model is as follows:

Governing equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dP}{dr}\right)=0$$

Inner boundary

$$\lim_{r\to r_w}\left(r\frac{dP}{dr}\right)=\frac{qB_o\mu}{2\pi kh}$$

Outer boundary

$$P=P_e\text{，}\quad r=r_e$$

Formulas of formation pressure distribution, average formation pressure and steady-state production can be easily obtained by solving the above model.

$$P(r)=P_e-\frac{qB_o\mu}{2\pi kh}\ln\frac{r_e}{r}$$

Average pressure:

$$P_{avg} = P_e - \frac{qB_o\mu}{2\pi kh}\left(\frac{1}{2} - \frac{r_w^2}{r_e^2 - r_w^2}\ln\frac{r_e}{r_w}\right) \approx P_e - \frac{qB_o\mu}{4\pi kh}$$

The steady-state productivity equation considering the effect of skin factor S is as follows:

$$q_{vss} = \frac{2\pi kh(P_{avg} - P_{wf})}{\mu B\left(\ln\frac{r_e}{r_w} - \frac{1}{2} + S\right)} \text{—Darcy}$$

According to the definition of oil productivity index, the steady state oil productivity index of ordinary vertical wells is as follows:

$$J_{vss} = \frac{542.87kh}{\mu B\left(\ln\frac{r_e}{r_w} - \frac{1}{2} + Skin\right)}$$

(3) Quasi-steady-state productivity equation

According to the quasi-steady seepage characteristics, the governing equation of plane radial seepage has the following simplified forms:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dP}{dr}\right)=\frac{\phi\mu c_t}{k_0}\frac{dP_{avg}}{dt}=-\frac{\phi\mu c_t}{k_0}\frac{qB}{Ah\phi c_t}=-\frac{\mu}{k_0}\frac{qB}{Ah}$$

$$\frac{dP_{avg}}{dt}=-\frac{(qB\rho)_w}{V\phi\rho_{avg}c_t}=-\frac{qB\exp[-c_t(P_{avg}-P_w)]}{Ah\phi c_t}\approx-\frac{qB}{Ah\phi c_t}$$

Its dimensionless form is:

$$\frac{1}{r_D}\frac{d}{dr_D}\left(r_D\frac{dP_D}{dr_D}\right)=\frac{dP_{avgD}}{dt_D}=\frac{2\pi r_w^2}{A}=\frac{2}{r_{eD}^2-1}$$

$$\left[r_D\frac{dP_D}{dr_D}\right]_{r_D=1}=-1$$

$$\left[\frac{dP_D}{dr_D}\right]_{r_D=r_{eD}}=0$$

External boundary condition integral method is applied to solve the problem.

$$r_D \frac{dP_D}{dr_D} = \frac{r_D^2 - r_{eD}^2}{r_{eD}^2 - 1} \; ; \quad P(r_D, t_D) = P_{wD}(t_D) + \frac{1}{r_{eD}^2 - 1}\left(\frac{r_D^2 - 1}{2} - r_{eD}^2 \ln r_D \right)$$

The average pressure is:

$$P_{avg}(t_D) - P_{wD}(t_D) = \frac{r_{eD}^2}{(r_{eD}^2 - 1)} - \frac{r_{eD}^4 \ln r_{eD}}{(r_{eD}^2 - 1)^2} - \frac{r_{eD}^4 - 1}{4(r_{eD}^2 - 1)^2} = \frac{3r_{eD}^2 - 1}{4(r_{eD}^2 - 1)} - \frac{r_{eD}^4 \ln r_{eD}}{(r_{eD}^2 - 1)^2}$$

The $r_{eD} >> 1$ condition is applied to obtain:

$$P_{wD}(t_D) = P_{avg}(t_D) + \ln r_{eD} - \tfrac{3}{4}$$

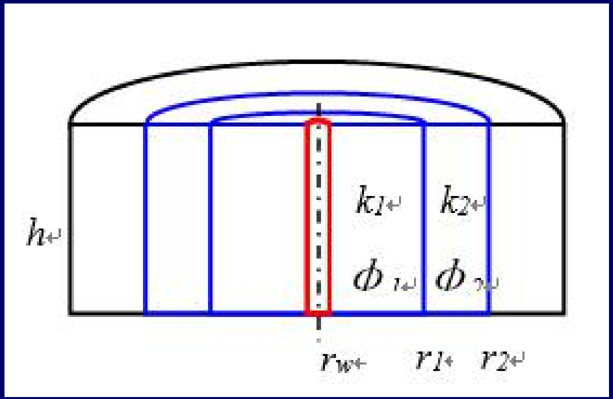
Quasi-steady-state productivity equation is obtained by dimensionalization.

$$q = \frac{2\pi kh[P_{avg}(t) - P_{wf}]}{\mu B\left(\ln \frac{r_e}{r_w} - \frac{3}{4} \right)}$$

二、Production Calculation of Ordinary Vertical Wells in Planar Composite Reservoirs

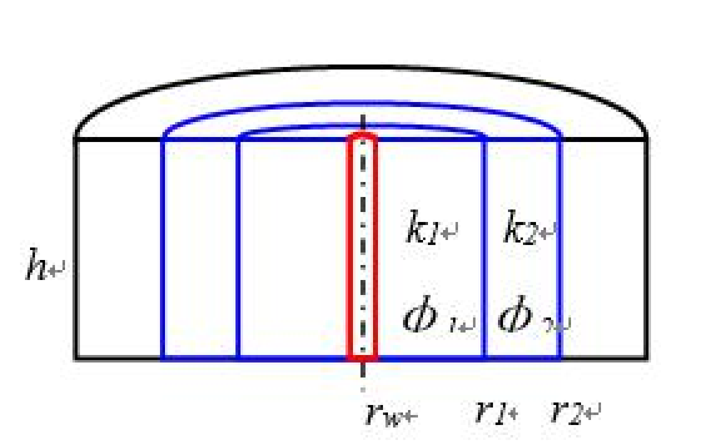
(1) Steady-state productivity equation

For the sake of generality, we first consider a horizontal heterogeneous formation with H thickness and N regions. Its permeability and porosity are k_i and φ_i, radius is r_i, and the supply pressure is P_e on the outer boundary of radius is re. The radius of a single well with constant production q is r_w, and the incompressible fluid i n the formation follows Darcy's law to produce stable plane radial seepage. The flow physical model is shown in the figure.



The governing equation of steady seepage is:

$$\frac{1}{r}\frac{d}{dr}\left[k(r)\cdot r\frac{dP}{dr}\right]=0$$



The physical equation and internal boundary can be written as follows:

$$\phi=\phi_j,\quad k=k_j,\quad r_{i-1}<r<r_i,\quad r_0=r_w,\quad r_n=r_e,\quad i=1,2,\Lambda,n$$

$$\left[r\frac{\partial P(r)}{\partial r}\right]_{r=r_w}=\frac{qB\mu}{2\pi kh}$$

According to the boundary conditions, the pressure distribution is obtained firstly.

$$P(r)=P_e-\frac{qB\mu}{2\pi h}\left(\int_{r_w}^{r_e}\frac{1}{rk(r)}dr-\int_{r_w}^{r}\frac{1}{rk(r)}dr\right)=P_e-\frac{qB\mu}{2\pi h}\int_{r}^{r_e}\frac{1}{rk(r)}dr$$

Substituting the physical property equation, the pressure distribution is as follows:

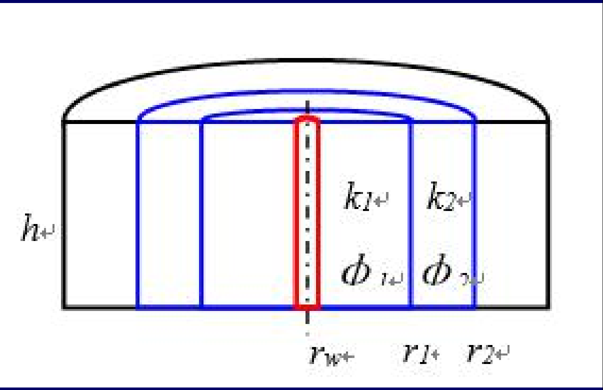
$$p_j(r)=p_e-\frac{qB\mu}{2\pi h}\left[\frac{1}{k_j}\int_{r}^{r_j}\frac{1}{r}dr+\sum_{i=j+1}^{N}\frac{1}{k_i}\int_{i-1}^{i}\frac{1}{r}dr\right]\qquad r_{j-1}<r<r_j$$

Compound reservoirs for two regions（$N=2$）：

$$P_1(r)=P_e-\frac{qB\mu}{2\pi h}\left(\frac{1}{k_2}\ln\frac{r_e}{r_m}+\frac{1}{k_1}\ln\frac{r_m}{r}\right)$$

$$P_2(r)=P_e-\frac{qB\mu}{2\pi k_2 h}\ln\frac{r_e}{r}$$

$$P_{avg}=\frac{2}{r_e^2-r_w^2}\int_{r_w}^{r_e} rP(r)dr$$



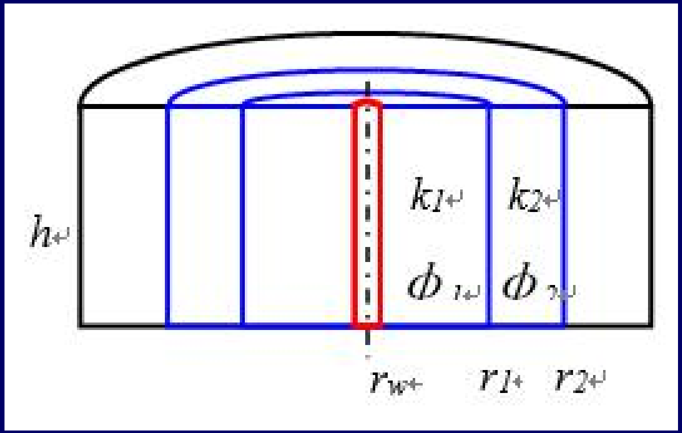
$$P_{avg}=P_e-\frac{qB\mu}{2\pi h}\left(\frac{k_1r_e^2-k_2r_w^2+(k_2-k_1)r_m^2}{2k_1k_2(r_e^2-r_w^2)}-\frac{r_w^2}{r_e^2-r_w^2}\left(\frac{1}{k_2}\ln\frac{r_e}{r_m}+\frac{1}{k_1}\ln\frac{r_m}{r_w}\right)\right)$$

Using the condition of re > > rw:

$$P_{avg}\approx P_e-\frac{qB\mu}{2\pi h}\left(\frac{k_1r_e^2+(k_2-k_1)r_m^2}{2k_1k_2r_e^2}\right)$$

If the bottom hole pressure is known to be Pwf, the condition of re >> rw is used:

$$q \approx \frac{2\pi h(P_{avg} - P_{wf})}{B\mu\left[\left(\frac{1}{k_2}\ln\frac{r_e}{r_m} + \frac{1}{k_1}\ln\frac{r_m}{r_w}\right) - \frac{k_1 r_e^2 + (k_2 - k_1) r_m^2}{2k_1 k_2 r_e^2}\right]}$$



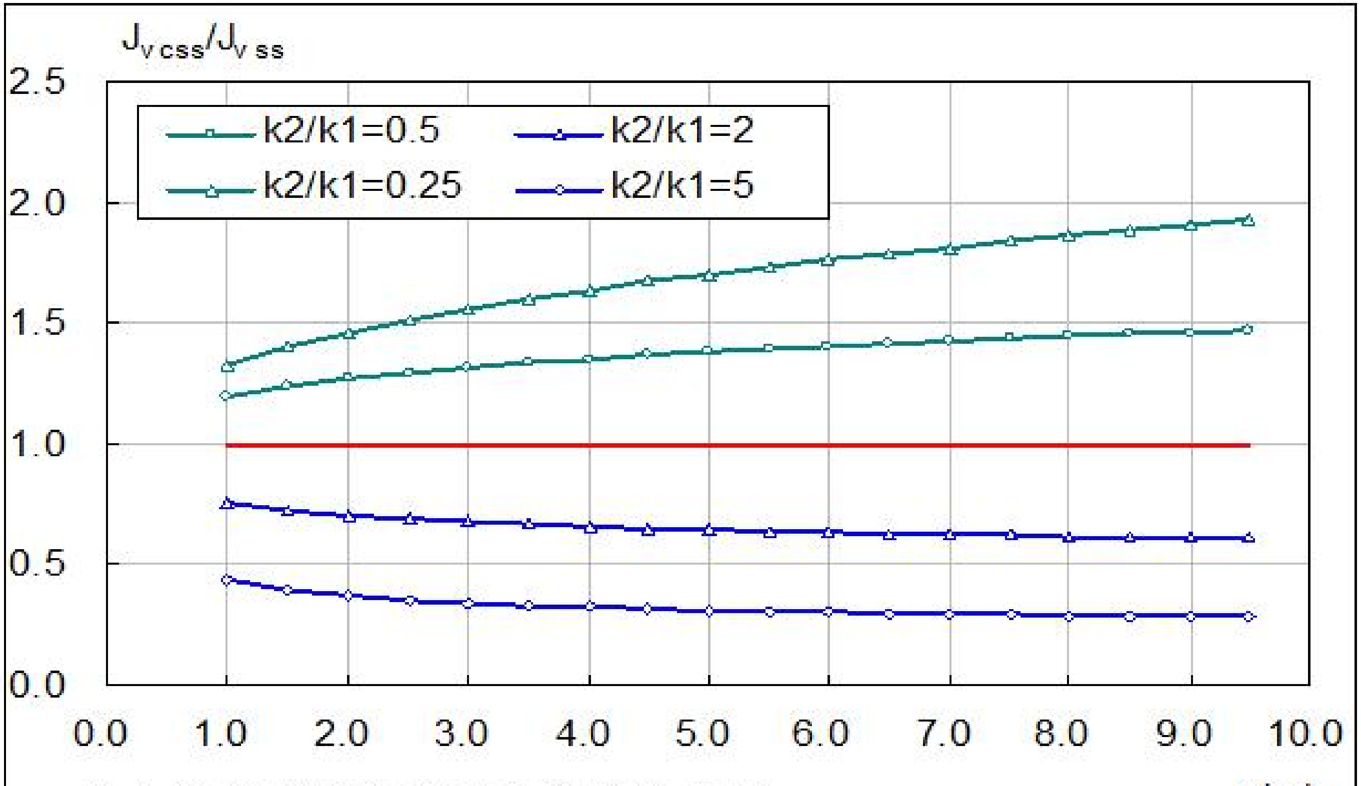
Accordingly, the steady-state productivity index is (SI unit system):

$$J_{vcss} \approx \frac{542.87h}{B\mu\left[\left(\frac{1}{k_2}\ln\frac{r_e}{r_m} + \frac{1}{k_1}\ln\frac{r_m}{r_w}\right) - \frac{k_1 r_e^2 + (k_2 - k_1) r_m^2}{2k_1 k_2 r_e^2}\right]}$$

In order to investigate the influence characteristics of main parameters of composite formation, without considering the epidermal factors, the following oil recovery index ratios are made, where $K_2 = k$, the results are as follows:

$$\frac{J_{vcss}}{J_{vss}} \approx \frac{\ln(r_e / r_w) - 1/2}{\ln\frac{r_e}{r_w} - \frac{1}{2} + \left(\frac{k_2}{k_1} - 1\right)\ln\frac{r_m}{r_w} - \frac{1}{2}\left(\frac{k_2}{k_1} - 1\right)\frac{r_m^2}{r_e^2}}$$

（1）If the radius of drainage is r_e= 200m, the radius of oil well is r_w = 0.1m and the permeability ratio k_2/k_1 is 0.25, 0.5, 1.0, 2.0 and 5.0 respectively, the composite radius r_m will change. The calculation results are shown in the figure.



