Advanced Reservoir Engineering

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Chapter 4 Decline Curve Analysis

Decline curve analysis is an important method of reservoir engineering, it is a traditional means of identifying well production problems and predicting well performance and life based on production data. in fact, it is a kind of statistical analysis.

These models include:

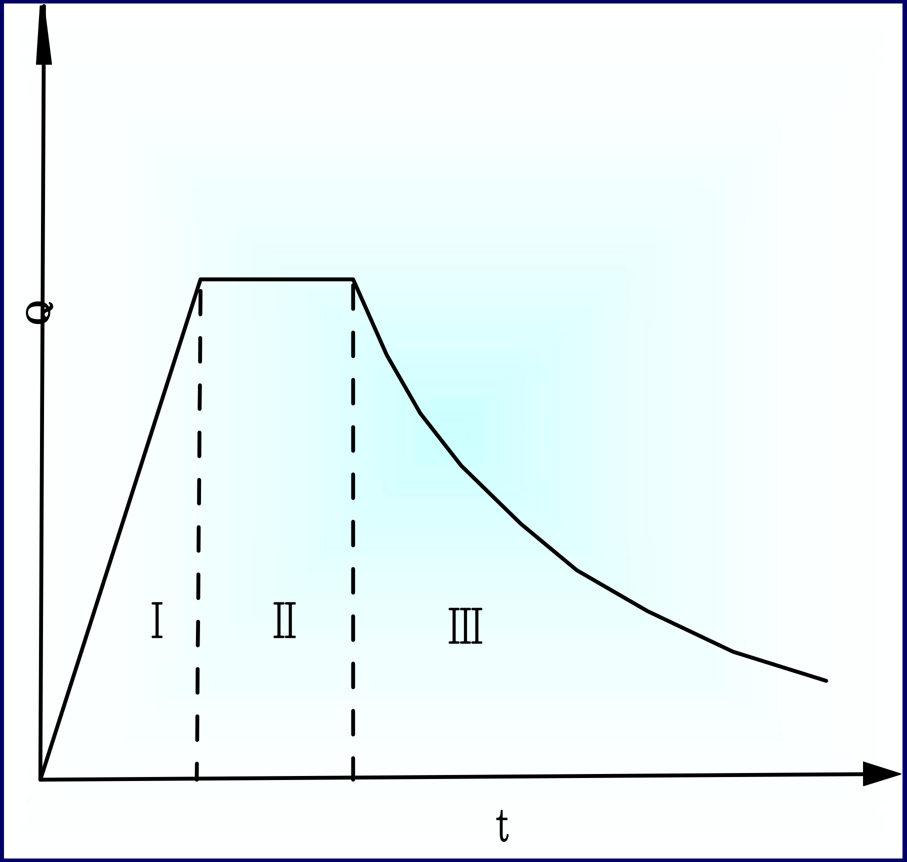
- Exponential decline： $N=0$
- Hyperbolic decline： $0<N<1$
- Harmonic decline： $N=1$

When the oilfield has reached certain recovery percent and enters production decline stage, we can judge the decline type , to predict its future production and to determine its recoverable reserves(N_R), recovery factor(E_R) and life time (T).

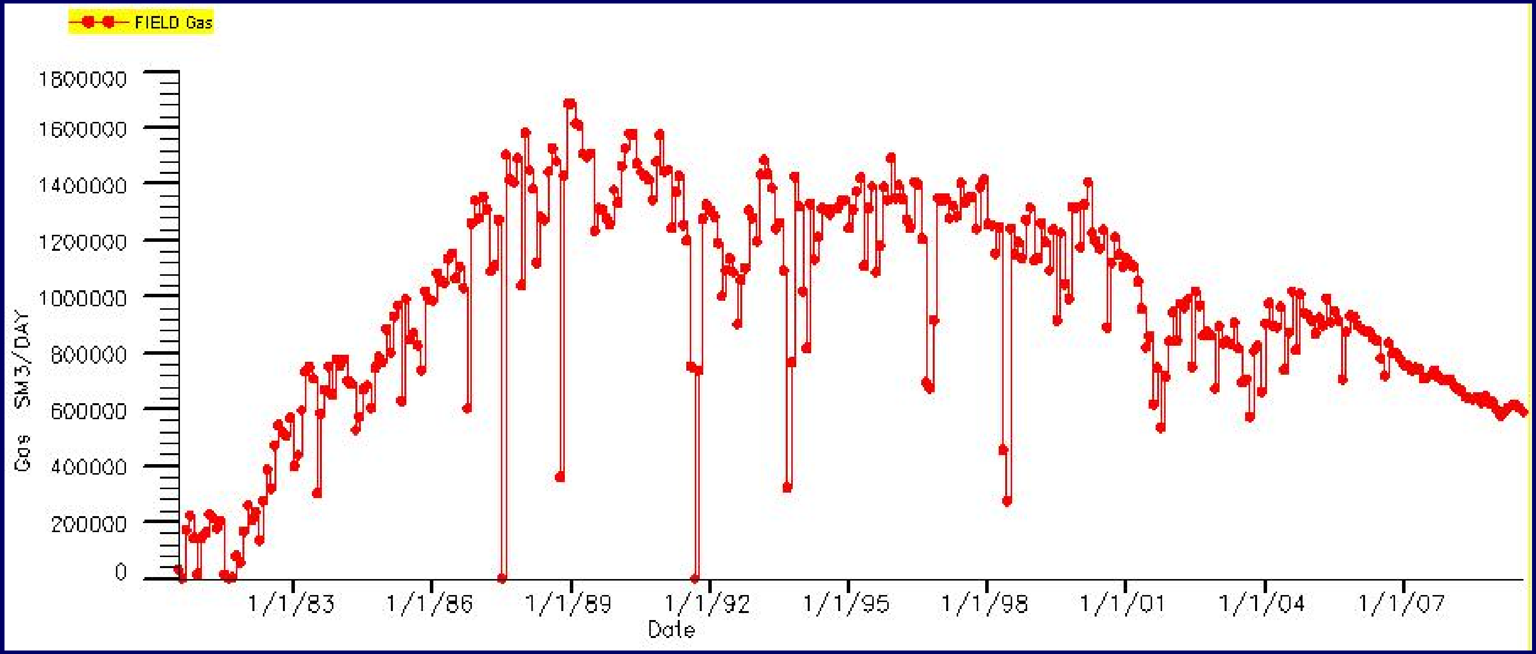
This method can be used in calculating both oilfield and single well, both oil and gas wells.

With a strong regular form, it could be used to compile corresponding computer programs according to mathematical formula to evaluate results quickly, thus having a widely application in oilfields.

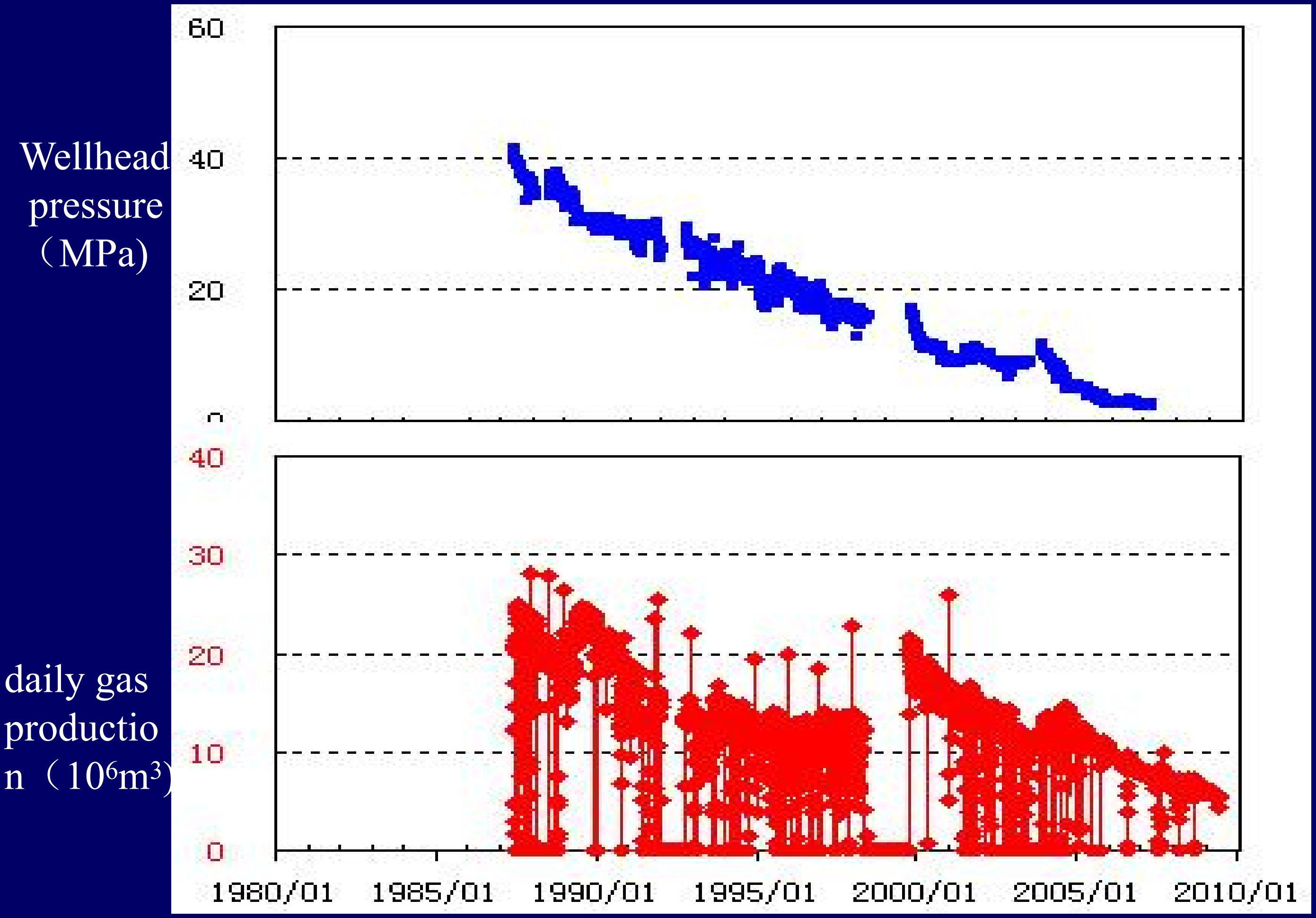
When an oilfield has brought into development, its production often undergoes three periods: rising, stable, and decline.