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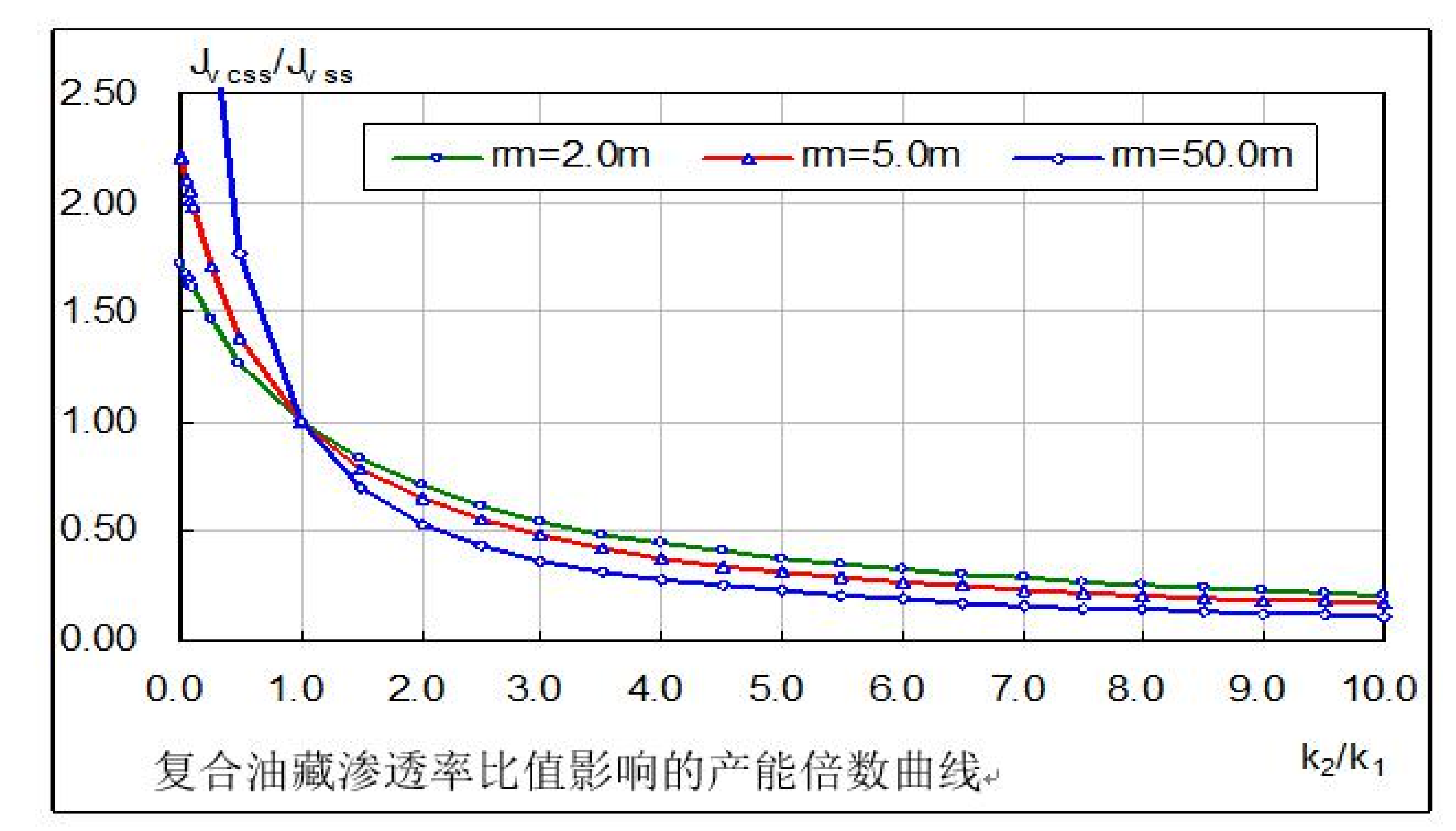
(2) If the radius of drainage is r_e= 200m and the radius of oil well is r_w = 0.1m, the composite radius r_m is 2.0m, 5.0m and 50.0m, respectively, and the permeability ratio k_2/k_1 is changed. The calculation results are shown in the figure.

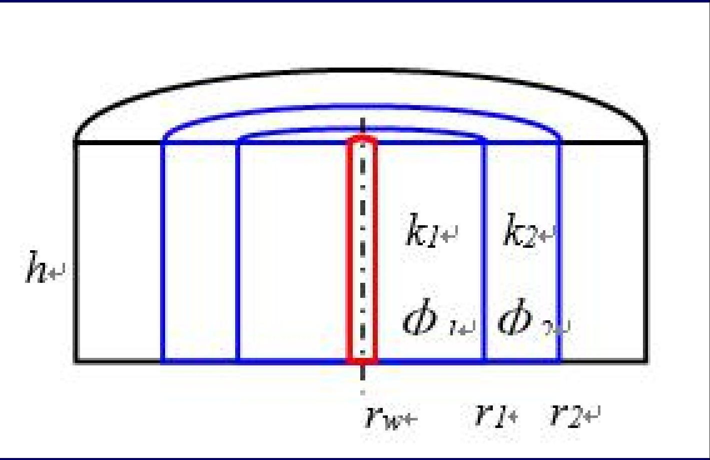


(2) Quasi-steady-state productivity equation

The governing equations of quasi-steady seepage flow are as follows:

$$\frac{1}{r}\frac{d}{dr}\left(k_D(r)\cdot r\frac{dP}{dr}\right)=\frac{\phi\mu c_t}{k_2}\frac{dP_{avg}}{dt}=-\frac{\phi\mu c_t}{k_2}\frac{qB}{Ah\phi c_t}=-\frac{\mu}{k_2}\frac{qB}{Ah}$$

$$k_D(r)=\begin{cases}k_1/k_2 & , \quad r_D\le r_{mD}\\ 1 & , \quad r_D>r_{mD}\end{cases}$$



Its corresponding dimensionless form:

$$\frac{1}{r_D}\frac{d}{dr_D}\left(k_D(r_D)\cdot r_D\frac{dP_D}{dr_D}\right)=\frac{dP_{avgD}}{dt_D}=\frac{2\pi r_w^2}{A}=\frac{2}{r_{eD}^2-1}$$

$$\left[k_D(r_D)r_D\frac{dP_D}{dr_D}\right]_{r_D=1}=-1$$

$$\left[\frac{dP_D}{dr_D}\right]_{r_D=r_{eD}}=0$$

If $r < r_m$ is considered, the pressure distribution formula can be written as follows:

$$P(r_D,t_D)-P_{wD}(t_D)=\frac{1}{r_{eD}^2-1}\int_1^{r_D}k_D^{-1}(v)\left(v-\frac{r_{eD}^2}{v}\right)dv$$
$$=\frac{k_1/k_0}{r_{eD}^2-1}\int_1^{r_D}\left(v-\frac{r_{eD}^2}{v}\right)dv=\frac{k_1/k_0}{r_{eD}^2-1}\left(\frac{r_D^2-1}{2}-r_{eD}^2\ln r_D\right)$$

If $r > r_m$ is considered, the pressure distribution formula is as follows:

$$P_2(r_D,t_D)-P_{wD}(t_D)=\frac{1}{r_{eD}^2-1}\int_1^{r_D}k_D^{-1}(v)\left(v-\frac{r_{eD}^2}{v}\right)dv$$
$$=\frac{1}{r_{eD}^2-1}\left[\frac{k_1}{k_2}\left(\frac{r_{mD}^2-1}{2}-r_{eD}^2\ln r_{mD}\right)+\left(\frac{r_D^2-r_{mD}^2}{2}-r_{eD}^2\ln\frac{r_D}{r_{mD}}\right)\right]$$

The average pressure is:

$$P_{avgD}(t_D) - P_{wD}(t_D) = \frac{1}{(r_{eD}^2 - 1)^2}\left[\int_1^{r_{eD}} k_D^{-1}(v)\left(2r_{eD}^2 v - \frac{r_{eD}^4}{v} - v^3\right)dv\right]$$

$$= \left(\frac{k_2}{k_1} - 1\right)\left(\frac{r_{eD}^2(r_{mD}^2 - 1)}{(r_{eD}^2 - 1)^2} - \frac{r_{eD}^4 \ln r_{mD}}{(r_{eD}^2 - 1)^2} - \frac{r_{mD}^4 - 1}{4(r_{eD}^2 - 1)^2}\right)$$

$$+ \frac{3r_{eD}^2 + 1}{4(r_{eD}^2 - 1)} - \frac{r_{eD}^4 \ln r_{eD}}{(r_{eD}^2 - 1)^2}$$

When $r_e >> r_w$, $r_e >> 10r_m$, we have:

$$P_{avgD}(t_D) - P_{wD}(t_D) = \left(\frac{k_2}{k_1} - 1\right)\left(\frac{r_{mD}^2}{r_{eD}^2} - \ln r_{mD} - \frac{r_{mD}^4}{4r_{eD}^4}\right) + \frac{3}{4} - \ln r_{eD}$$

$$\approx \left(1 - \frac{k_2}{k_1}\right)\ln r_{mD} + \frac{3}{4} - \ln r_{eD}$$

Dimensionalization of the above formula yields the productivity equation as follows:

$$q = \frac{2\pi k_2 h[P_{avg}(t) - P_w(t)]}{\mu B\left[\left(\frac{k_2}{k_1} - 1\right)\ln\frac{r_m}{r_w} + \ln\frac{r_e}{r_w} - \frac{3}{4}\right]}$$

The pseudo-steady-state productivity evaluation method of composite formation is given. These results can be used to evaluate the effectiveness of acidizing measures.

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一、Research status

Mepky o(1958) of the former Soviet Union first proposed an analytical formula for calculating horizontal well production.

Борисов（1964）published the theoretical monograph "Developing Oilfields with Horizontal and Inclined Wells". It systematically summarized the development history and production principle of Horizontal and Inclined Wells, and put forward the formula for calculating steady flow production of Horizontal Wells, but did not report its detailed derivation process.

In the 1980s, foreign scholars Giger (1984) and Jourdan (1984) took the lead in using electrical simulation to study the engineering principle of horizontal well reservoirs. On the basis of previous research results, they deduced the production calculation formula of horizontal wells.

Joshi (1987), an American scholar, further expounded the production principle of horizontal wells by electric simulation, and deduced the steady-state production calculation of horizontal wells in detail.

Babu (1989) et al. first proposed a horizontal well production formula for calculating quasi-steady flow in finite reservoirs by solving Green function of asymptotic unstable seepage in horizontal wells.

Liu Ciqun (1991), a Chinese scholar, took the lead in meaningfully studying the production formula of horizontal wells by using quasi-three-dimensional method, which aroused the interest of other domestic scholars. A number of research topics based on this method were launched immediately (Fan Zifei, 1993; Cheng Linsong, 1994, Wang Demin, 1995, Wang Xiaodong, 1998).

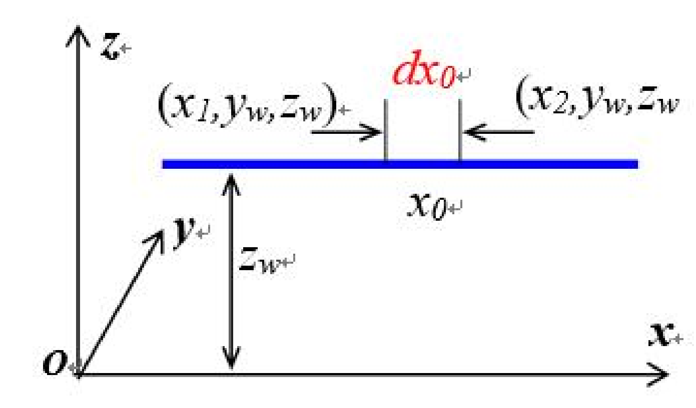
In recent years, the economic advantages of horizontal well production technology have been noticeable. However, there are many facts that the actual production is far from the predicted production, and the research on horizontal well productivity in low permeability, pressure sensitive reservoirs and other situations has not even been paid attention to. This naturally leads to the intensification of research on horizontal well reservoir engineering and horizontal well production calculation formula.

At present, a few productivity calculation formulas for horizontal wells are being applied in practice. But in order to make it more close to the actual production of oil field, more complex factors must be studied. Many oilfields and relevant departments in China have attached great importance to Zizi and made gratifying progress.

二、Three Dimensional Potential Field of Horizontal Well with Uniform Flow in Unbounded Homogeneous Formation

For homogeneous isotropic unbounded formation, the potential function satisfies the Laplace equation under Darcy steady seepage condition.

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0\ ,\quad \Phi=\frac{k}{\mu}P$$



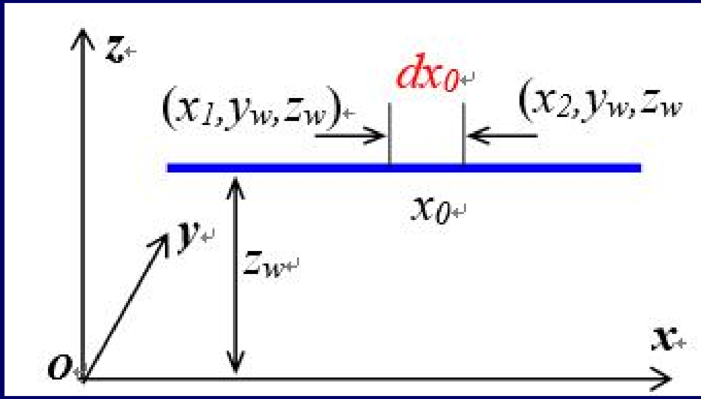
Horizontal wells are assumed to be homogeneous sinks, with output of Q and output of q/L per unit length. Take a micro element dx0 at x0 on the online sink and regard it as a spatial point source. The velocity potential of any point (x, y, z) in the stratum is (Lv Jin, 1994):

$$d\Phi=-\frac{q}{4\pi L}\frac{dx_0}{\sqrt{(x-x_0)^2+(y-y_w)^2+(z-z_w)^2}}$$

$$\frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{d\Phi}{dR}\right)=0$$

$$4\pi R^2\frac{d\Phi}{dR}=\frac{q}{L}dx_0$$

According to the principle of potential superposition, the velocity potential caused by the whole horizontal well is as follows:



$$\Phi = \int_L d\Phi = -\frac{q}{4\pi L}\int_{x1}^{x2}\frac{dx_0}{\sqrt{(x-x_0)^2+(y-y_w)^2+(z-z_w)^2}} + C$$

$$= \frac{q}{4\pi L}\int_{x-x_1}^{x-x_2}\frac{dv}{\sqrt{v^2+(y-y_w)^2+(z-z_w)^2}} + C$$

$$= \frac{q}{4\pi L}\left[\ln\left(2\sqrt{v^2+(y-y_w)^2+(z-z_w)^2}+2v\right)\right]_{x-x_1}^{x-x_2} + C$$

$$= \frac{q}{4\pi L}\ln\frac{\sqrt{(x-x_2)^2+(y-y_w)^2+(z-z_w)^2}+(x-x_2)}{\sqrt{(x-x_1)^2+(y-y_w)^2+(z-z_w)^2}+(x-x_1)} + C$$

The above formula is simplified as follows:

$$\Phi = -\frac{q}{4\pi L}\ln\frac{r_2 + (x_2 - x)}{r_1 + (x_1 - x)} + C$$

$$r_j = \sqrt{(x - x_j)^2 + (y - y_w)^2 + (z - z_w)^2}, \quad j = 1,2$$

$$r_1^2 - (x_1 - x)^2 = r_2^2 - (x_2 - x)^2$$

$$\left[r_1 - (x_1 - x)\right]\left[r_1 + (x_1 - x)\right]$$

$$= \left[r_2 - (x_2 - x)\right]\left[r_2 + (x_2 - x)\right]$$

$$\Phi = -\frac{q}{4\pi L}\ln\frac{r_1 - (x_1 - x)}{r_2 - (x_2 - x)} + C, \quad \Phi = -\frac{q}{4\pi L}\ln\frac{r_1 + r_2 + L}{r_1 + r_2 - L} + C$$

如果 $\frac{a}{b} = \frac{c}{d}$

那么 $\frac{\mathrm{a+c}}{\mathrm{b+d}} = \frac{a}{b} = \frac{c}{d}$

Equipotential equation:

$$\frac{r_1 + r_2 + L}{r_1 + r_2 - L} = d$$

The above formula can be written as follows:

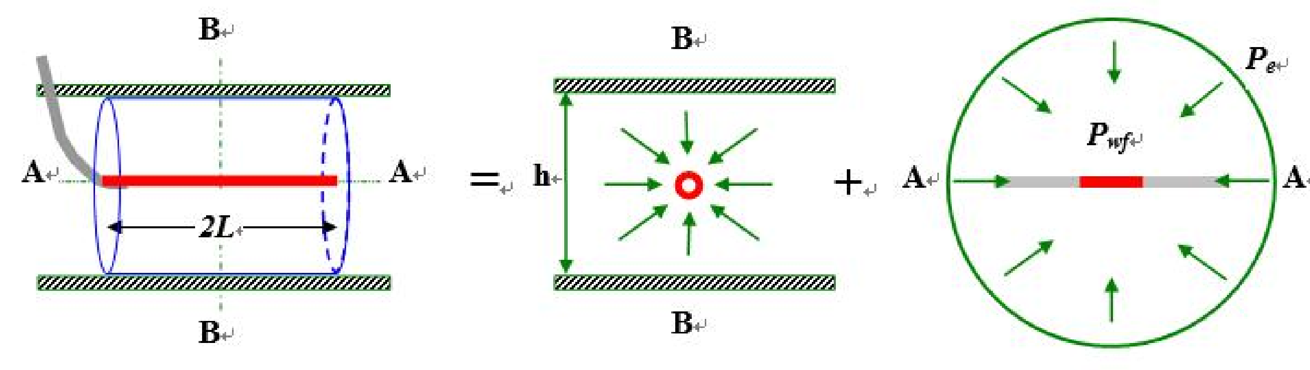
$$\frac{\left(x - \dfrac{x_1 + x_2}{2}\right)^2}{\left[\dfrac{(d+1)L}{2(d-1)}\right]^2} + \frac{(y - y_w)^2}{\dfrac{dL^2}{(d-1)^2}} + \frac{(z - z_w)^2}{\dfrac{dL^2}{(d-1)^2}} = 1$$

The above formula shows that the isopotential surface of steady seepage in horizontal wells in homogeneous and unbounded formation is a revolving ellipsoid with a center at [0.5 (x1 + x2), yw, zw]. Its projections on XOZ and XOY planes are ellipses, and on YOZ planes are circles, elliptical focal length and circular radius:

$$R_x = 2\sqrt{\left[\frac{(d+1)L}{2(d-1)}\right]^2 - \frac{dL^2}{(d-1)^2}} = L\,, \quad R_z = \frac{\sqrt{d}\,L}{|d-1|}$$

三、 Борисов Production Formula of Two-dimensional Steady-state Seepage

Former Soviet scholar Меркулов first used the equivalent resistance method to discuss the production calculation method of horizontal wells (groups) in 1958, and then Борисов(1964) gave a more concise formula for the production calculation of single horizontal wells.



Борисов 水平井近井三维流场简化示意图

Assuming homogeneous anisotropy of reservoir, horizontal well is located in the center of reservoir, length 2L, radius rw, supply edge radius Reh, boundary pressure Pe, bottom hole pressure Pwf, and liquid in reservoir is incompressible.

（1）Horizontal Radial Seepage in External Flow Field

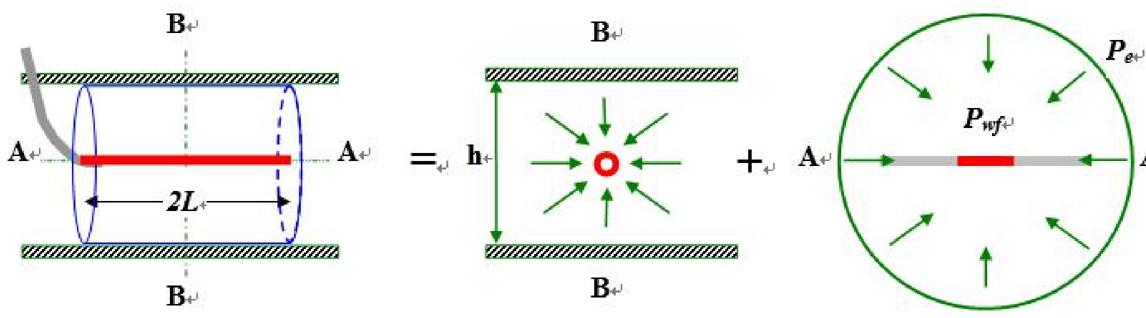
From the circular drainage zone with Reh radius and constant boundary pressure Pe to the "ordinary vertical well" with L/2 diameter and Pm bottom hole flow pressure, the reservoir undergoes planar radial Darcy steady-state seepage, and the production formula is as follows:

$$Q_1 = \frac{542.87 k_h h (P_e - P_m)}{\mu B_o \ln \frac{R_{eh}}{L/2}} = \frac{(P_e - P_m)}{\frac{\mu B_o}{542.87 k_h h} \ln \frac{2R_{eh}}{L}} = \frac{(P_e - P_m)}{\Omega_1}$$

（2）Internal flow field

In the vertical plane, the horizontal well profile is regarded as a "normal vertical well" whose radius is rw, and its discharge radius is a circular area with h/2π. According to Dupuit formula, the vertical flow distribution in the horizontal well is obtained as follows:

$$q_2 = \frac{542.87k \cdot 2L \cdot \left(p_m - p_{wf}\right)}{\mu B \ln\left(\frac{h/2\pi}{r_w}\right)} = \frac{(p_m - p_{wf})}{\frac{\mu B_o}{542.87kh} \cdot \frac{h}{2L} \cdot \ln\frac{h}{2\pi r_w}} = \frac{(p_m - p_{wf})}{\Omega_2}$$



Борисов 水平井近井三维流场简化示意图

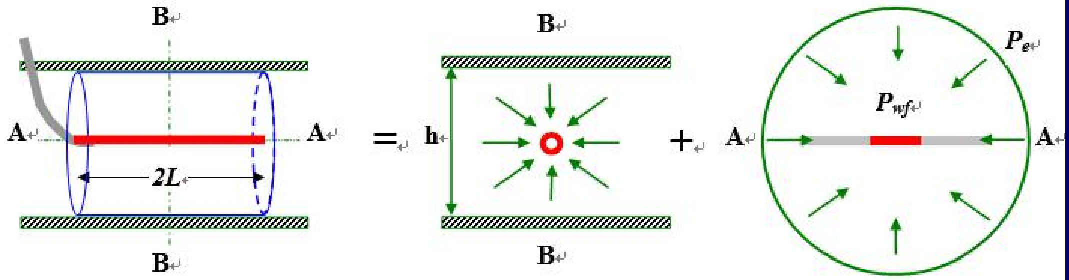
（3）Production Formula of Horizontal Well

According to the principle of hydroelectric similarity, the external and internal flow fields are connected in series to supply oil to horizontal wells. The total seepage Ωt can be obtained by the equivalent seepage resistance method as follows:

$$\Omega_t = \Omega_1 + \Omega_2 = \frac{\mu B}{542.87 k_h h}\left(\ln \frac{2R_{eh}}{L} + \frac{h}{2L} \cdot \frac{k_h}{k_v} \ln \frac{\beta h}{\pi r_w (\beta + 1)} \right)$$

Considering the modification of skin factor S and Muskat anisotropy:

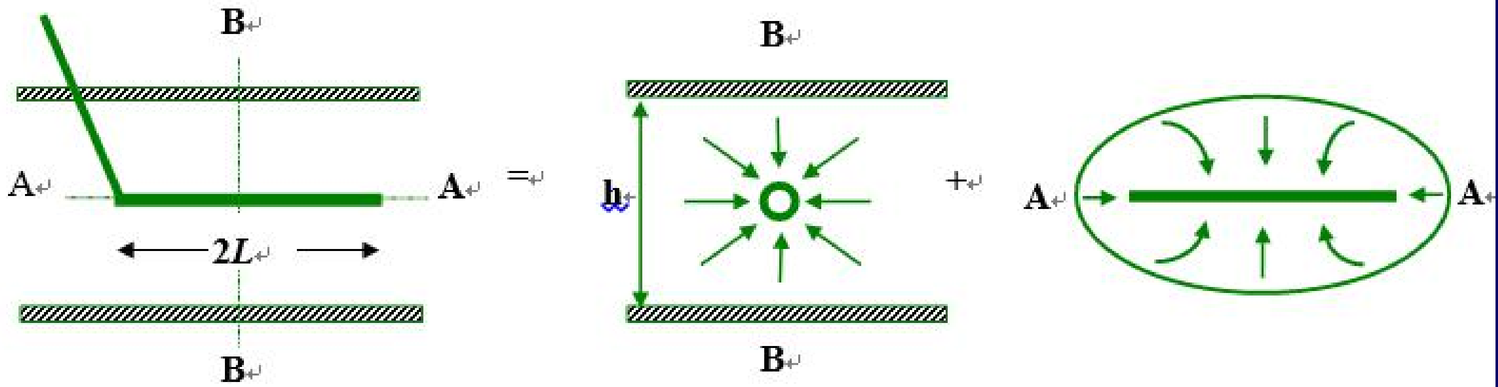
$$q_{\text{Bop}} = \frac{(P_e - P_w)}{\Omega_2} = \frac{542.87 k_h h (P_e - P_w)}{\mu B \left(\ln \frac{4R_{eh}}{2L} + \frac{h\beta}{2L} \ln \frac{\beta h}{\pi r_w (\beta + 1)} + S \right)}$$



Борисов 水平井近井三维流场简化示意图

四、Joshi Production Formula for Two-Dimensional Steady-State Seepage

Joshi (1988) simplified the three-dimensional flow field in horizontal wells. It was considered that the real three-dimensional flow in horizontal wells could be approximated by the internal vertical plane radial flow plus the external horizontal elliptical flow. The final production formula was obtained by using the equivalent seepage resistance method after solving respectively.



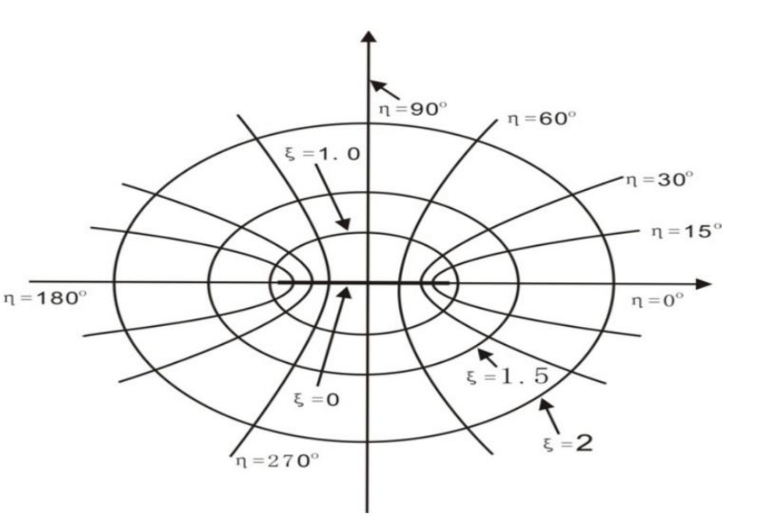
Joshi 等人水平井近井三维流场简化示意图

(1) Horizontal Elliptical Seepage in External Flow Field

$$\frac{\partial^2 P}{\partial \xi^2} + \frac{\partial^2 P}{\partial \eta^2} = 0$$

If it is assumed that all ellipses are confocal and equipotential (the outer boundary is an equipotential ellipse boundary), then there are:

$$\frac{\partial P}{\partial \eta} = 0$$



The governing equation becomes:

$$\frac{\partial^2 P}{\partial \xi^2} = 0$$

Volume flow:

$$q = \frac{4kh}{\mu B} \int_0^{\pi/2} \left. \frac{\partial P}{\partial \xi} \right|_{\xi = const.} dv$$

The pressure distribution formula can be obtained by using the internal boundary condition to determine the integral constant B.

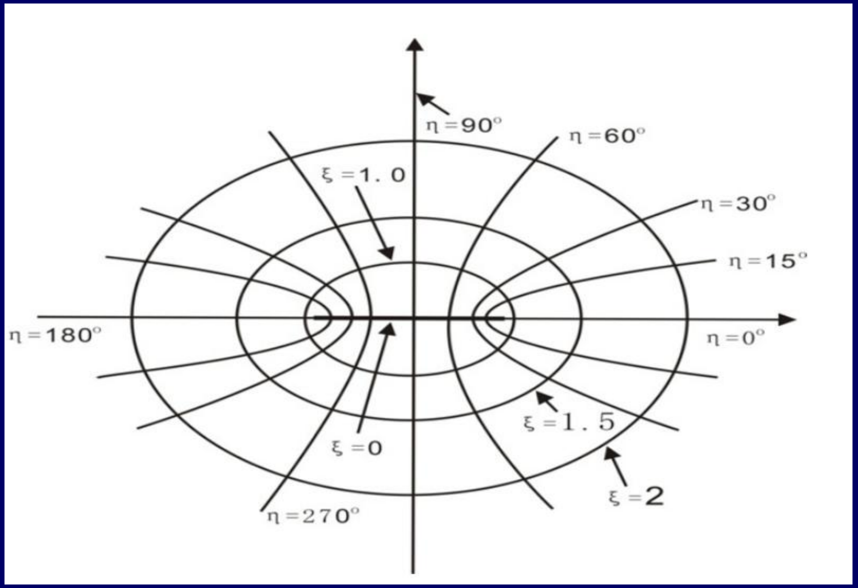
$$P(\xi) = P_{wf} + \frac{q\mu B}{2\pi\sqrt{k_x k_y}\, h}(\xi - \xi_w)$$

$$P(\xi_e) = P_{wf} + \frac{q\mu B}{2\pi\sqrt{k_x k_y}\, h}(\xi_e - \xi_w)$$

The relationship between rectangular coordinates and elliptical coordinates is known as:

$$x' = L\cosh\xi\cos\eta\,,\quad y' = L\sinh\xi\sin\eta$$

Solving coordinate transformation equation simultaneously:



$$\frac{x'^2}{L^2\cosh^2\xi}+\frac{y'^2}{L^2\sinh^2\xi}=1,\quad \cosh^2\xi-\sinh^2\xi=1$$

$$L^2\cosh^4\xi-\left(L^2+x'^2+y'^2\right)\cosh^2\xi+x'^2=0$$

$$\cosh^2\xi=\frac{\left(x'^2+y'^2+L^2\right)\pm\sqrt{\left(x'^2+y'^2+L^2\right)^2-4L^2x'^2}}{2L^2}$$

$$\xi=\cosh^{-1}H^*,\quad H^*=\sqrt{\frac{x'^2+y'^2+L^2\pm\sqrt{\left(x'^2+y'^2+L^2\right)^2-4L^2x'^2}}{2L^2}}$$

$$\xi_e=\cosh^{-1}H^*\Big|_{x=a}^{y=0}=\cosh^{-1}\sqrt{\frac{a^2+L^2+\sqrt{\left(a^2+L^2\right)^2-4L^2a^2}}{2L^2}}=\ln\frac{a+\sqrt{a^2-L^2}}{L}$$

$$\xi_w=\cosh^{-1}\left(H^*\Big|_{x=0}^{y=b\to 0}\right)=\cosh^{-1}\left[\lim_{b\to 0}\sqrt{\frac{b^2+L^2+\sqrt{\left(b^2+L^2\right)^2}}{2L^2}}\right]=\cosh^{-1}1=0$$

$$r_e^2=ab,\quad b=\sqrt{a^2-L^2}$$

$$a=L\left[\frac{1}{2}+\sqrt{\frac{1}{4}+\left(\frac{r_e}{L}\right)^4}\right]^{0.5}$$

There is a production formula of plane elliptical seepage:

$$Q_1 = \frac{2\pi\sqrt{k_x k_y}\, h \Delta P_w}{\mu B \ln \dfrac{a + \sqrt{a^2 - L^2}}{L}} = \frac{\Delta P_w}{\Omega_1}$$

（2）Internal flow field

In the vertical plane, the horizontal well profile is regarded as a vertical well with radius rw and its drainage area is a circular area with radius h/2. The anisotropic effective diameter is corrected by Peaceman method, and the vertical flow distribution in the horizontal well is obtained according to Dupuit formula.

$$Q_2 = \frac{2\pi k_z (2L)\Delta P_w}{\mu B \ln\left(\dfrac{\beta h}{\pi r_w (\beta + 1)}\right)} = \frac{\Delta P_w}{\Omega_2}$$

（3）Production Formula of Horizontal Well

According to the series equivalent seepage resistance method, the external horizontal flow resistance and the internal vertical flow resistance are connected in series.

$$\Omega = \Omega_1 + \Omega_2 = \frac{\ln \dfrac{a + \sqrt{a^2 - L^2}}{L} + \dfrac{k_h}{k_z} \dfrac{h}{2L} \ln\left(\dfrac{\beta h}{\pi r_w (\beta + 1)} \right)}{2\pi k_h h / \mu B}$$

Production Formula of Final Horizontal Well

$$q_{\text{Joshi}} = \frac{542.87 k_h h \Delta P / (\mu B)}{\ln\left(a/L + \sqrt{(a/L)^2 - 1}\right) + \frac{h\beta}{2L} \ln \frac{\beta h}{\pi r_w (\beta + 1)}}$$

$$a/L = \sqrt{0.5 + \sqrt{0.25 + (R_{eh}/L)^4}}$$

五、Giger and Renard-Dupuy yield formulas for two-dimensional steady seepage

Giger, a French scholar, thinks that the semi-long axis of horizontal elliptical drainage zone can be replaced by an equivalent radius reh. As a result, the Joshi formula becomes:

$$q_{\text{Giger}} = \frac{542.87 k_h h \Delta P / (\mu_o B_o)}{\ln \dfrac{r_{eh} + \sqrt{r_{eh}^2 - (L/2)^2}}{L} + \dfrac{h\beta}{2L}\left[\ln \dfrac{\beta h}{\pi r_w (\beta + 1)} + S\right]}$$

In a paper published by Renard and Dupuy in 1991, the horizontal well production formula was as follows:

$$q_{\text{R\&D}} = \frac{542.87 k_h h \Delta P / (\mu_o B_o)}{\cosh^{-1}\left(\dfrac{a}{L}\right) + \dfrac{h\beta}{2L}\left[\ln \dfrac{\beta h}{\pi r_w (\beta + 1)} + S\right]}$$

$$a = L\sqrt{0.5 + \sqrt{0.25 + (r_{eh}/L)^4}}$$

六、Formula for calculating horizontal well production in pressure sensitive medium

The so-called pressure-sensitive medium refers to the medium that is prone to partial or total irreversible deformation, which has a significant impact on the physical properties of the medium.