Development Model


FIELD Gas
1800000
1600000
1400000
1200000
1000000
800000
600000
400000
200000
0
Gas SM3/DAY
1/1/83
1/1/86
1/1/89
1/1/92
1/1/95
1/1/98
1/1/01
1/1/04
1/1/07
Date

But some fields are not rising and stable stage, at the beginning, the field has entered the stage of production decline.



The yield curve of a gas well

As the large amount of statistics of oilfield development suggest, when 60% of the recoverable reserves is produced, it means entering the decline period .

Chapter 4 Decline Curve Analysis

Section 1 Basic conception

Section 2 Decline type

Section 3 Comparison of decline types

Section 4 Determine the decline type

Section 5 Application of decline curve analysis

Section 1 Basic conception

1. Decline period

The decline period refers to that various measures imposed on oilfield failed to change the production decline trend. The length of decline period is mainly depend on the final economic and technical index requirement of oilfield development.

2. Decline rate （D）

It refers to the yield change rate or yield decline percentage in unit time. And it reflects the oilfields stable production situation .

The smaller the decline rate is, the better the stable production situation will be. The decline rate is the basis to make plans of oil production.

Represented as: $$D = -\frac{1}{Q}\frac{dQ}{dt} = -\frac{d\ln Q}{dt}$$

Where:

D=decline rate， %/mon or %/a

Q=production rate, ton per day, month, or year

t=The production time in decline period, day,month, or year

$\frac{dQ}{dt}$ = The production change rate in unit time

3. Decline index

The Decline index is a parameter measuring the rate of production decline , it is represented as a symbol “n” , the larger the “n” is, the faster it declines.

This index is very useful ,use it we can determine the decline type.

- Exponential decline——$n \to \infty$ infinite
- Hyperbolic decline—— $1 < n < \infty$
- Harmonic decline—— $n = 1$

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Section 2 Decline Type

In 1945, J.J.Arps put forward three typical decline types, that is, exponential decline, hyperbolic decline and harmonic decline.

Exponential decline——n→ ∞ infinite

Hyperbolic decline—— $1 < n < \infty$

Harmonic decline—— $n = 1$

The exponential and hyperbolic production declines occur in many reservoirs,while the third type, harmonic decline , is now believed to be uncommon.

The decline models are applicable to both oil and gas wells.

In the early decline period, these three decline types are somewhat similar, so the exponential decline is often adopted to study the real problem;

In the middle decline period, it usually accords with hyperbolic decline;

while in the late decline period, harmonic decline is more proper.

But the decline type is not invariable as it transforms under the influence of human and natural factor.

Exponential decline and harmonic decline are the particular decline type of hyperbolic decline. Through the theoretical study of exponential decline, harmonic decline and hyperbolic decline, Arps put forward the relation of production, decline rate and decline index as follows:

$$\frac{Q}{Q_i} = \left(\frac{D}{D_i}\right)^n$$

These three parameters are related through the formula (Arps,1945)

Where:

D_i——Initial decline rate, %/mon or %/a

Q_i——Initial production

1. Hyperbolic decline

I. The Conception of Hyperbolic Decline

II. The Production Formula of Hyperbolic Decline

III. The Cumulative Production of Hyperbolic Decline

IV. The Time Formula of Hyperbolic Decline

V. The Properties of Hyperbolic Decline

(1)The conception of hyperbolic decline

The equation that defines hyperbolic decline may be expressed as:

$$Q = \frac{Q_i}{\left(1 + \frac{D_i}{n} t\right)^n}$$

Where:

D_i——Initial decline rate, 1/time

Q_i——Initial production

n——Hyperbolic index

(2)Production formula of hyperbolic decline

definition

$$D=-\frac{1}{Q}\frac{dQ}{dt}$$

into

general formula

$$D=D_i\left(\frac{Q}{Q_i}\right)^{\frac{1}{n}}$$

$$-\frac{1}{Q}\frac{dQ}{dt}=D_i\left(\frac{Q}{Q_i}\right)^{\frac{1}{n}}$$

Separate variable and integra

$$-\int_{Q_i}^{Q}\frac{dQ}{Q^{\left(1+\frac{1}{n}\right)}}=\int_0^t\frac{D_i}{Q_i^{\frac{1}{n}}}dt$$

Collation

$$n\left(\frac{1}{Q^{\frac{1}{n}}}-\frac{1}{Q_i^{\frac{1}{n}}}\right)=\frac{D_i}{Q_i^{\frac{1}{n}}}t$$

$$Q=\frac{Q_i}{\left(1+\frac{D_i}{n}t\right)^{n}}$$

(3)Cumulative oil production formula of hyperbolic decline

$$N_p = E\int_0^t Q dt = E\int \frac{Q_i}{\left(1+\frac{D_i}{n}t\right)^n} dt$$

Integral

$$N_p = \frac{EQ_i}{D_i}\left(\frac{n}{n-1}\right)\left[1-\left(1+\frac{D_i}{n}t\right)^{1-n}\right]$$

Collation

$$N_p = \frac{EQ_i}{D_i}\left(\frac{n}{n-1}\right)\left[1-\left(\frac{Q_i}{Q}\right)^{\frac{1-n}{n}}\right]$$

"E" in the formula is a Time conversion coefficient , when the unit of production time and production is inconsistent.

(3)Time formula of hyperbolic decline

$$Q = \frac{Q_i}{\left(1 + \frac{D_i}{n} t\right)^n}$$

$$\left(\frac{Q_i}{Q}\right)^{\frac{1}{n}} = 1 + \frac{D_i}{n} t$$

$$t = \frac{n}{D_i}\left[\left(\frac{Q_i}{Q}\right)^{\frac{1}{n}} - 1\right]$$

(4)The properties of hyperbolic decline

The hyperbolic decline also called double logarithm decline

$$Q = \frac{Q_i}{\left(1+\frac{D_i}{n}t\right)^n}$$

logarithm

$$LgQ = LgQ_i - nLg\left(1+\frac{D_i}{n}t\right)$$

$$LgQ = \boxed{Lg\frac{Q_i}{\left(\frac{D_i}{n}\right)^n}} - \boxed{n}Lg\left(\boxed{\frac{n}{D_i}}+t\right)$$

$$LgQ = A_3 - B_3 Lg\left(t + C\right)$$

Where:

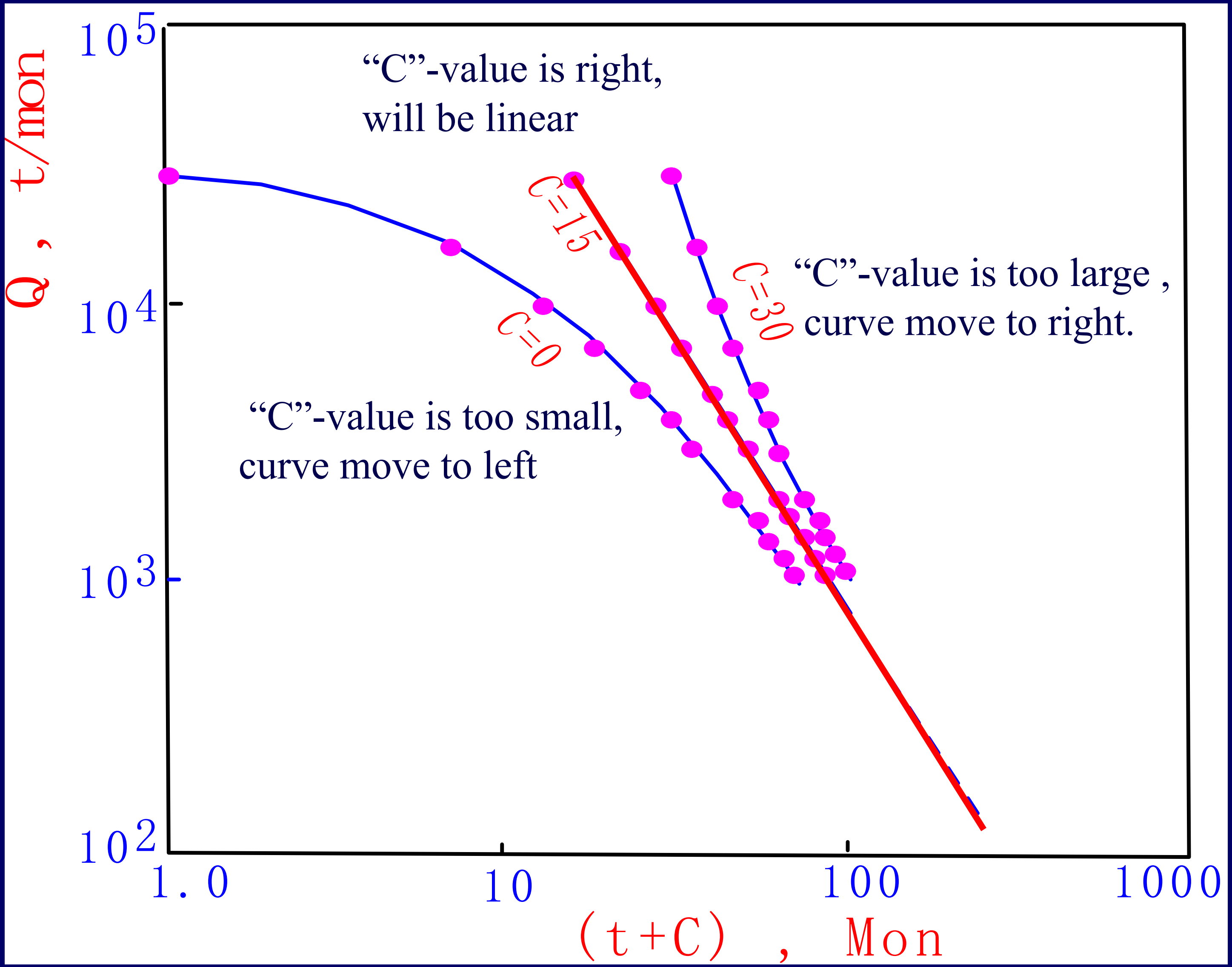
$$A_3 = Lg\frac{Q_i}{\left(\frac{D_i}{n}\right)^n} \qquad B_3 = n \qquad C = {}^{n}/_{D_i}$$

$$LgQ = A_3 - B_3 Lg(t + C)$$

Where: C—— Curve shift constant

For hyperbolic decline, it can be given a different value of C, use the curve shift method to get a best straight-line.

Given different "C"-value , to calculate the different value of (t+C) , if the "C"-value is right, a plot of lgQ~lg(t+C) will be linear , but if the "C"-value is small ,we will have a curve move to left ,whereas , if the "C"-value is big ,we will have a curve move to right.



Q， t/mon
10^5
10^4
10^3
10^2
1.0
10
100
1000
(t+C)， Mon
“C”-value is right,
will be linear
C=15
C=0
C=30
“C”-value is too large ,
curve move to right.
“C”-value is too small,
curve move to left

lg(t+C) ~ lg Q

2. Exponential decline

I. The conception of exponential decline

II. The production formula of exponential decline

III. The cumulative production of exponential decline

IV. The time formula of exponential decline

V. The properties of exponential decline

(1) The conception of exponential decline

Exponential decline refers to the decline rate (D) is equal to a constant. Thus, it is also called constant percent decline, and due to the semilog linear relation of lgQ vs t, it is called semilog decline as well.

The simplest type of decline curve is the exponential,the equation that defines exponential decline may be written:

$$Q = Q_i e^{-Dt}$$

Where:

Q——producing rate at time "t", ton/day

Q_i——Initial production, ton/day

D——Initial decline rate,1/time

t——time,days

(2) Production formula of exponential decline

definition

(I)

$$D = -\frac{1}{Q}\frac{dQ}{dt}$$

Separate variable and integral

$$\int_0^t D dt = -\int_{Q_i}^{Q} \frac{dQ}{Q}$$

$$Dt = -\ln\frac{Q}{Q_i}$$

$$Q = Q_i e^{-Dt}$$

(II)

$$Q = \frac{Q_i}{\left(1+\frac{D_i}{n}t\right)^n}$$

$n \to \infty$

infinite

$$\lim_{n\to\infty}\left(1+\frac{D_i}{n}t\right)^n = e^{D_i t}$$

$D_i = D = const$

$$Q = Q_i e^{-Dt}$$

(3)Cumulative oil production formula of exponential decline

$$N_p = E\int_0^t Qdt = E\int_0^t Q_i e^{-Dt} dt = \frac{EQ_i}{D}\left(1 - e^{Dt}\right)$$

$$Q = Q_i e^{-Dt}$$

$$N_p = \frac{E(Q_i - Q)}{D}$$

E in the formula is a Time conversion constant , when the unit of production time and production is inconsistent.

(4)Time formula of exponential decline

$$Q = Q_i e^{-Dt}$$

$$\ln Q = \ln Q_i - Dt$$

$$t = \frac{1}{D} \ln \frac{Q_i}{Q}$$

(5)The properties of exponential decline

(I)Exponential decline rate is equal to a constant

$$D_i = D = const$$

The exponential decline also called constant percentage decline

(II) A plot of the lgQ vs t is a straight-line relationship

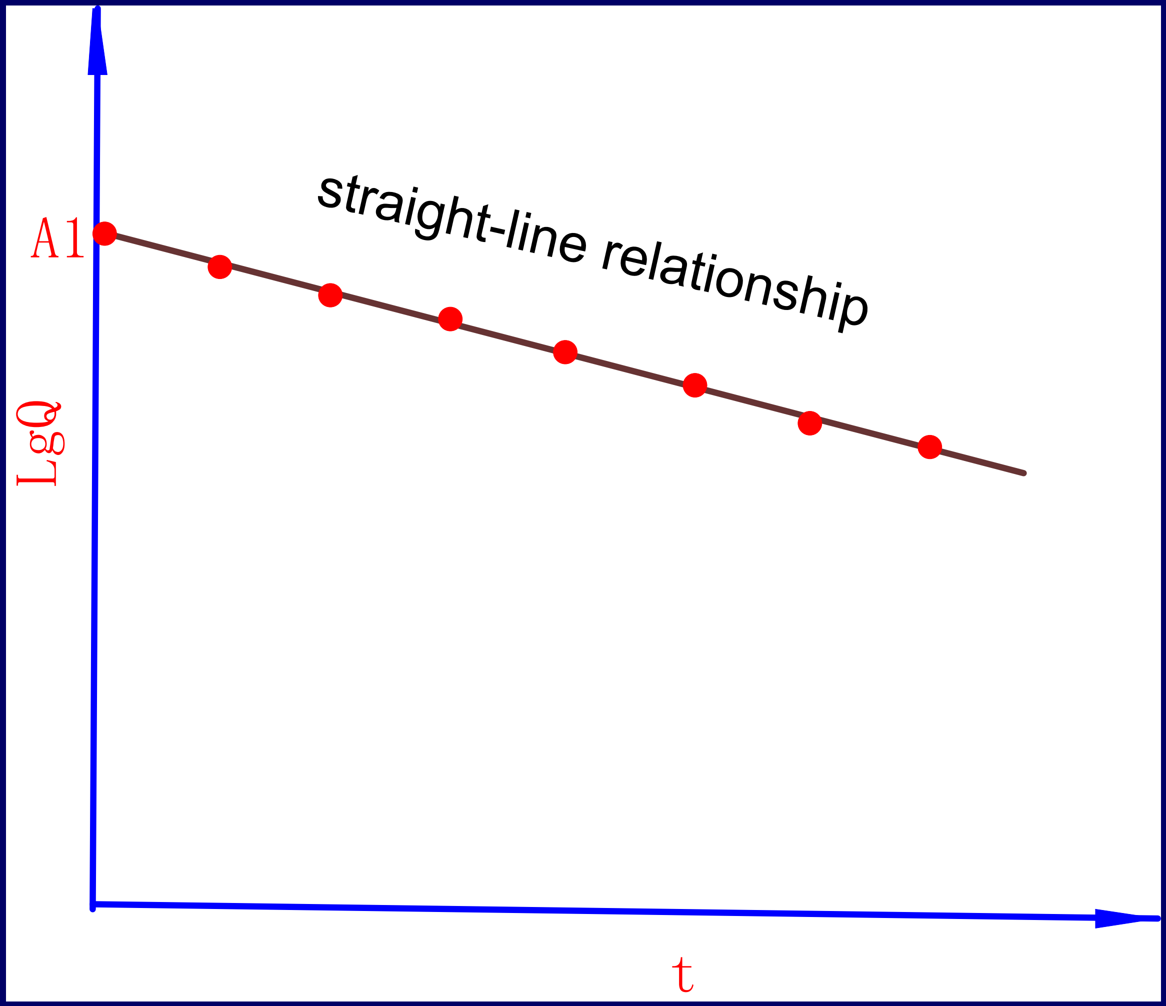
$$Q = Q_i e^{-Dt}$$

$$LgQ = A_1 - B_1 t$$

Where:

$$A_1 = LgQ_i$$

$$B_1 = D/2.303$$

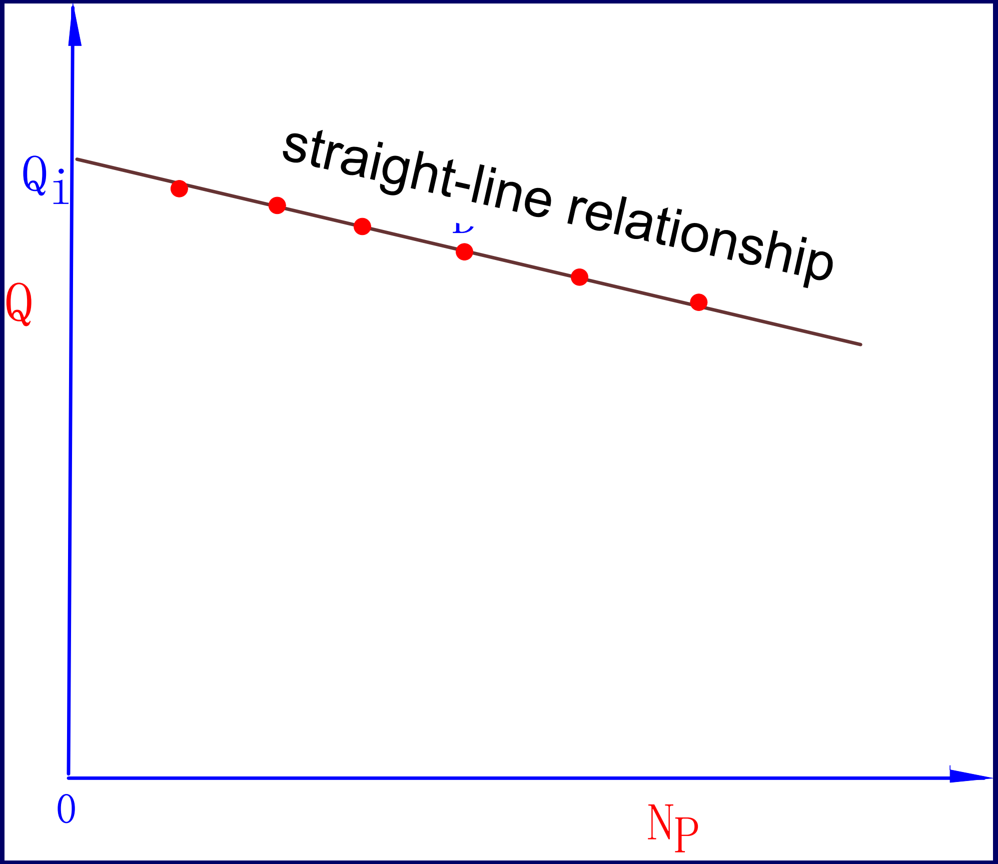


lgQ vs. time

(III) A plot of Q vs Np is a straight-line relationship

$$N_p = \frac{E(Q_i - Q)}{D}$$

$$Q = Q_i - \frac{D}{E} N_p$$



Q vs. Np

3. Harmonic decline

I. The conception of harmonic decline

II. The production formula of harmonic decline

III. The cumulative production of harmonic decline

IV. The time formula of harmonic decline

V. The properties of harmonic decline

(1) The conception of harmonic decline

Harmonic decline refers to the decline rate (D) with time is not a constant in decline period , but it's decline rate is proportional to production. The smaller the production is, the smaller the decline rate will be, and the trend of production decline is slowing down gradually.

The equation that defines harmonic decline may be written:

$$D = D_i \frac{Q}{Q_i}$$

Where:

Q——producing rate at time "t", ton/day

Q_i——Initial production, ton/day

D——Initial decline rate, 1/time

(2)Production formula of harmonic decline

$$Q = \frac{Q_i}{\left(1 + \frac{D_i}{n} t\right)^n}$$

$$\xrightarrow{n=1}$$

$$Q = \frac{Q_i}{1 + D_i t}$$

(3) Cumulative oil production formula of harmonic decline

$$N_p = E\int_0^t Q\,dt = E\int_0^t \frac{Q_i}{1+D_i t}dt = \frac{EQ_i}{D_i}\ln(1+D_i t)$$

$$Q = \frac{Q_i}{1+D_i t}$$

$$N_p = \frac{EQ_i}{D_i}\ln\frac{Q_i}{Q}$$

(4)Time formula of harmonic decline

$$Q = \frac{Q_i}{1 + D_i t}$$

$$t = \frac{Q_i - Q}{D_i Q}$$

(5)The properties of harmonic decline

A plot of lgQ vs Np is a straight-line relationship

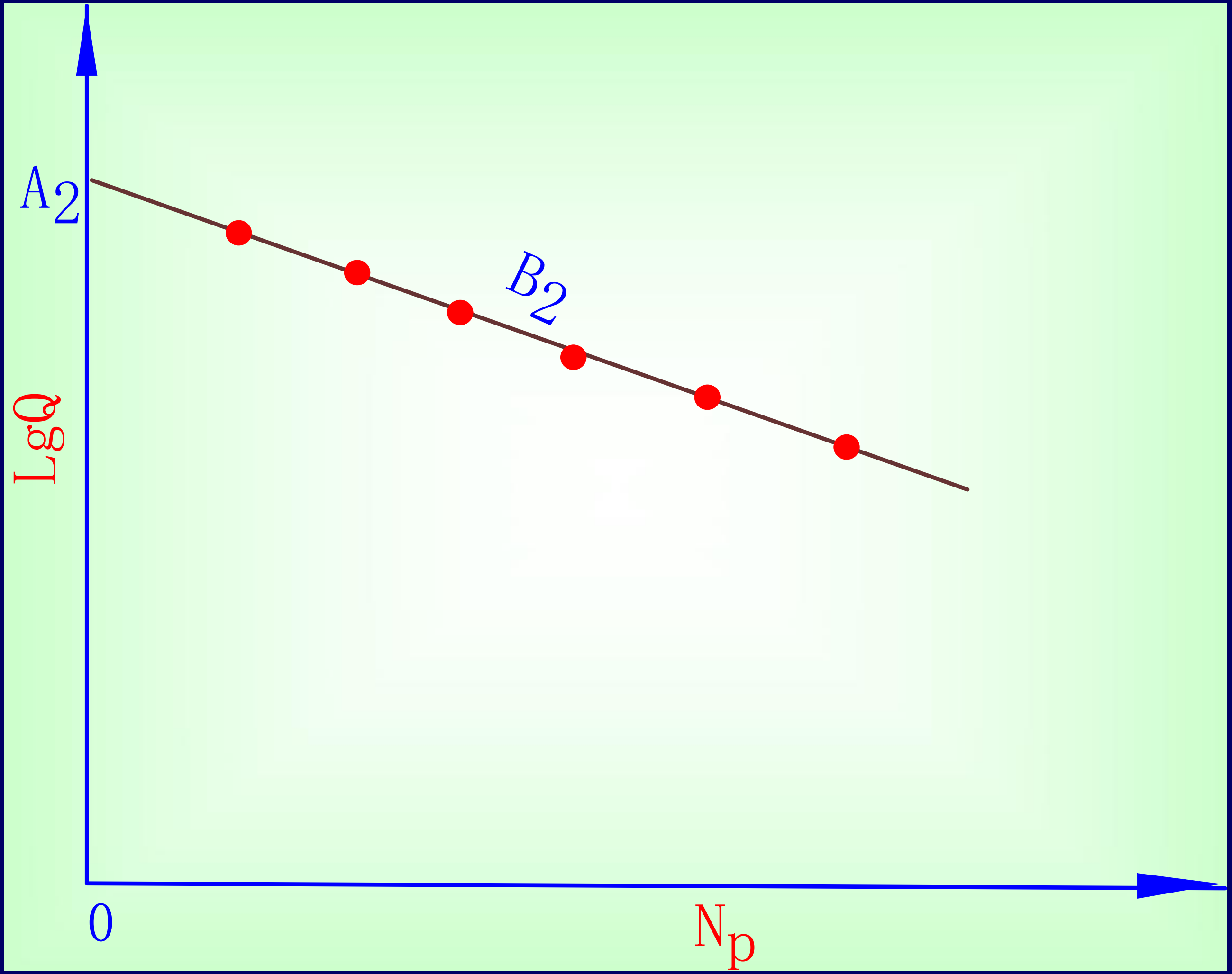
$$N_p = \frac{EQ_i}{D_i}\ln\frac{Q_i}{Q}$$