Meinzer (1928) and Jacob (1940) have pointed out that plastic deformation exists in porous media under certain conditions. Raghavan and Miller (1969) first described the mathematical model of non-linear unsteady seepage flow in pressure-sensitive media completely. He gave the steady seepage solution and used numerical method to solve the unsteady seepage flow. The calculation results are not ideal； Samaniego (1977, 1980, 1989) and others used numerical methods to study the seepage of pressure sensitive.

Pedrosa (1986) introduced the permeability variation formula and solved the non-linear seepage mathematical model of pressure-sensitive medium by small perturbation method. The first-order approximate solution of point source was given， Kikani and Pedrosa (1991) gave the second order approximate solution by small perturbation method； Based on the results of Kikani and Pedrosa, Zhang (1994) et al. extended the permeability variation model.

(1) Nonlinear seepage mathematical model

Assuming that the variation of fluid viscosity and permeability due to pressure sensitivity conforms to Hooker's elastic rheological law, according to the research results of Raghavan and Miller (1969), the quasi-pressure function can be defined to generalize the seepage problem of pressure-sensitive media. The quasi-pressure function is defined as:

$$m(P) = \int_{P_0}^{P} \frac{k(P)}{\mu(P)} dP$$

In the formula, P_0 is the reference pressure. Using Darcy's law, the plane radial unsteady seepage equation can be expressed as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m(P)}{\partial r}\right)=\frac{\mu(P)}{k(P)}\phi c_t\frac{\partial m(P)}{\partial t}$$

Under steady seepage condition in circular reservoir, pressure approximately obeys the following equation:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dm(P)}{dr}\right]=0$$

For ordinary vertical wells, the boundary conditions are as follows:

$$\left[\frac{1}{r}\frac{dm(P)}{dr}\right]_{r=r_w} = -\frac{qB}{2\pi h}$$

$$\left[\frac{1}{r}\frac{dm(P)}{dr}\right]_{r=r_e} = 0$$

The boundary conditions are used to integrate directly.

$$m(P_e) - m(P_w) = \frac{qB}{2\pi h}\ln\left(\frac{r_e}{r_w}\right)$$

$$q = \frac{2\pi h[m(P_e) - m(P_w)]}{B\ln(r_e / r_w)} = \frac{m(P_e) - m(P_w)}{\Omega}$$

Using the equivalent seepage resistance method, the production formula of horizontal wells is obtained.

$$q_{\text{Mepm}} = \frac{542.87h[m(P_e) - m(P_{wf})]}{B\left(\ln\frac{2R_{eh}}{L} + \frac{h\beta}{2L}\ln\frac{\beta h}{\pi r_w(\beta+1)} + S\right)}$$

If the composite index of seepage parameters changes, that is:

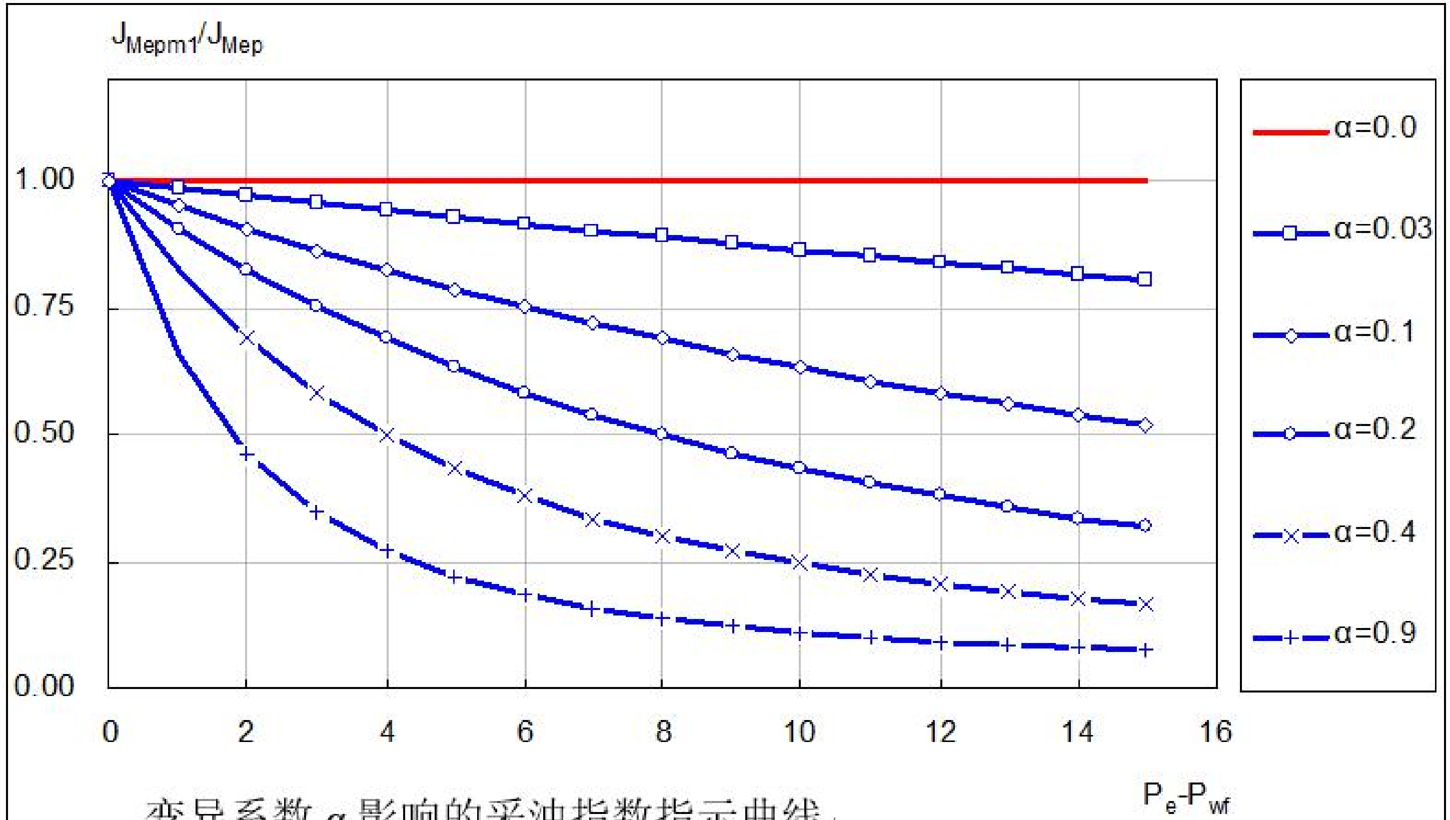
$$\frac{k(P)}{\mu(P)} = \frac{k_0}{\mu_0}\exp[-\alpha(P_e - P)], \quad 0 \le \alpha \le 1$$

At this time, the horizontal well production formula can be written as follows:

$$q_{\text{Mepm1}} = \frac{542.87 k_0 h\{1 - \exp[-\alpha(P_e - P_{wf})]\}}{\mu_0 B \alpha \left(\ln \frac{2R_{eh}}{L} + \frac{h\beta}{2L} \ln \frac{\beta h}{\pi r_w (\beta + 1)} + S \right)}$$

Compared with the Меркулов horizontal well formula without pressure sensitivity, the ratio of oil recovery index is:：

$$\frac{J_{\text{Mepm1}}}{J_{\text{Mep}}} = \frac{1 - \exp[-\alpha(P_e - P_{wf})]}{\alpha(P_e - P_{wf})}$$



变异系数 α 影响的采油指数指示曲线

If the composite power law of seepage parameters changes, then:

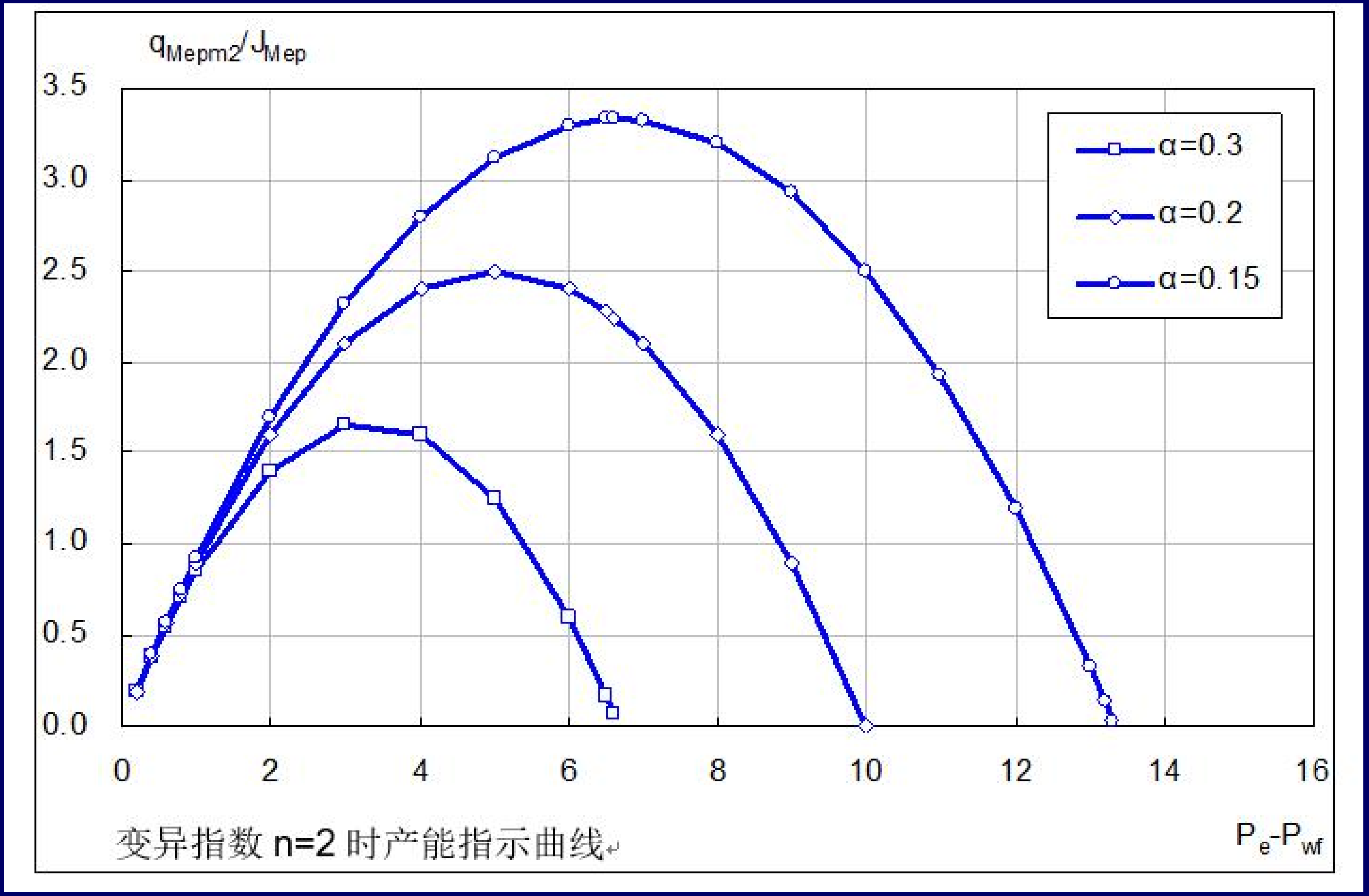
$$\frac{k(P)}{\mu(P)} = \frac{k_0}{\mu_0}[1-\alpha(P_e - P)]^{n-1}$$

The production formula of horizontal wells is as follows:

$$q_{\text{Мерm2}} = \frac{542.87 k_0 h\{1-[1-\alpha(P_e - P_{wf})]^n\}}{\mu_0 B n\alpha\left(\ln\frac{2R_{eh}}{L} + \frac{h\beta}{2L}\ln\frac{\beta h}{\pi r_w(\beta+1)} + S\right)}$$

Compared with the Меркулов horizontal well formula without pressure sensitivity, the specific oil recovery index is as follows:

$$\frac{q_{\text{Мерm2}}}{J_{Мер}} = \frac{1-[1-\alpha(P_e - P_{wf})^n]}{n\alpha}$$

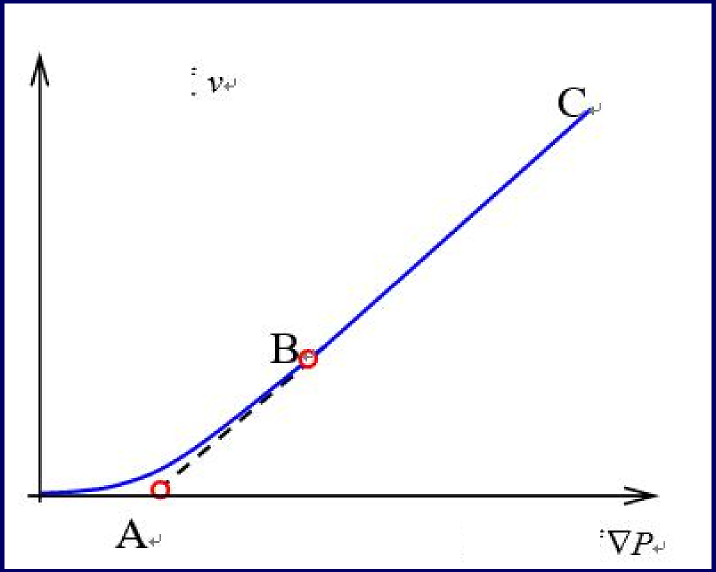


q_Mepm2/J_Mep
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
14
16
α=0.3
α=0.2
α=0.15
变异指数 n=2 时产能指示曲线
Pe-Pwf

七、Formula for Calculating Horizontal Well Production in Low Permeability Reservoir

Low permeability oilfields account for a very large proportion of proven onshore crude oil reserves in China. At present, nearly 300 low permeability oilfields (reservoirs) have been discovered and proved, which are widely distributed in more than 20 oil fields.

The main forms of seepage flow in low permeability formation are: when the pressure gradient is small, the seepage velocity and pressure gradient are convex downward; when the pressure gradient is large, the relationship between them is approximately linear.



（1）Mathematical Model of Steady-state Seepage with Start-up Pressure Gradient

Set up a production well in a circular constant pressure formation and steady seepage occurs in the formation. The governing equations are as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial P}{\partial r}-G\right)\right]=0\,,\quad \frac{\partial P}{\partial r}>G$$

$$\left[r\left(\frac{\partial P}{\partial r}-G\right)\right]_{r=r_w}=\frac{q\mu B}{2\pi kh}$$

$$P(r_e)=P_e$$

By direct integration, the pressure distribution formula is obtained as follows:

$$\Delta P(r)=P(r)-P_{wf}=\frac{q\mu B}{2\pi kh}\ln\left(\frac{r}{r_w}\right)+G(r-r_w)$$

The steady-state productivity formula can be written as follows

$$q=\frac{2\pi kh[P_e-P_{wf}-G(r_e-r_w)]}{\mu B\ln(r_e/r_w)}=\frac{2\pi kh(P_e-P_{wf})}{\mu B\ln(r_e/r_w)}\left[1-\frac{G(r_e-r_w)}{P_e-P_{wf}}\right]$$

（2）Productivity model of horizontal wells in low permeability reservoirs

Still according to the practice of Борисов, the production formula of horizontal wells is written as follows:

$$q = \frac{(P_e - P_w)}{\Omega} = \frac{P_e - P_w}{\frac{\mu B}{2\pi kh}\left((1+G_{r1})\ln\frac{2R_{eh}}{L} + (1+G_{r2})\frac{h}{2L}\ln\frac{h}{2\pi r_w} + S\right)}$$

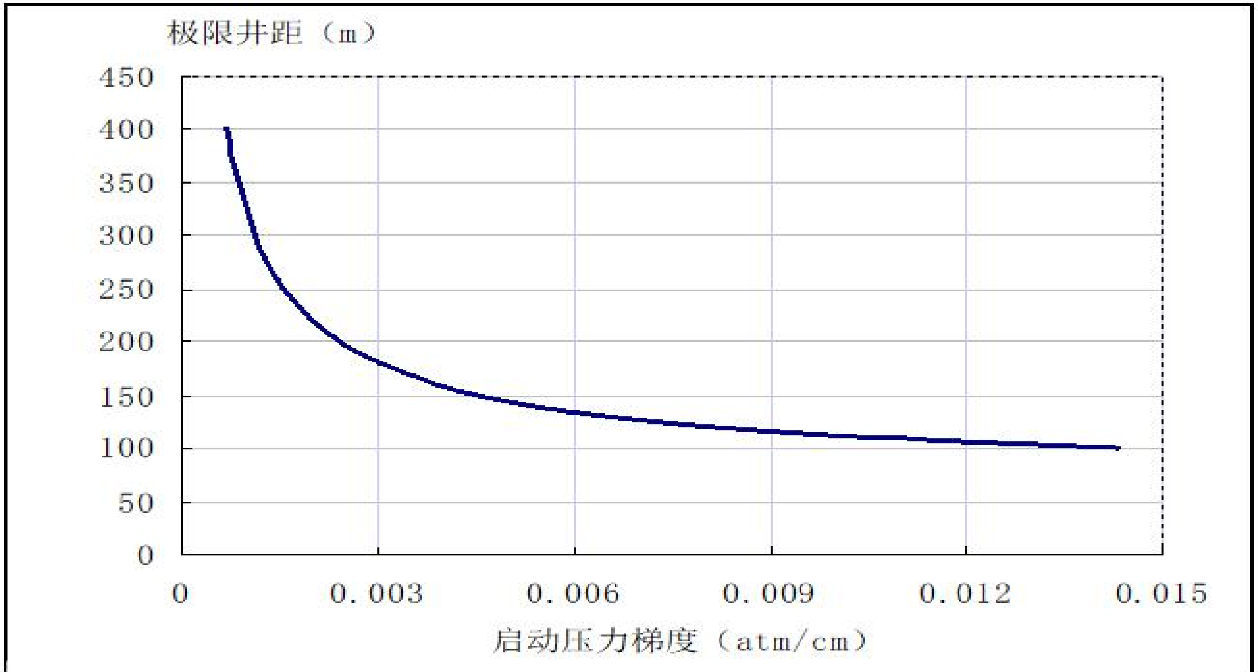
$$G_{r1} = \frac{2\pi kh(R_{eh} - L/2)}{q\mu B}G\text{，}\quad G_{r2} = \frac{2\pi kh(h/2\pi - r_w)}{q\mu B}G$$

Further arrangement is as follows:

$$q = \frac{P_e - P_w}{\frac{\mu B}{2\pi kh}\left(\ln\frac{2R_{eh}}{L} + \frac{h}{2L}\ln\frac{h}{2\pi r_w} + S\right)} - \frac{(R_{eh} - \frac{L}{2})\ln\frac{2R_{eh}}{L} + (\frac{h}{2\pi} - r_w)\frac{h}{2L}\ln\frac{h}{2\pi r_w}}{\frac{\mu B}{2\pi kh}\left(\ln\frac{2R_{eh}}{L} + \frac{h}{2L}\ln\frac{h}{2\pi r_w} + S\right)}G$$

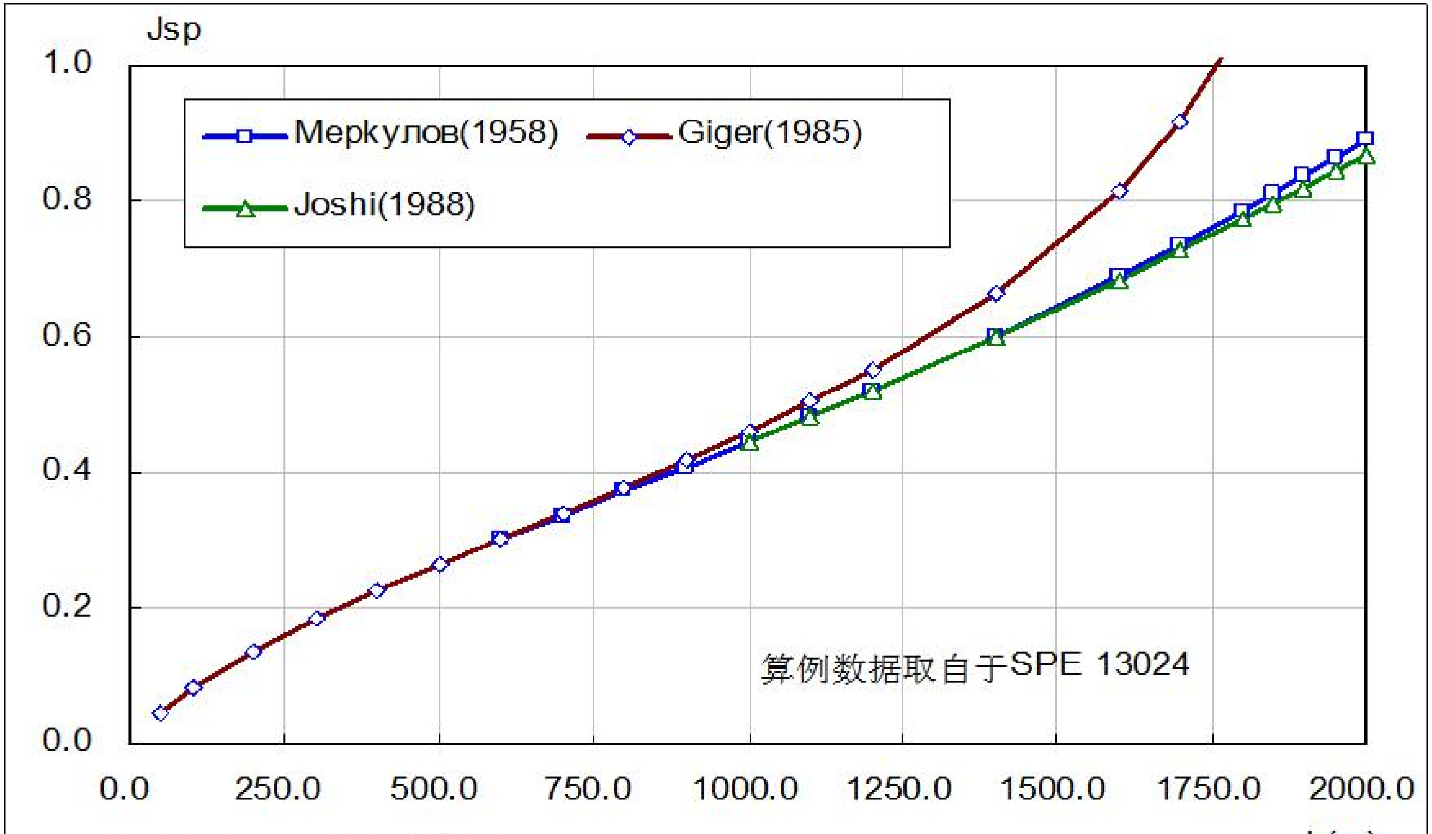
When G = 0, it can degenerate to the original formula of Борисов.

Suppose the horizontal well length is 200 thickness meters, the production pressure difference is 5 MPa, the reservoir is 10 meters, and the wellbore radius is 0.1 meters. Calculate the limit well spacing under different starting pressure gradients as shown in Figure 1.