$$LgQ = LgQ_i - \frac{D_i}{2.303EQ_i}N_p$$

$$LgQ = A_2 - B_2N_p$$

Where:

$$A_2 = LgQ_i \qquad B_2 = \frac{D_i}{2.303Q_iE}$$



lgQ vs. Np

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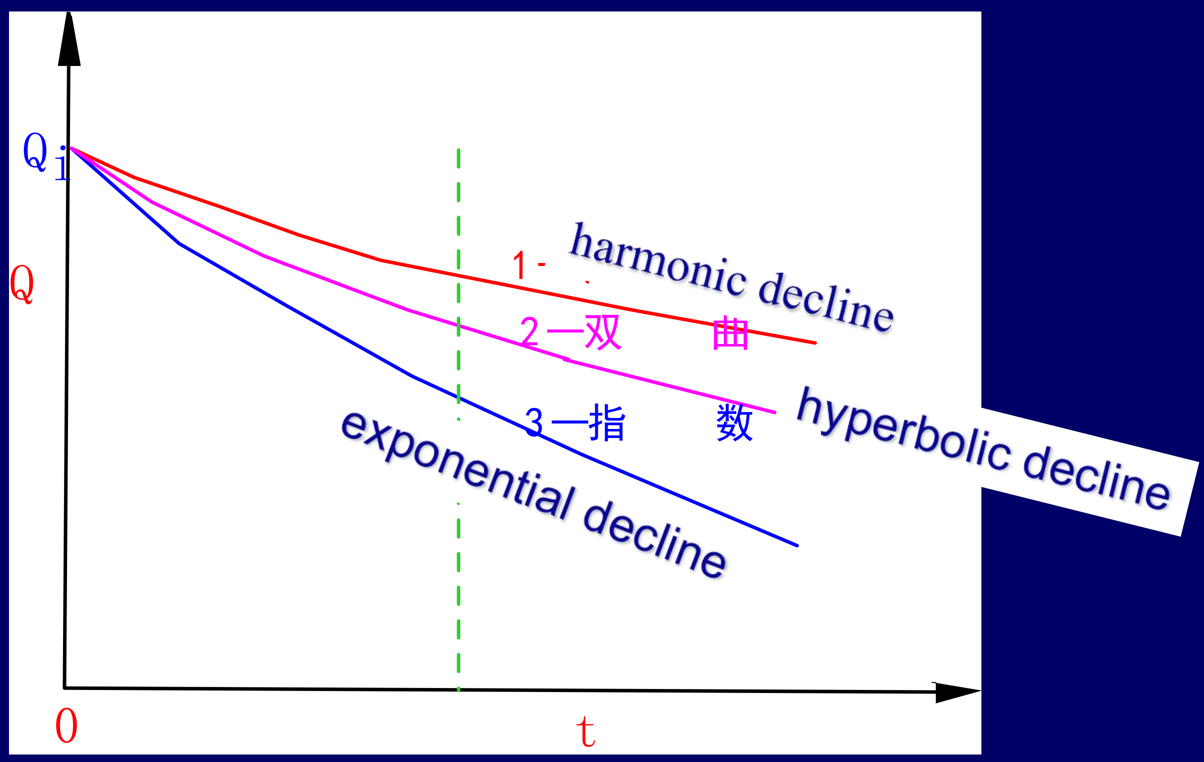
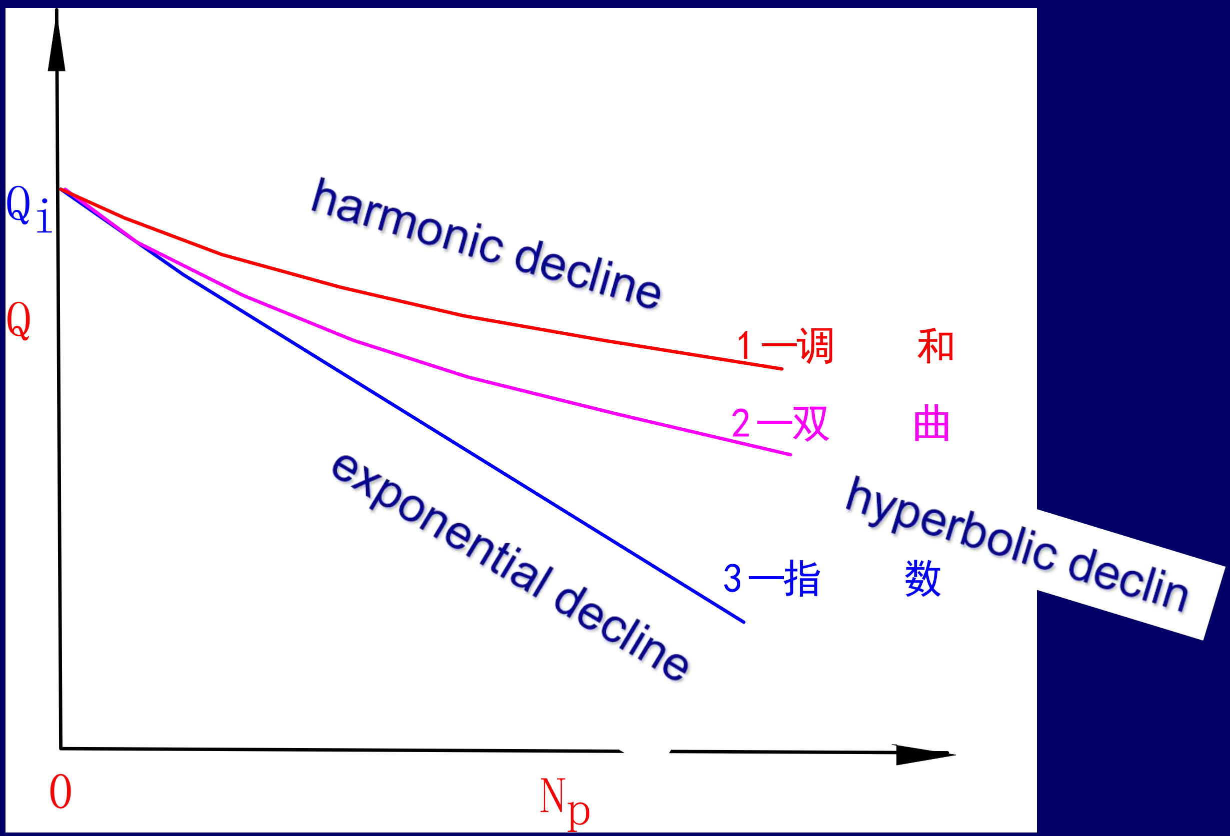
3. In different coordinate system, three types decline have difference to each other

(1)On the curve of Q vs. t relationship , three types of decline all present curve,but the decline speed is different , the exponential decline speed is the fastest, harmonic decline speed is the lowest, hyperbolic decline speed is between two.