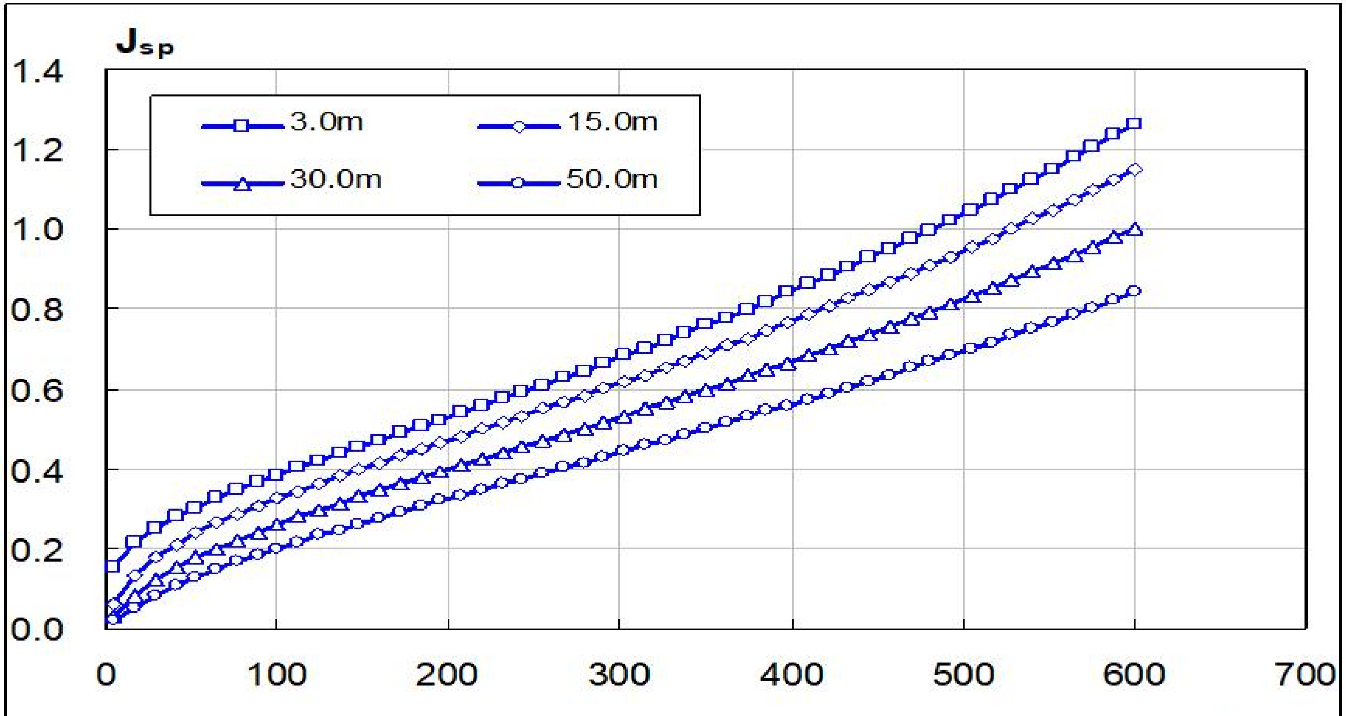
水平井极限井距与启动压力关系图

（3）Comparison of productivity formulas for horizontal wells

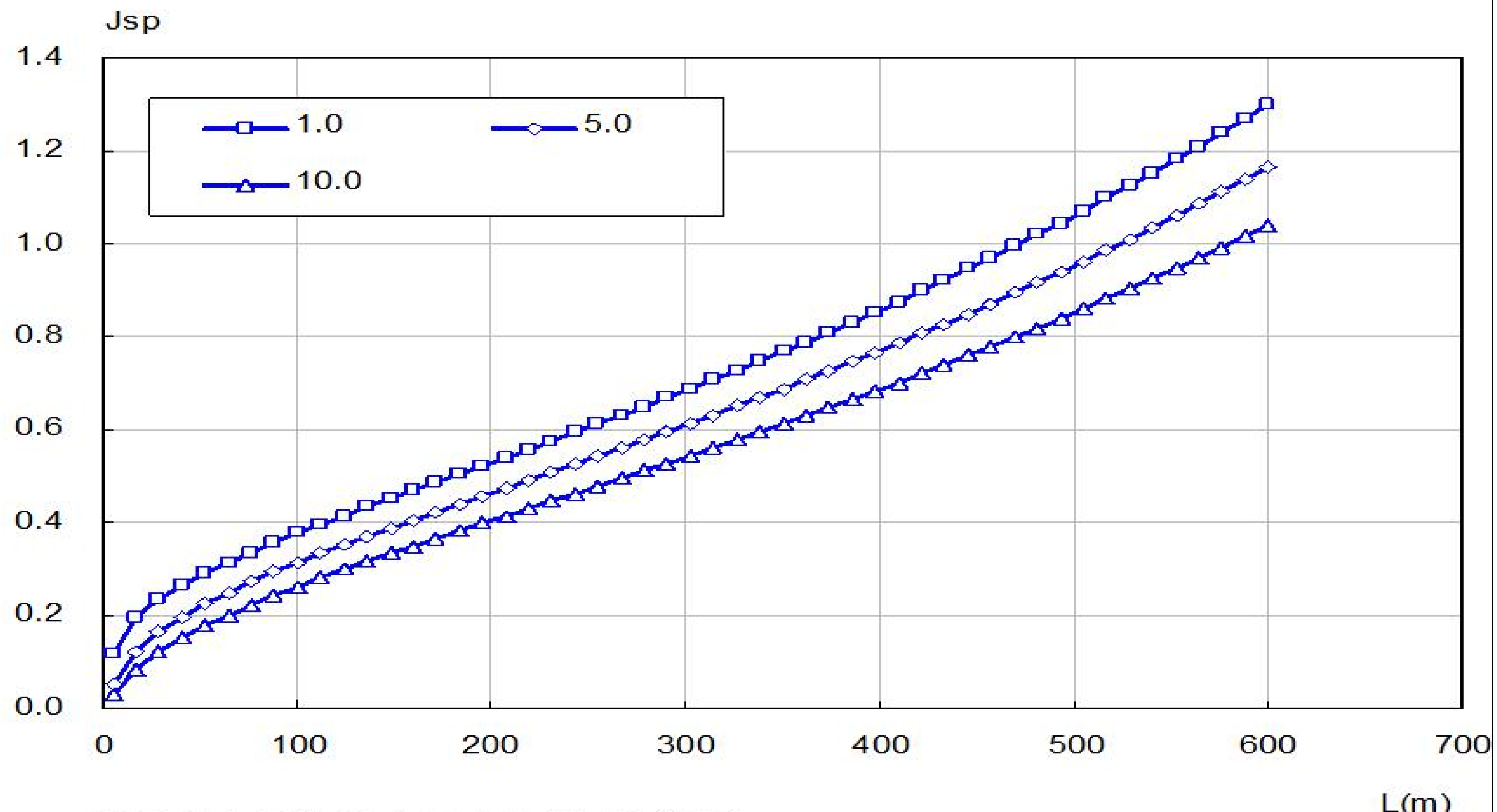


图水平井产量计算结果对比图

（4）Analysis of factors affecting productivity



地层厚度影响水平井产量曲线图



地层各向异性影响水平井产量曲线图

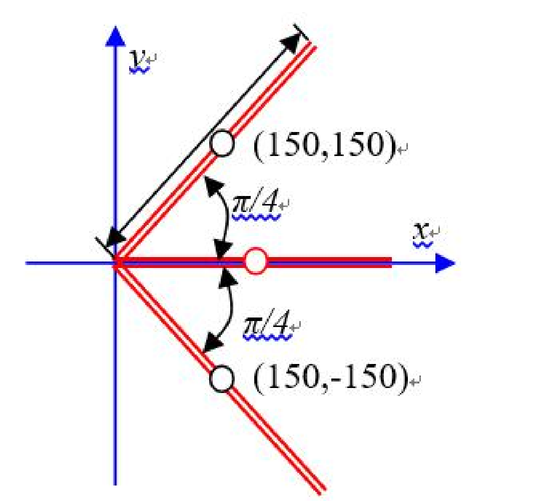
名称	参数	扶平-2	Z53-P37	Z55-P46	Z57-P33	Z57-P35	Z59-P55	Z60-P33	Z60-P54	州 62-P61	州 66-P61
储层参数	h	8.000	0.800	1.200	1.200	1.200	1.200	1.200	1.200	1.100	1.100
	k_y	0.070	0.020	0.198	0.035	0.025	0.025	0.020	0.023	0.044	0.044
	k_x	0.070	0.020	0.198	0.035	0.025	0.025	0.020	0.023	0.044	0.044
	r_e	500.000	293.000	293.000	293.000	293.000	293.000	293.000	293.000	293.000	293.000
	k_z	0.007	0.005	0.033	0.005	0.005	0.005	0.005	0.005	0.007	0.007
	z_w	4.000	0.600	0.550	0.600	0.550	0.600	0.550	0.600	0.550	0.550
生产参数	$2L$	37.500	103.000	426.200	346.000	526.000	296.000	443.000	318.000	462.200	482.500
	ΔP	2.082	10.090	7.270	7.410	5.520	8.950	8.760	7.590	8.710	9.660
	r_w	0.110	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062
流体参数	μ	43.900	9.770	9.770	9.770	9.770	9.770	9.770	9.770	9.000	9.000
	B	1.032	1.069	1.069	1.069	1.069	1.069	1.069	1.069	1.071	1.071
分析结果 (m^3/d)	Joshi	2.171	3.420	86.832	13.014	10.369	10.000	11.009	8.234	24.620	28.643
	Borisov	2.171	3.420	87.554	13.054	10.614	10.150	11.120	8.251	24.925	28.981
	Giger	2.171	3.431	105.027	14.217	17.820	10.552	13.778	8.808	32.254	39.377
	Renard-Dupuy	2.353	3.429	87.149	13.072	10.400	10.031	11.033	8.257	24.702	28.639
	三维稳态	2.338	3.270	79.072	12.006	9.333	9.286	9.986	7.616	22.307	25.802
	实际产量	2.315	3.210	60.020	13.890	9.450	10.700	10.090	8.010	7.590	10.300

八、Productivity Analysis of Complex Multi-Branch Horizontal Wells

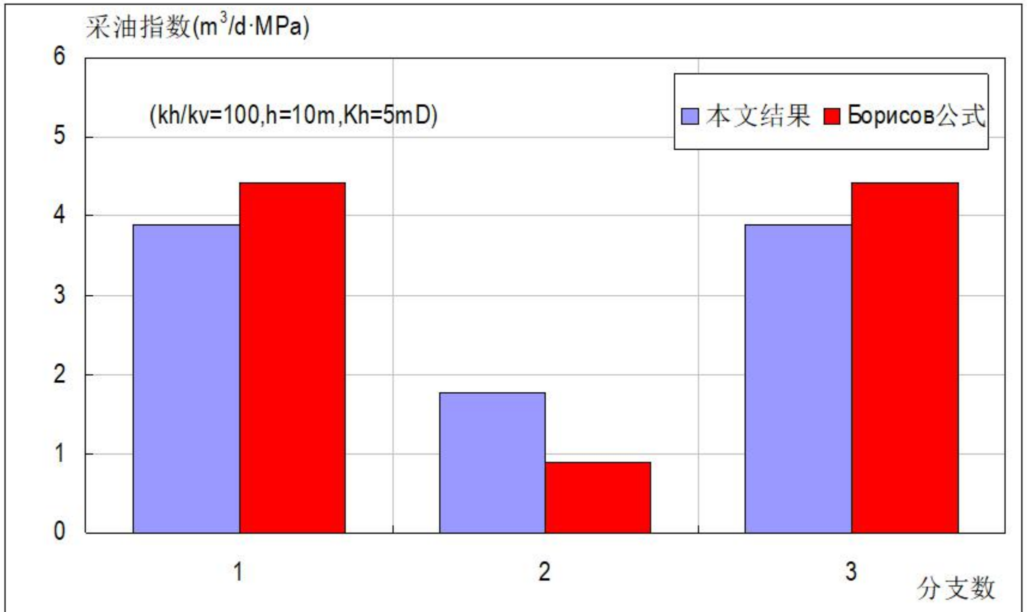
Branch horizontal wells refer to several horizontal wells drilled from the bottom of a main vertical well in all directions. There are two main types of branch horizontal wells: one is multi-branch horizontal wells in the same layer, which can be regarded as plane branch wells; the other is stratified branch horizontal wells. Branch horizontal well development is a very promising technology, which can multiply the volume of reservoir oil release, effectively suppress water or gas cones, and improve oil and gas recovery of low permeability reservoirs. Especially for offshore oil and gas field development, using branch horizontal well technology on a platform can make more effective use of this platform, reduce oil and gas development costs and improve economic benefits.

Since 1985, many foreign scholars have studied the problem of horizontal well pressure analysis and obtained some valuable results. Around 1990, China began to study the theory of horizontal well seepage. Among many researchers, the work of F. Kuchuk (1987), Ozkan and Raghavan (1991), Wang Xiaodong and Liu Ciqun (1996) is both innovative and practical. Researches on seepage in branch horizontal wells have only begun in the past 20 years. Kong Xiangyan and Xu Xianzhi (1996), Wang Xiaodong and Liu Ciqun (1997) have made theoretical studies on pressure analysis in horizontal branch horizontal wells. They have given some new solutions.

（1）Equal-length three-branch horizontal well

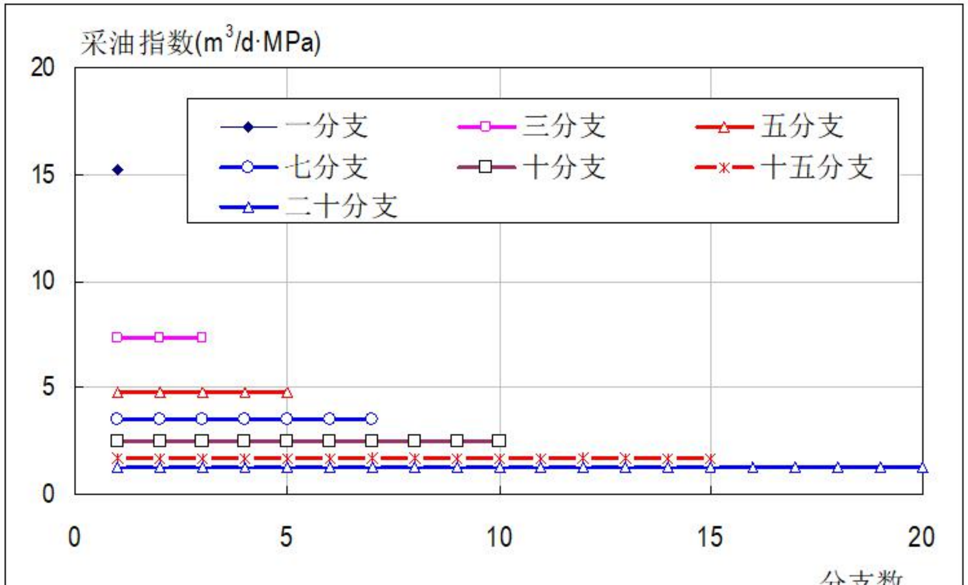


三个分支对称分支水平井分布图

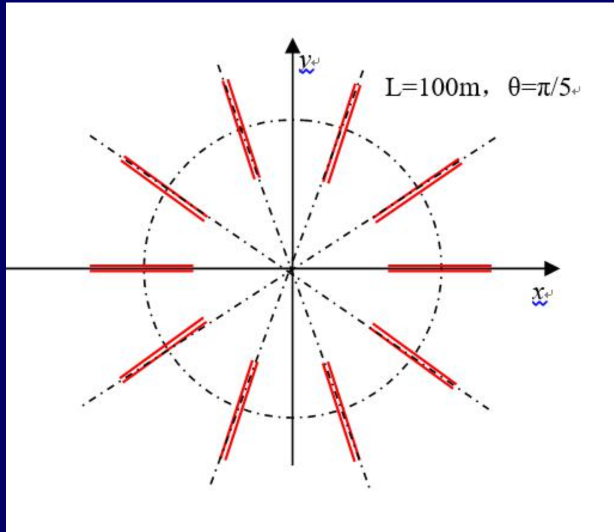


三分支对称水平井采油指数分布图

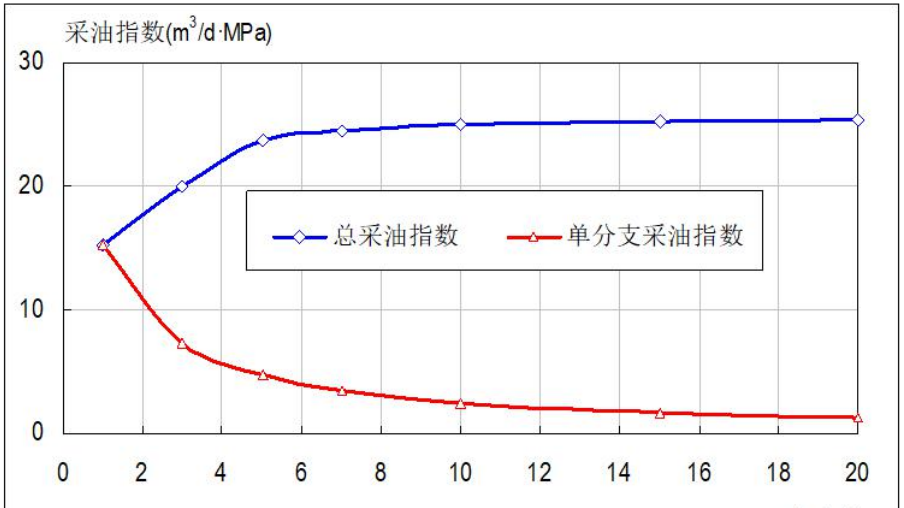
（2）Horizontal Well with Multi-Branch and Uniform Distribution



分支数对单分支采油指数的影响曲线

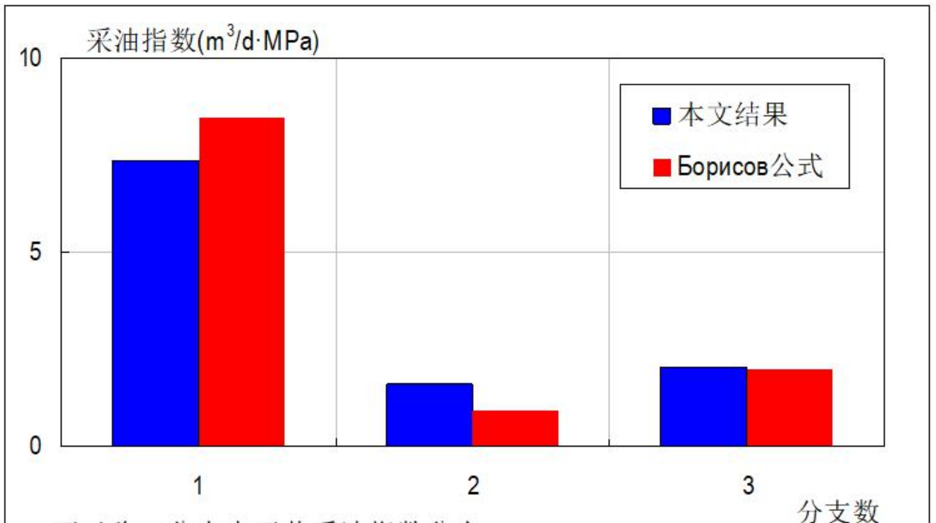


十个分支对称分支水平井分布图

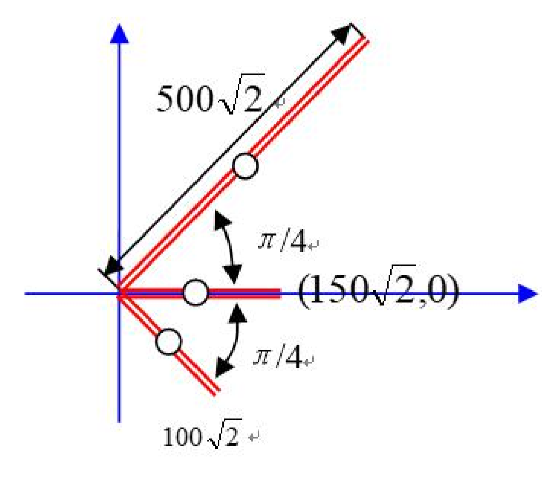


分支数对采油指数的影响

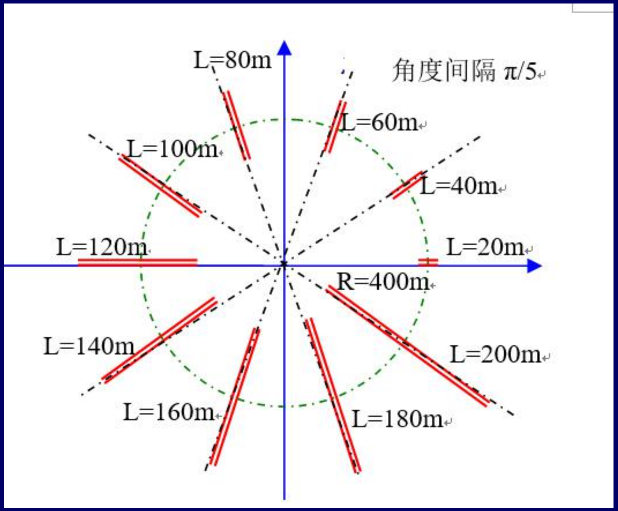
（3）Three-Branch Horizontal Wells with Unequal Length

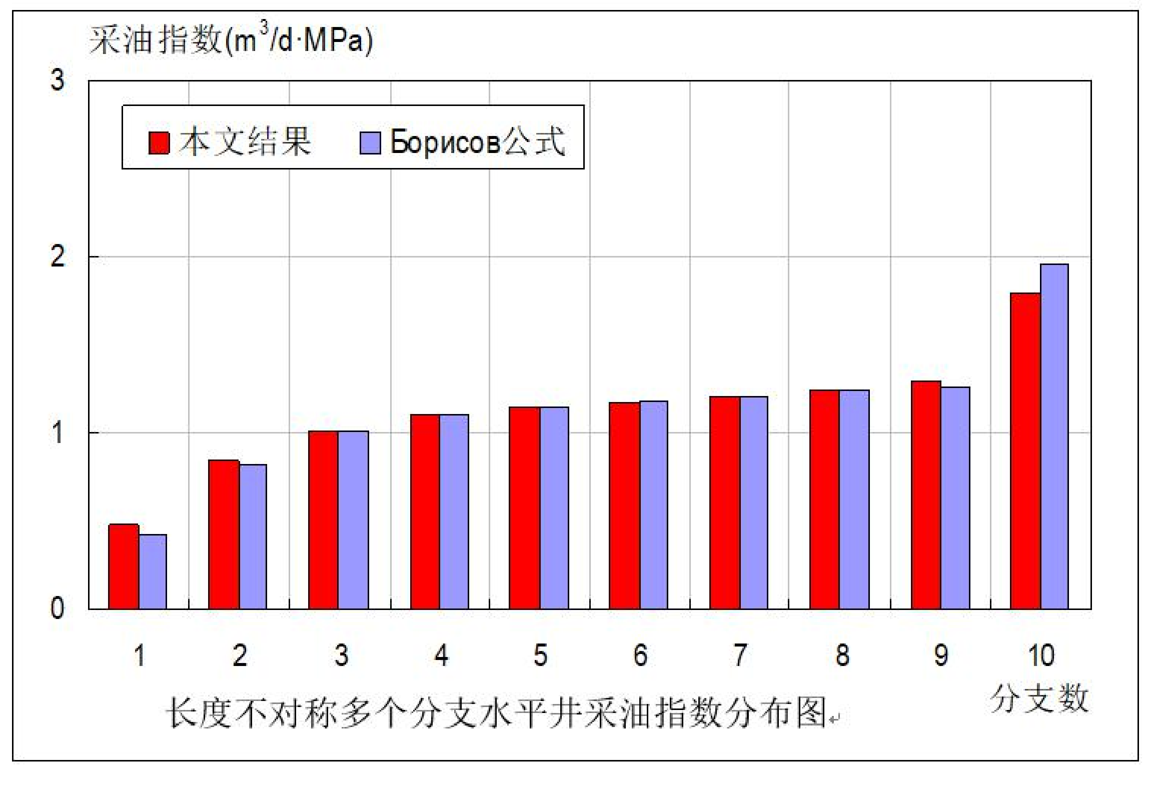


不对称三分支水平井采油指数分布



长度不对称三个分支水平井分布图




采油指数(m3/d·MPa)
3
2
1
0
本文结果
Борисов公式
1
2
3
4
5
6
7
8
9
10
分支数
长度不对称多个分支水平井采油指数分布图

Chapter 5 Steady-state productivity evaluation of oil wells

Section 1 Productivity evaluation method for vertical wells

Section 2 Productivity evaluation method for horizontal wells

Section 3 Productivity evaluation method for fractured wells

French scholar Giger (1985) predictably pointed out in his article "Production Technology of Horizontal Wells in Heterogeneous Reservoirs": "Horizontal wells are a good way to develop heterogeneous reservoirs. Therefore, as long as cementing technology reaches industrial application, the idea of horizontal well fracturing will be implemented", and his viewpoint pointed out the development potential of this technology.

Later, Brown (1992) pointed out that the Beihai Danish oilfield had the best fracturing effect. The formation permeability of the oilfield was 1.0 $*10^{-3}$ um^2. Ten horizontal wells were drilled. Five fractures were fractured in each horizontal well, and then up to 10 fractures were fractured. Practice has proved that fracturing is very effective.

Modern hydraulic fracturing process is to inject fracturing fluid into the target formation pump to form a fracture at high pressure, then inject sand-carrying fluid mixed with proppant to continue to extend the fracture, while carrying proppant deep into the fracture for support, and then inject gel breaker to degrade the fracturing fluid into a low-viscosity fluid, finally release the blowout and unload at the wellhead, and reverse drain the injected fluid to the bottom of the well. In this way, a hydraulic fracture can be left in the formation, which is the oil and gas passage with high conductivity.

（1）Classification of hydraulic fractures

There are generally three types of artificial fracture in horizontal wells: transverse fracture, longitudinal fracture and horizontal fracture.Transverse fracture refers to a fracture that is perpendicular to the horizontal wellbore, and can generally produce multiple transverse fractures. Longitudinal fracture refers to a fracture that extends along the horizontal wellbore direction. Horizontal fracture refers to a fracture that extends along the horizontal direction. For a horizontal well, what kind of fracture will occur after actual fracturing depends on the in-situ stress.

Opening Principle:

Hydraulic cracks generally open along the direction of minimum stress and extend along the direction of maximum principal stress. The fracture morphology mainly depends on the relative magnitude of vertical stress_v and horizontal stress_h in the formation.

（2）Fracture conductivity

Quantitative description of hydraulic fracture conductivity is to define hydraulic fracture conductivity factor C_f, which is the product of fracture permeability k_f and fracture width w (Prats, 1960):

$$C_f = k_f w$$

It is a fluid with a viscosity of 1. Under the action of unit pressure gradient, it passes through the cross section of the fracture at unit height. Dimensionless form is defined as follows:

$$C_{fD} = \frac{k_f w}{k x_f}$$

$$C_{FD} = \frac{k_f w_f}{k x_f} \cdot \frac{h_f / \mu}{h_f \mu} = \frac{w_f h_f \dfrac{k_f}{\mu}}{x_f h_f \dfrac{k}{\mu}} = \frac{q_{inf}}{q_{outf}}$$

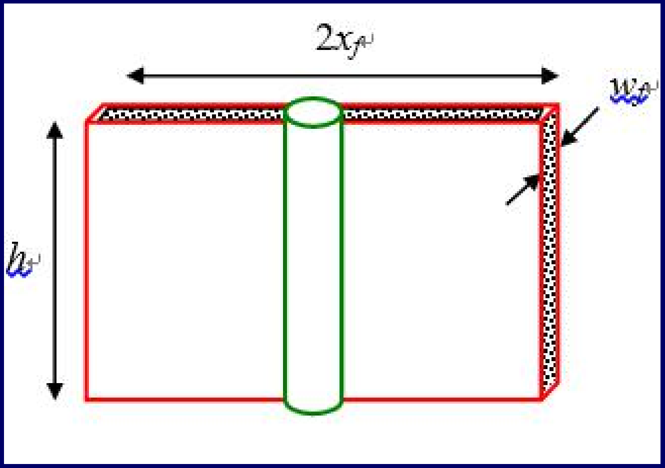


Diagram of Vertical Fractured Wells

（3）Equivalent radius model of infinite conductivity

Considering a production well with vertical fracture in the center of the discharging zone with circular supply boundary, the vertical fracture is abstracted as a straight line sink with uniform flow rate. A small element dα is selected at the position (0,a) of the straight line sink, and the small element dalpha is regarded as a point sink. The steady-state pressure is as follows:

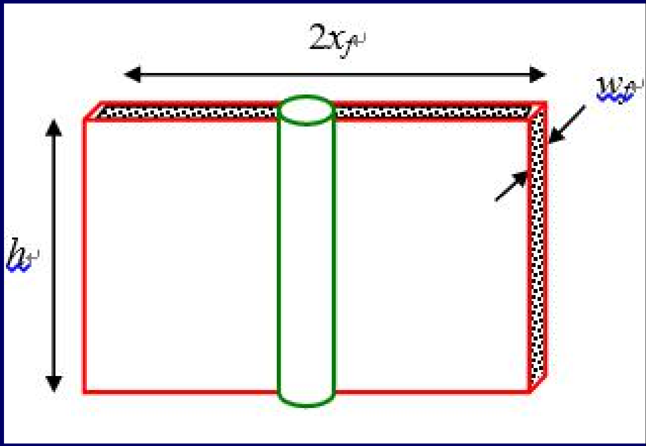


Diagram of Vertical Fractured Wells

Integral results:

$$\Delta P_w = \Delta P(x,0) = \int_{-x_f}^{+x_f} \frac{q_f \mu B}{4\pi kh} \{\ln r_e^2 - \ln[(x-\alpha)^2\} \cdot d\alpha$$

$$\Delta P_w = \frac{q\mu B}{2\pi kh}\left\{\ln\frac{r_e}{x_f} + 1 - \sigma(x)\right\}, \quad \int \ln x dx = x\ln x - x + C$$

$$\sigma(x) = \frac{1}{2}\left[(1 - x/x_f)\ln\left|1 - x/x_f\right| + (1 + x/x_f)\ln\left|1 + x/x_f\right|\right]$$

For uniform flow fracture, the integral average of wellbore pressure along the fracture is as follows:

$$I = \frac{1}{2}\int_{-1}^{1}[\sigma(x)-1]d\alpha = \ln 2 - \frac{1}{2}\lim_{u\to 0}\left[\frac{u^2}{2}\ln u\right] - \frac{3}{2} = \ln 2 - \frac{3}{2}$$

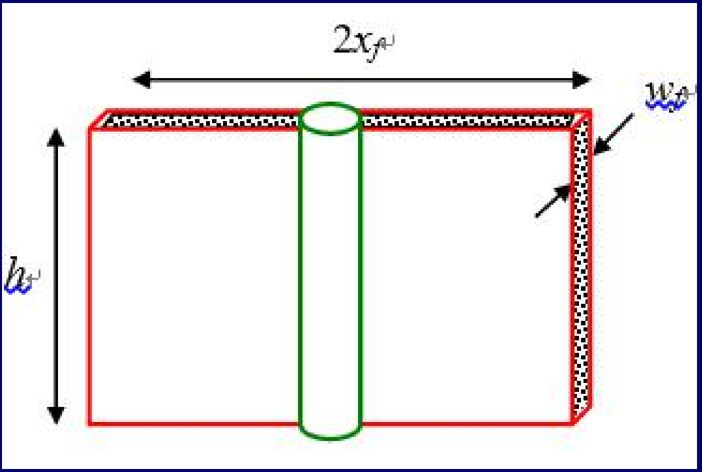


Diagram of Vertical Fractured Wells

The effective radius is

$$r_{equ} \approx x_f e^{-\left(\frac{3}{2}-\ln 2\right)} = 0.44626 x_f$$

For infinite conductivity fractures, the equivalent diameter formula can also be obtained from the results of Gringarten et al. (1974) when $x = 0.738\ x_f$ is chosen.

$$r_{equ} = 0.499 x_f$$

(4) Effective Radius Model of Finite Conduction

The governing equations that strictly describe fluid seepage in fractures can be written as follows:

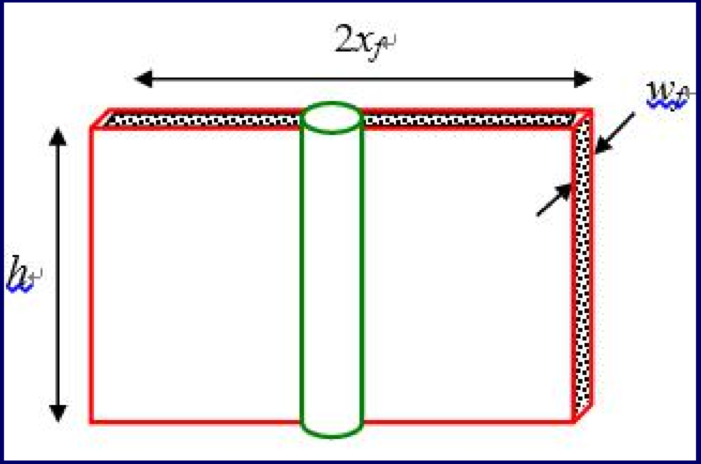


Diagram of Vertical Fractured Wells

$$\frac{\partial^2 P_f}{\partial x^2}+\frac{\partial^2 P_f}{\partial y^2}=\frac{\phi_f \mu_f c_{ft}}{k_f}\frac{\partial P_f}{\partial t}, \quad 0<x<x_f, \quad 0<y<w_f/2$$

$$P_f(x,y,t)=P_i$$

$$\frac{\partial P_f(x_f,y,t)}{\partial x}=0$$

$$\frac{k_f}{\mu}\frac{\partial P_f(x,w_f/2,t)}{\partial y}=\frac{k}{\mu}\frac{\partial P(x,w_f/2,t)}{\partial y}$$

$$\int_0^{\frac{w_f}{2}}\frac{\partial P_f(0,y,t)}{\partial x}dy=\frac{qB\mu}{4hk_f}$$

Considering that the fracture size is smaller than the drainage area, the integral average of the above equation is simplified in the fracture.

$$\frac{\partial^2 P_f}{\partial x^2}+\frac{2k}{w_f k_f}\left.\frac{\partial P}{\partial y}\right|_{y=\frac{w}{2}}=\frac{\phi_f \mu_f c_{ft}}{k_f}\frac{\partial P_f}{\partial t}, \quad 0<x<x_f$$

For steady seepage:

$$\frac{d^2 P_f}{dx^2} + \frac{2k}{w_f k_f} \left.\frac{dP}{dy}\right|_{y=\frac{w}{2}} = 0\,, \quad 0 < x < x_f$$

The dimensionless pressure distribution along the fracture is obtained by solving the above dimensionless model.

$$P_{fD}(0) - P_{fD}(x_D) = \frac{\pi}{C_{FD}} \left(x_D - \int_0^{x_D} \int_0^{v} q_{fD}(u)\,du\,dv \right)$$

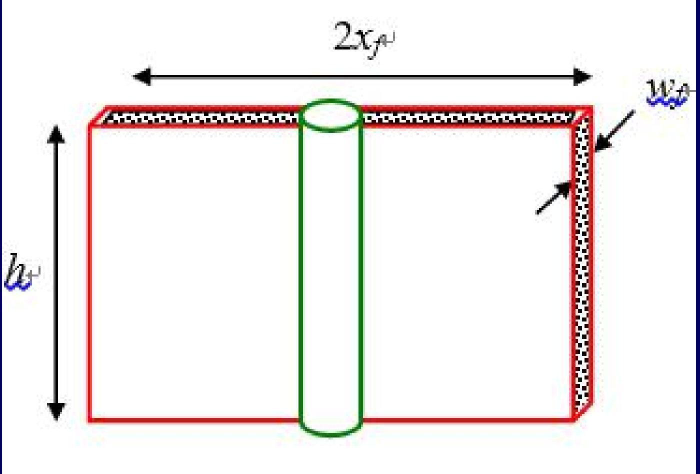


Diagram of Vertical Fractured Wells

When a vertical fracture exists, the pressure distribution in the formation can be written as follows:

$$P_D(x_D, y_D) = \ln r_{eD} - \frac{1}{4}\int_{1}^{-1} q_{fD}(\alpha)\ln\left[(x_D - \alpha)^2 + y_D^2\right]\cdot d\alpha$$

The reservoir pressure is equal to the fracture pressure at the fracture surface.

$$P_D(x_D, 0) = P_{fD}(x_D)\,,\quad P_{wD} = P_{fD}(0)$$

There are the following integral equations when they are combined:

$$P_{wD} + \frac{1}{4}\int_0^1 q_{fD}(\alpha)\left\{\ln\left[(x_D-\alpha)^2\right] + \ln\left[(x_D+\alpha)^2\right]\right\}\cdot d\alpha + \frac{\pi}{C_{FD}}\int_0^{x_D}\int_0^{v} q_{fD}(u)\,du\,dv = \ln r_{eD} + \frac{\pi x_D}{C_{FD}}$$

The double integral term can be discretized as follows:

$$\int_0^{x_D}\int_0^{v} q_{fD}(u)\,du\,dv = \sum_{i=1}^{j-1} q_{fDi}\left[0.5\Delta x^2 + \Delta x\left(x_{Dj} - i\Delta x\right)\right] + 0.125\,\Delta x^2 q_{fDj}$$

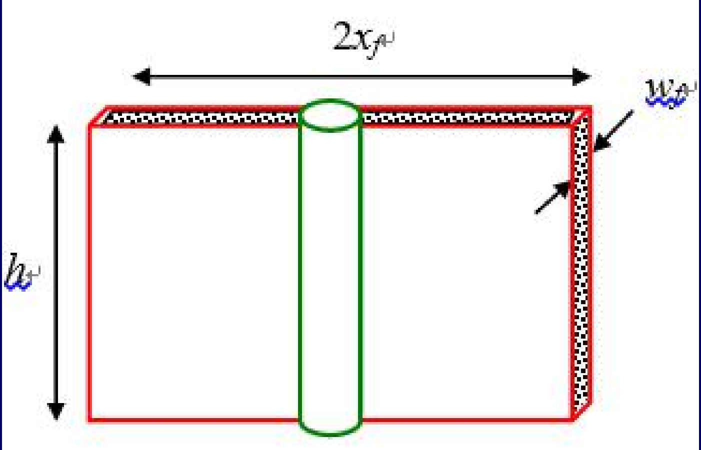


Diagram of Vertical Fractured Wells

Here, x_{Dj} is the midpoint of the discrete element for each fracture in j, and there is a mass conservation equation at the same time.

$$\Delta x_D \sum_{i=1}^{n} q_{fDi} = 1$$

Finally, a set of linear equations about P_{wD} and q_{fDi} is formed. The steady flow distribution and pressure distribution can be calculated after solving the equations.