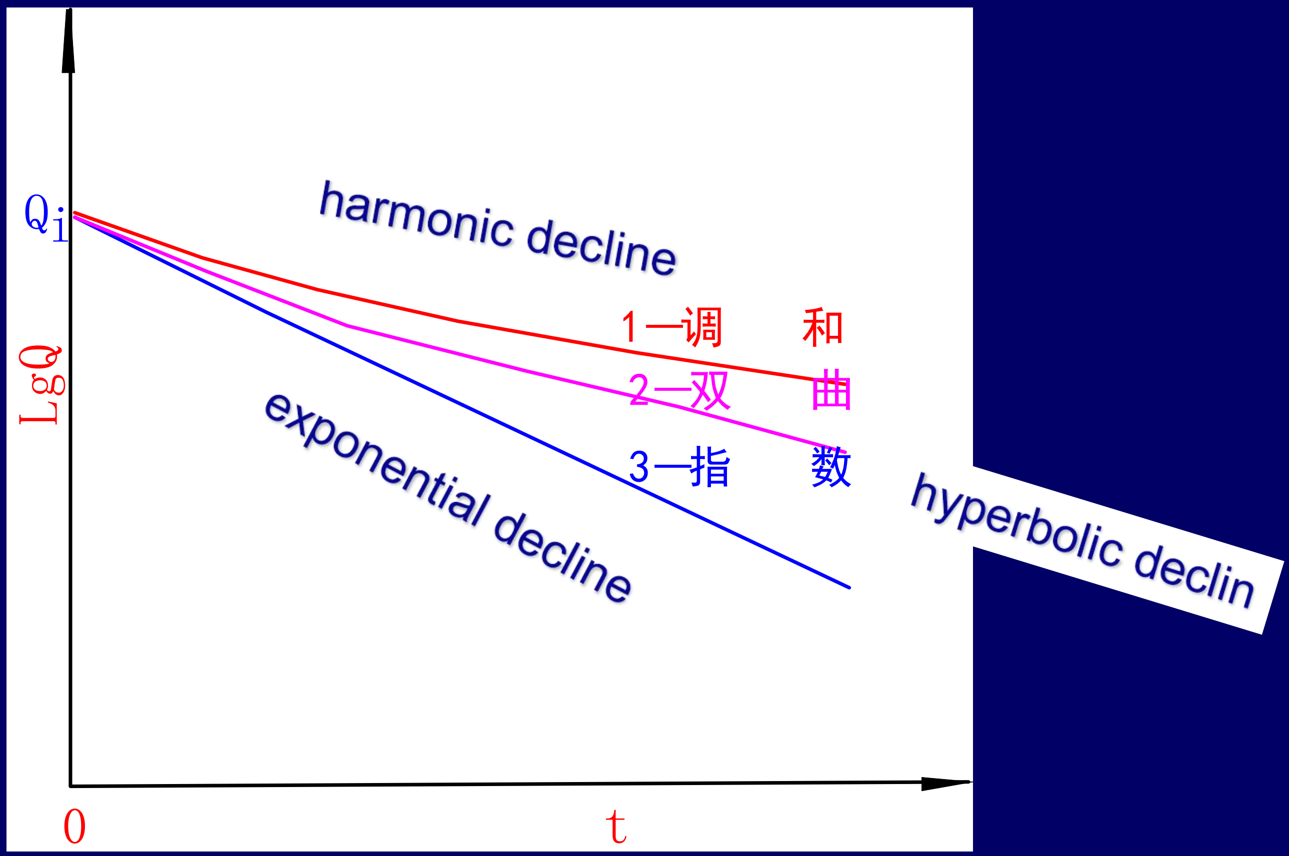
Q vs. t

(2) On the curve of Q vs. Np relationship , only exponential decline present straight line, the other decline present curve.



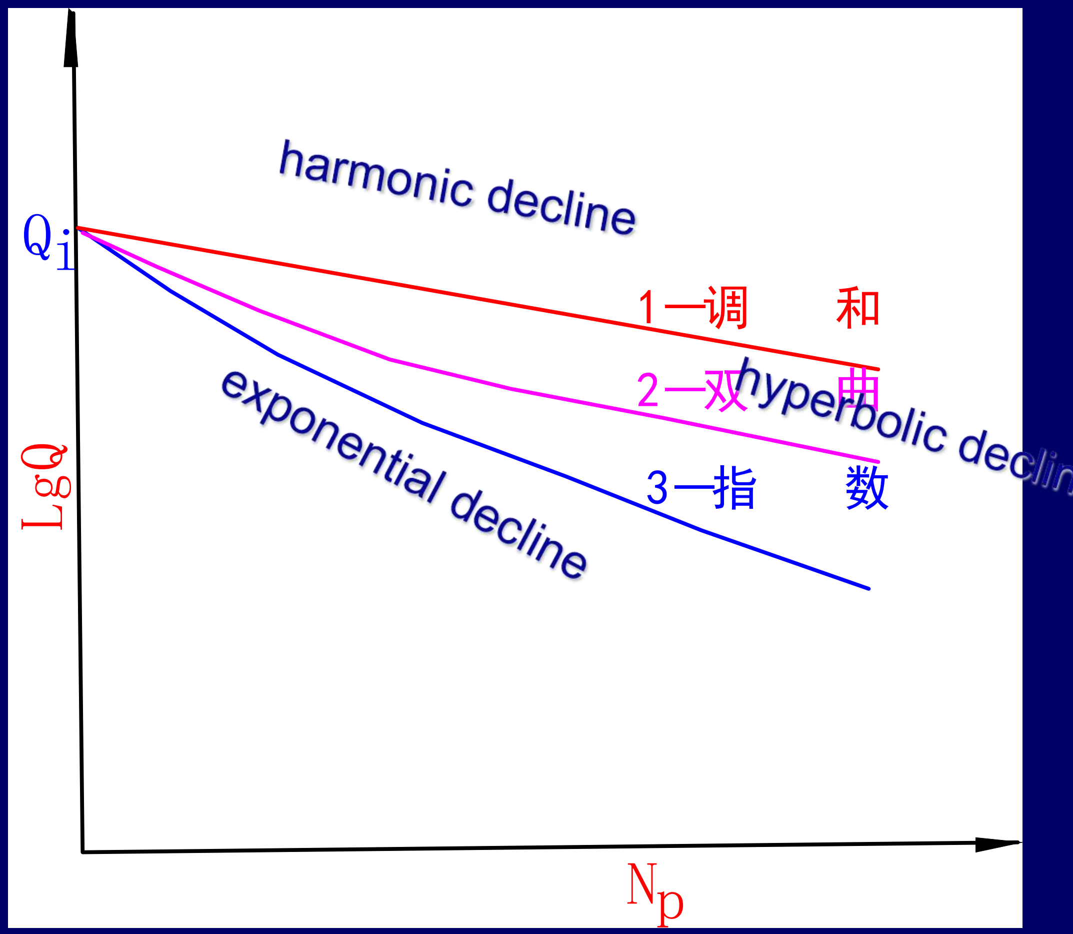
Q vs. Np

(3)On the curve of lgQ vs. t relationship , only exponential decline present straight line, the other decline present curve.



lgQ vs. t

(4) On the curve of lgQ vs. Np relationship , only harmonic decline present straight line, the other decline present curve



lgQ vs. Np

4. Each type of decline has its own correlation equation

decline type	exponential decline	harmonic decline	hyperbolic decline
decline exponent	$n \to \infty$	$n = 1$	$1 < n < \infty$
decline ratet	$D = D_i = const$	$D = D_i\left(\frac{Q}{Q_i}\right)$ $D = D_i(1 + D_i)^{-1}$	$D = D_i\left(\frac{Q}{Q_i}\right)^{\frac{1}{n}}$ $D = D_i\left(1 + \frac{D_i}{n}t\right)^{-n}$
production	$Q = Q_i e^{-Dt}$	$Q = Q_i(1 + D_i t)^{-1}$	$Q = Q_i\left(1 + \frac{D_i}{n}t\right)^{-n}$
cumulative production	$N_p = \frac{E(Q_i - Q)}{D}$	$N_p = \frac{EQ_i}{D_i}\ln\frac{Q_i}{Q}$	$N_p = \frac{EQ_i}{D_i}\left(\frac{n}{n-1}\right)\left[1 - \left(\frac{Q_i}{Q}\right)^{\frac{1-n}{n}}\right]$
Q VS. t	$\ln Q = \ln Q_i - Dt$	$\frac{1}{Q} = \frac{1}{Q_i} + \frac{D_i}{Q_i}t$	$\left(\frac{1}{Q}\right)^{\frac{1}{n}} = \left(\frac{1}{Q_i}\right)^{\frac{1}{n}} + \frac{D_i t}{nQ_i^{\frac{1}{n}}}$
time	$t = \frac{1}{D}\ln\frac{Q_i}{Q}$	$t = \frac{Q_i - Q}{D_i Q}$	$t = \frac{n}{D_i}\left[\left(\frac{Q_i}{Q}\right)^{\frac{1}{n}} - 1\right]$
Q VS. Np	$Q = Q_i - \frac{D}{E}N_p$	$\ln Q = \ln Q_i - \frac{D_i}{EQ_i}N_p$	$Q = \frac{Q_i}{\left\{1 - \left[\frac{D_i(n-1)N_p}{EnQ_i}\right]\right\}^{\frac{n}{1-n}}}$

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Section 4 Determine the decline type

When the oilfield came into decline period , we should according to the production data , make use of difference methods to determine the decline type and the decline parameter（D、D_i、n）, Therefore, establish empirical formula to forecast oil producing rate in the future .

Section 4 Determine the decline type

Generally speaking, we will see whether it belongs to exponential decline or not. If it is not exponential decline, then we will consider if it is harmonic decline. Finally, we will use the hyperbolic decline to conduct judgment.

Methods

When the oilfield is in decline stage, we can use different methods to determine the decline type.

At present, the following methods are common to use.

1.Graphic method

2." Cut-and-try " method （Trial and error）

3. Curve shift method

4. Two variable regression method

5. The loss-ratio method

6. Newton iterative method

7. Correlation coefficient comparison method

8. Typical curve fitting method

1.Graphic method

Production data can be plotted in different ways to determine the decline type.

This job can be easily performed with computer.

(1) If it is exponential decline type, a plot of lgQ vs. t and Q vs.Np will be linear.