Asymptotic analysis of the results of the previous section shows that the bottom hole pressure of quasi-steady flow in the middle and late Laplace transform space is:

$$s\widetilde{P}_{wD}(s) = \frac{2}{r_{eD}^2 s} + \ln\frac{r_{eD}}{2} + \frac{3}{4} + f(C_{FD})$$

After inversion, the asymptotic formula of quasi-steady wellbore pressure is obtained.

$$P_{wD}(t_{Df}) = \frac{2t_{Df}}{r_{eD}^2} + \ln\frac{r_{eD}}{2} + \frac{3}{4} + f(C_{FD})$$

Thus, the quasi-steady state productivity formula of vertical fractured wells with limited conductivity can be obtained.

$$q_f = \frac{kh\left(P_{avg} - P_{wf}\right)}{1.842 \times 10^{-3} \mu B \left[\ln \frac{r_e}{2x_f} + \frac{3}{4} + f(C_{FD}) \right]}$$

Comparing the above formula with the quasi-steady-state production formula of ordinary vertical wells, we can see that:

$$r_{equ} = 2x_f \exp\left\{ -\left[\frac{3}{2} + f(C_{FD}) \right] \right\}$$

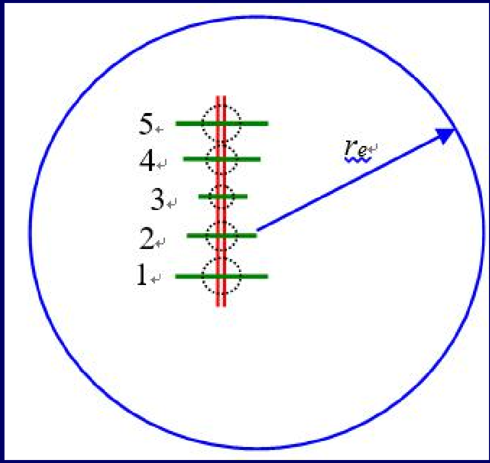
（5）Productivity formula of multi-stage fractured horizontal wells

According to the equivalent borehole diameter model, each fracture is equivalent to a vertical well, and its radius is respectively:

$$r_{equ1} = r_{equ5} = 0.499x_{f1}$$

$$r_{equ2} = r_{equ4} = 0.499x_{f2}$$

$$r_{equ3} = 0.499x_{f3}$$



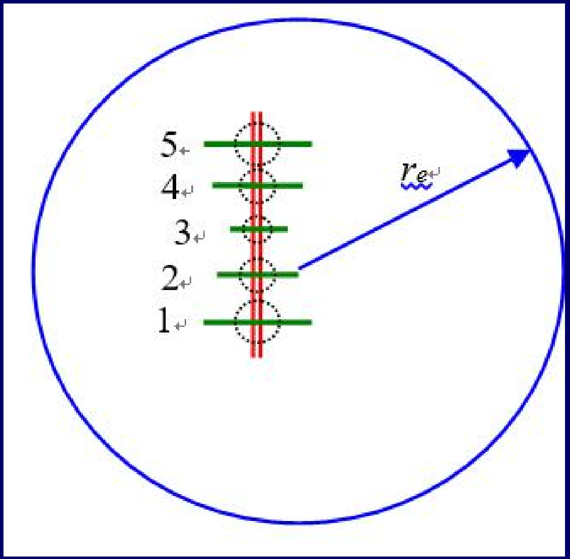
Five fractures for fractured horizontal wells

If q1 is assumed to be the flow rate of fracture (1,5), q_2 is the flow rate of fracture (2,4), q_3 is the flow rate of fracture 3, and P_w is the flow pressure of horizontal wellbore with uniform distribution, according to the principle of pressure drop superposition, the following conclusions can be obtained:

$$\begin{cases} p_w - C = \dfrac{\mu B}{2\pi kh}\left(q_1 \ln 4r_{equ1} + q_2 \ln 3d^2 + q_3 \ln 2d\right) \\ p_w - C = \dfrac{\mu B}{2\pi kh}\left(q_1 \ln 3d^2 + q_2 \ln 2dr_{equ2} + q_3 \ln d\right) \\ p_w - C = \dfrac{\mu B}{2\pi kh}\left(q_1 \ln 4d^2 + q_2 \ln d^2 + q_3 \ln r_{equ3}\right) \\ p_e - C = \dfrac{q_t \mu B}{2\pi kh} \ln r_e \end{cases}$$

It is further obtained that the total flow q_t is:

$$q_t = \frac{542.87kh(P_e - P_w)}{\mu B\left[\ln \dfrac{r_e}{(4dr_{equ1})^{\beta_1}(3d^2)^{\beta_2}(2d)^{\beta_3}}\right]}$$



Five fractures for fractured horizontal wells

$$\beta_1 = \frac{q_1}{q_t} = \frac{\alpha_1}{2\alpha_1 + 2\alpha_2 + \alpha_3}$$

$$\beta_2 = \frac{q_2}{q_t} = \frac{\alpha_2}{2\alpha_1 + 2\alpha_2 + \alpha_3}$$

$$\beta_3 = \frac{q_3}{q_t} = \frac{\alpha_3}{2\alpha_1 + 2\alpha_2 + \alpha_3}$$

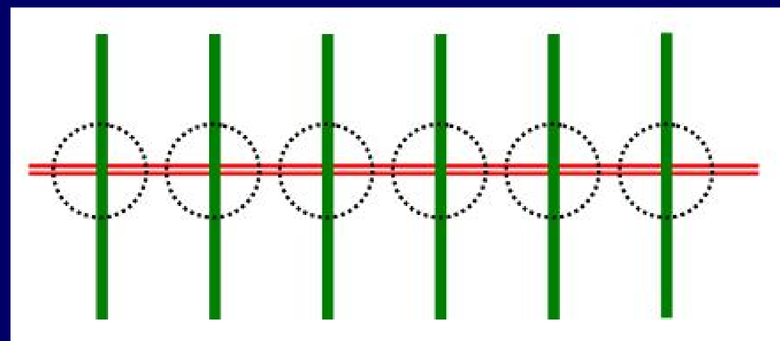
$$\alpha_1 = \ln 2 \ln 3 - \ln \frac{2r_{equ2}}{3d} \ln \frac{r_{equ3}}{2d}$$

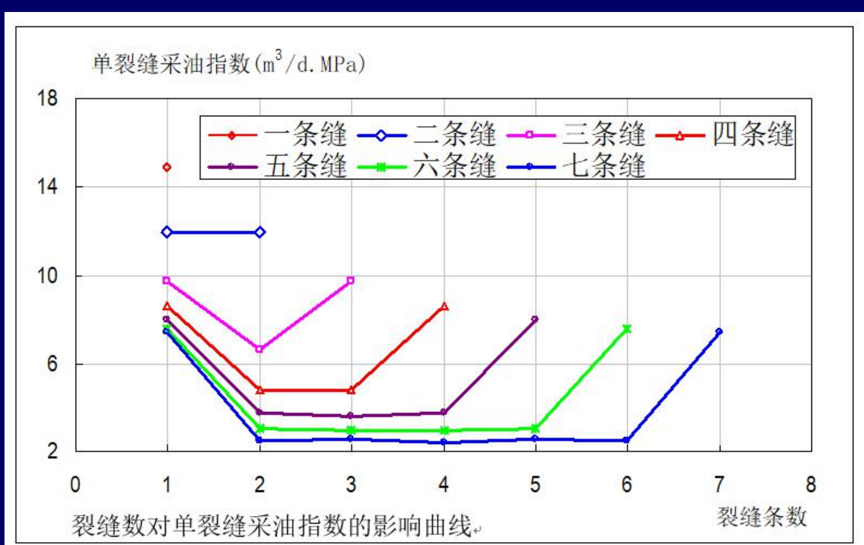
$$\alpha_2 = \ln \frac{r_{equ3}}{2d} \ln \frac{3d^2}{4r_{equ1}} + \ln 2 \ln \frac{d^2}{r_{equ1}}$$

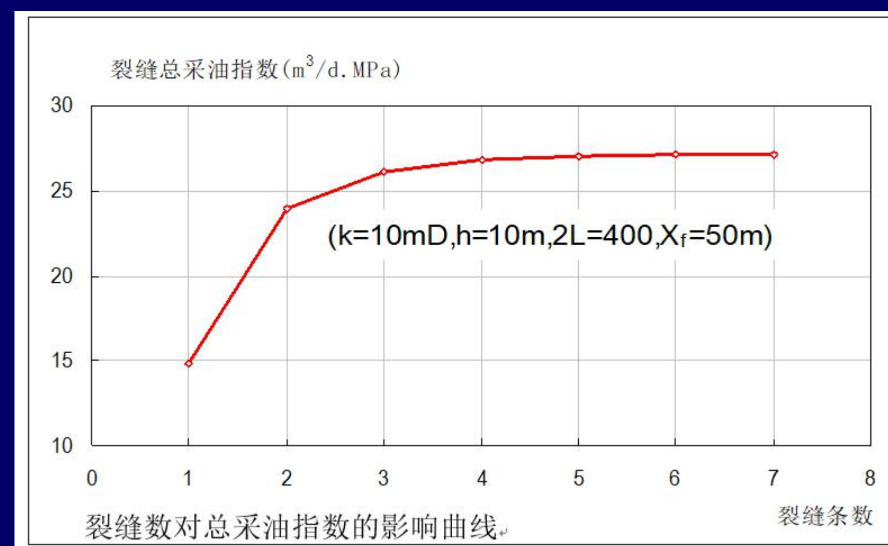
$$\alpha_3 = \ln \frac{d^2}{r_{equ1}} \ln \frac{2r_{equ2}}{3d} + \ln 3 \ln \frac{3d^2}{4r_{equ1}}$$

In fact, according to this idea, we can solve the problem of arbitrary fracture distribution.

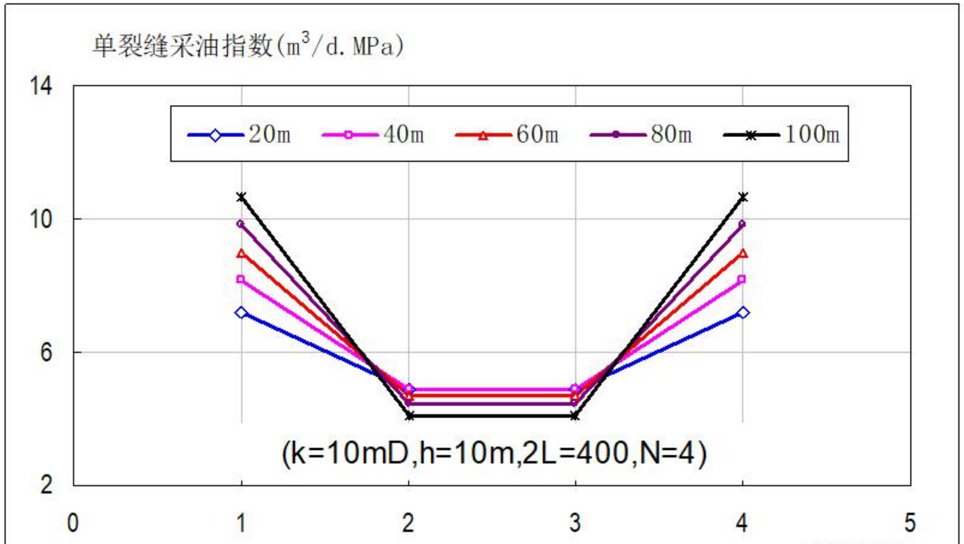
（6）Effect of number of fractures on productivity index



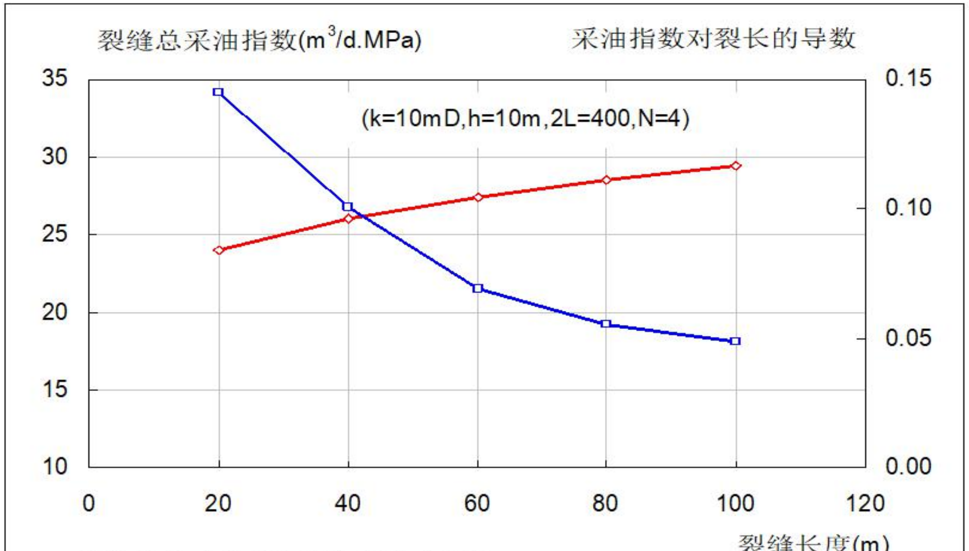




（6）Influence of crack length on production index

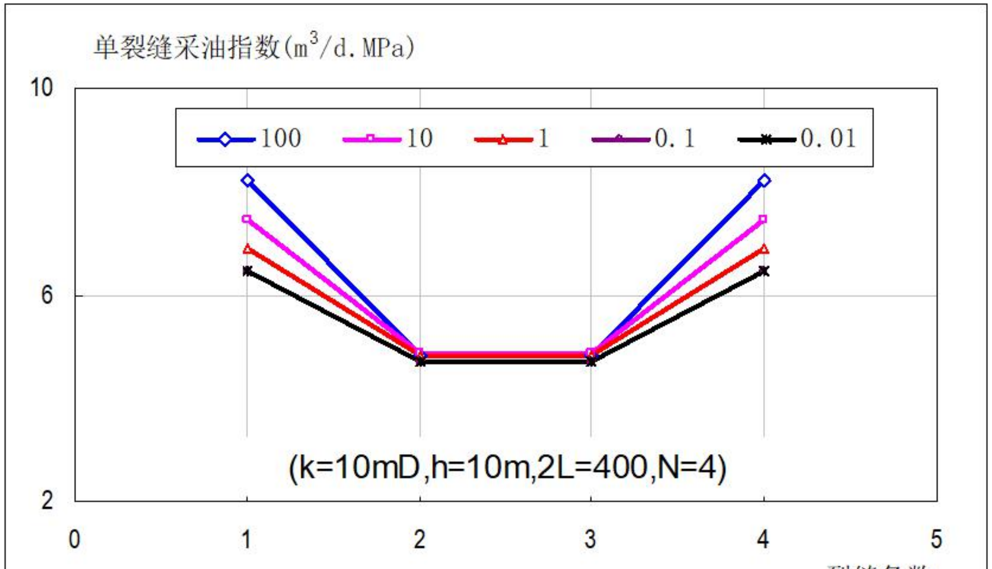


裂缝长度对单裂缝采油指数的影响曲线

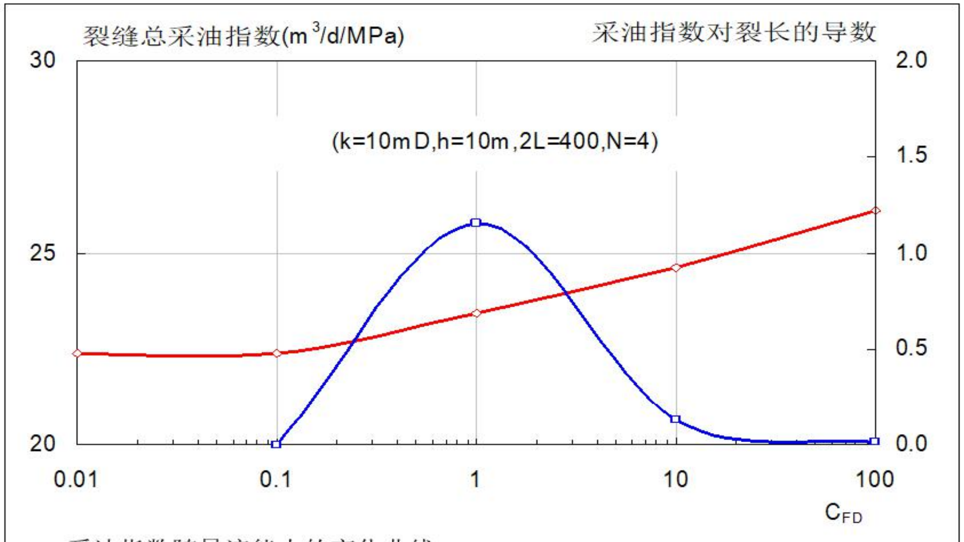


裂缝长度对总采油指数的影响曲线

（7）Effect of fracture conductivity on productivity index



导流能力对单裂缝采油指数的影响曲线



采油指数随导流能力的变化曲线