For the exponential decline type,it has the following formula:

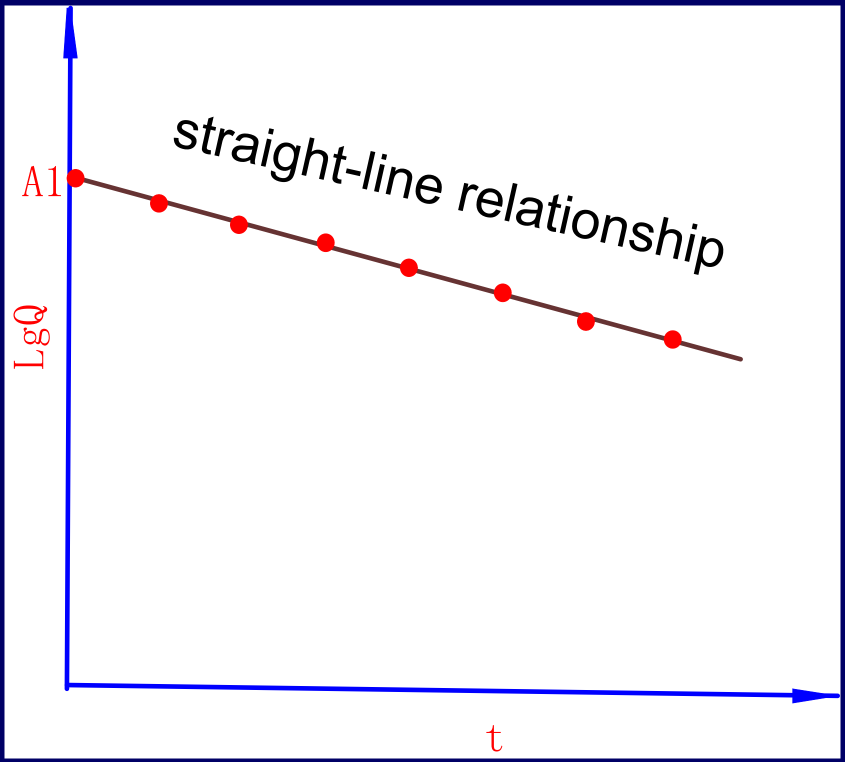
$$Q = Q_i e^{-Dt} \rightarrow LgQ = A_1 - B_1 t$$

Where: $A_1 = LgQ_i$ $\quad B_1 = D/2.303$

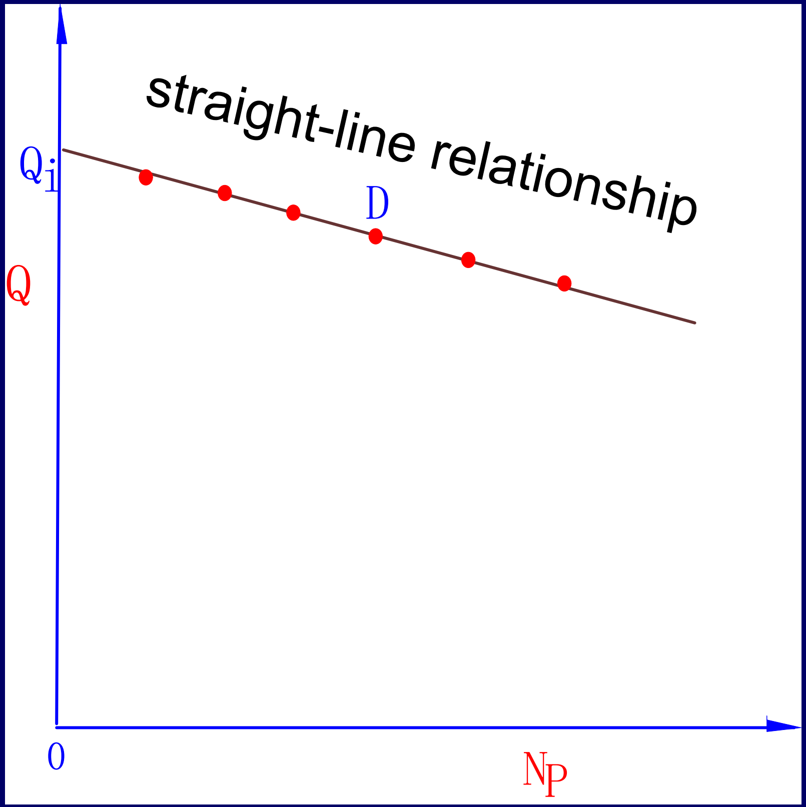
$$N_p = \frac{E(Q_i - Q)}{D} \rightarrow Q = Q_i - \frac{D}{E} N_p$$

"E" Time conversion coefficient , when the unit of production time and production is inconsistent.

Exponential decline



lgQ vs. t



Q vs. Np

(2)If it is the harmonic decline type, a plot of lgQ vs.Np will be linear

For the harmonic decline type,it has the following formula:

$$\boxed{N_p = \frac{EQ_i}{D_i} \ln \frac{Q_i}{Q}}$$

$$\downarrow$$

$$\boxed{LgQ = LgQ_i - \frac{D_i}{2.303EQ_i} N_p}$$

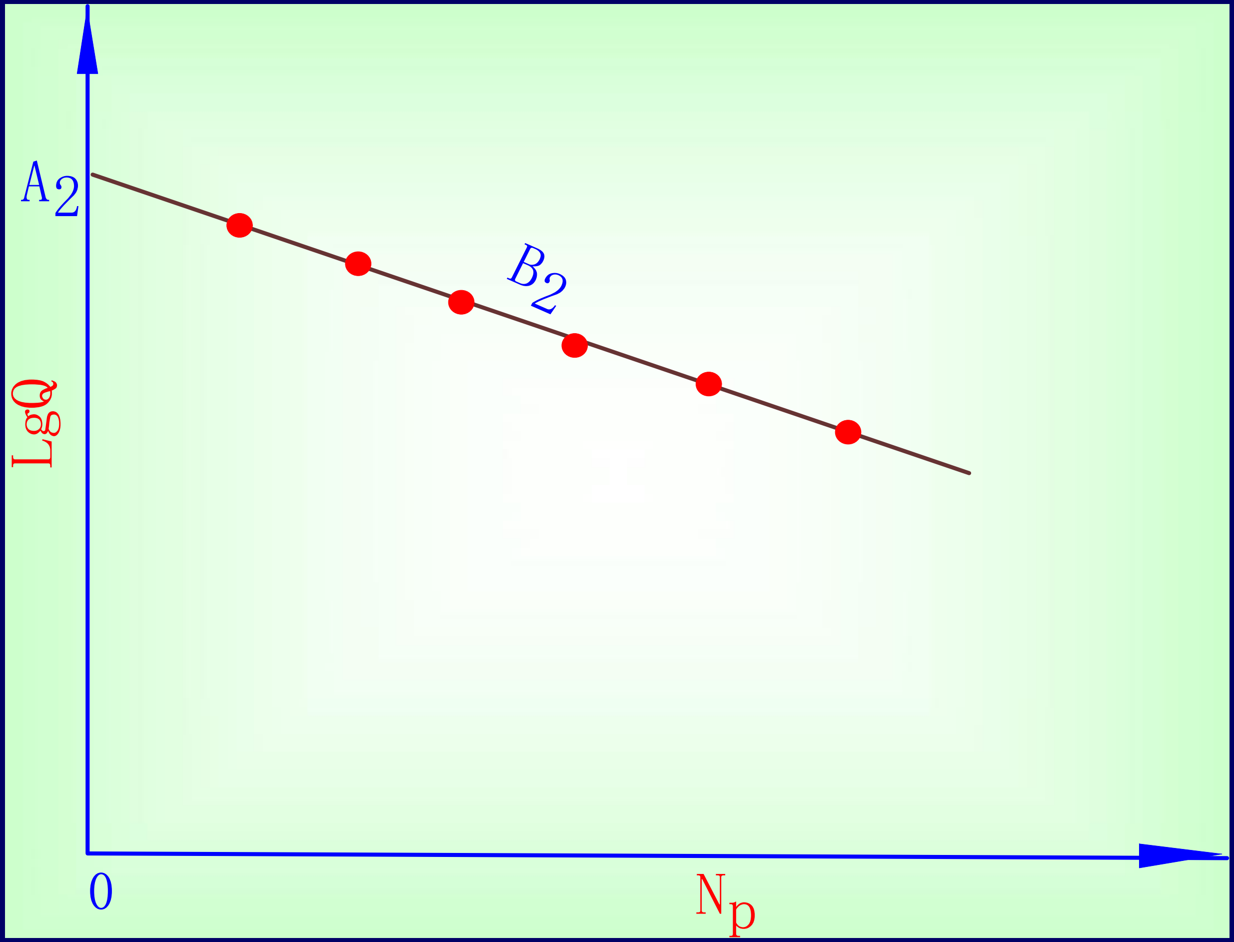
$$\downarrow$$

$$\boxed{LgQ = A_2 - B_2 N_p}$$

Where:

$$A_2 = LgQ_i \qquad B_2 = \frac{D_i}{2.303Q_iE}$$

Harmonic decline



lgQ vs. Np

(3) If no straight line is seen in these plots,the hyperbolic decline type may be verified.

This job can be easily performed with computer program.

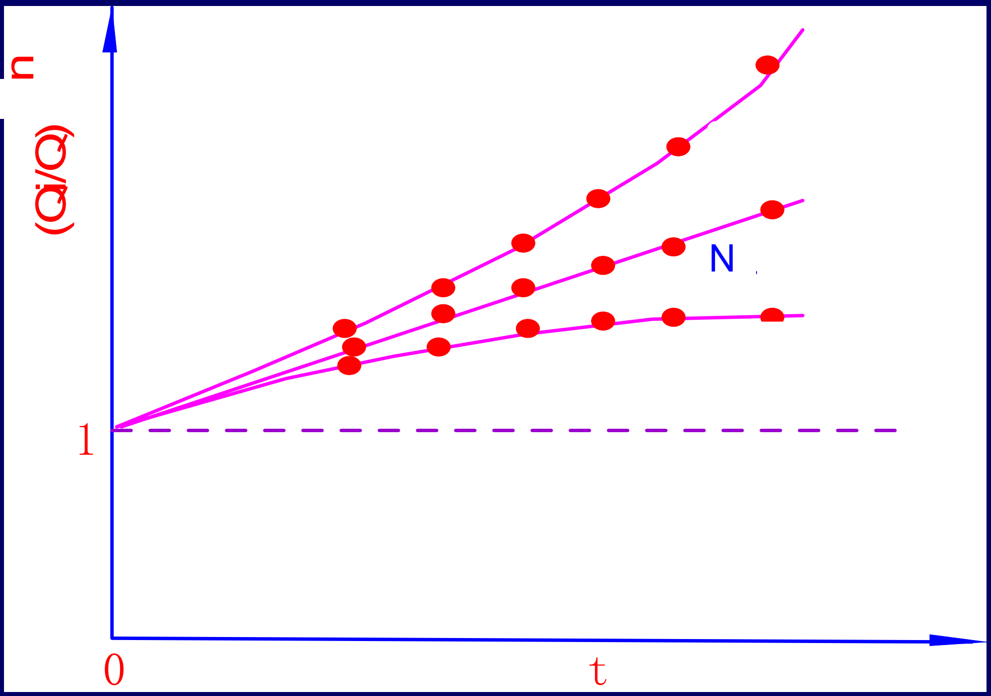
2." Cut-and-try " (Trial and error)

This method is given different n-value , then having repeatly trial calculation , until obtain a satisfying result.

We know :

$$Q = \frac{Q_i}{(1 + nD_i t)^{\frac{1}{n}}}$$

$$\therefore \left(\frac{Q_i}{Q}\right)^n = 1 + \boxed{nD_i} t = A + \boxed{B} t$$



" Cut-and-try " method

According to the production data ,given different “n”-value , to calculate the different value of $(\frac{Q_i}{Q})^n$. if “n”-value is right , a plot of $(\frac{Q_i}{Q})^n \sim t$ will be linear , but if the “n”-value is low , we will have a downbent curve , whereas , if the “n”-value is high, we will have a upswept curve.

For this method , first we can give a suitable initial “n ”-value , then according to the accuracy requirment to select a suitable step ,use computer to calculate the correlation coefficient (R) to determine the linearity.

3. Curve shift method

$$Q = \frac{Q_i}{\left(1 + \frac{D_i}{n} t\right)^n}$$

logarithm

$$LgQ = LgQ_i - nLg\left(1 + \frac{D_i}{n} t\right)$$

$$LgQ = Lg\frac{Q_i}{\left(\frac{D_i}{n}\right)^n} - nLg\left(\frac{n}{D_i} + t\right)$$

$$LgQ = \underline{A} - \underline{B}Lg\left(t + \underline{C}\right)$$

Where:

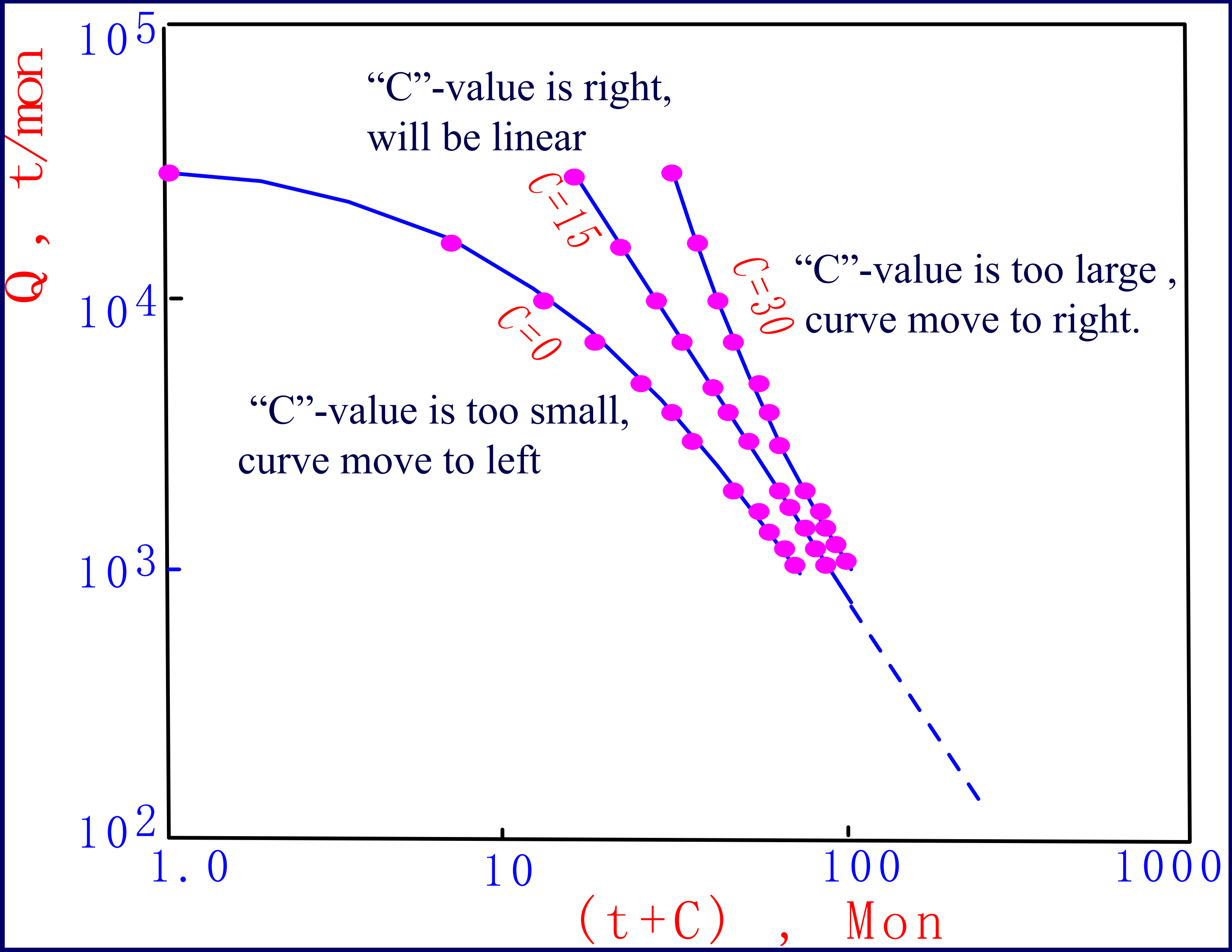
$$A = Lg\frac{Q_i}{\left(\frac{D_i}{n}\right)^n} \qquad B = n \qquad C = n/D_i$$

$$LgQ = A - BLg(t + C)$$

Where: C—— Curve shift constant

For hyperbolic decline, it can be given a different value of C, use the curve shift method to get a best straight-line.

Given different "C"-value , to calculate the different value of (t+C) , if the "C"-value is right, a plot of lgQ~lg(t+C) will be linear , but if the "C"-value is small ,we will have a curve move to left ,whereas , if the "C"-value is big ,we will have a curve move to right.



Q , t/mon
10^5
10^4
10^3
10^2
1.0
10
100
1000
(t+C) , Mon
"C"-value is right,
will be linear
C=15
C=30
C=0
"C"-value is too large ,
curve move to right.
"C"-value is too small,
curve move to left

lg(t+C) ~ lg Q

When we have the right “C”-value ,and the value A,B ,then we can get “n”-value and “D_i” , if we know A,B,C , we can solve the value of n ,D_i ,Q_i

4. Two variable regression method

This method transform the cumulative oil production equation into two variable regression equation .use two variable regression computer program , we can get the three constant coefficients a_0, a_1 and a_2 .

According to the value of a_0, a_1 and a_2 , we could obtain the value of "n" "Q_i" "D_i" at once .

$$N_p = \frac{EQ_i}{D_i}\left(\frac{n}{n-1}\right)\left[1-\left(\frac{Q_i}{Q}\right)^{\frac{1-n}{n}}\right]$$

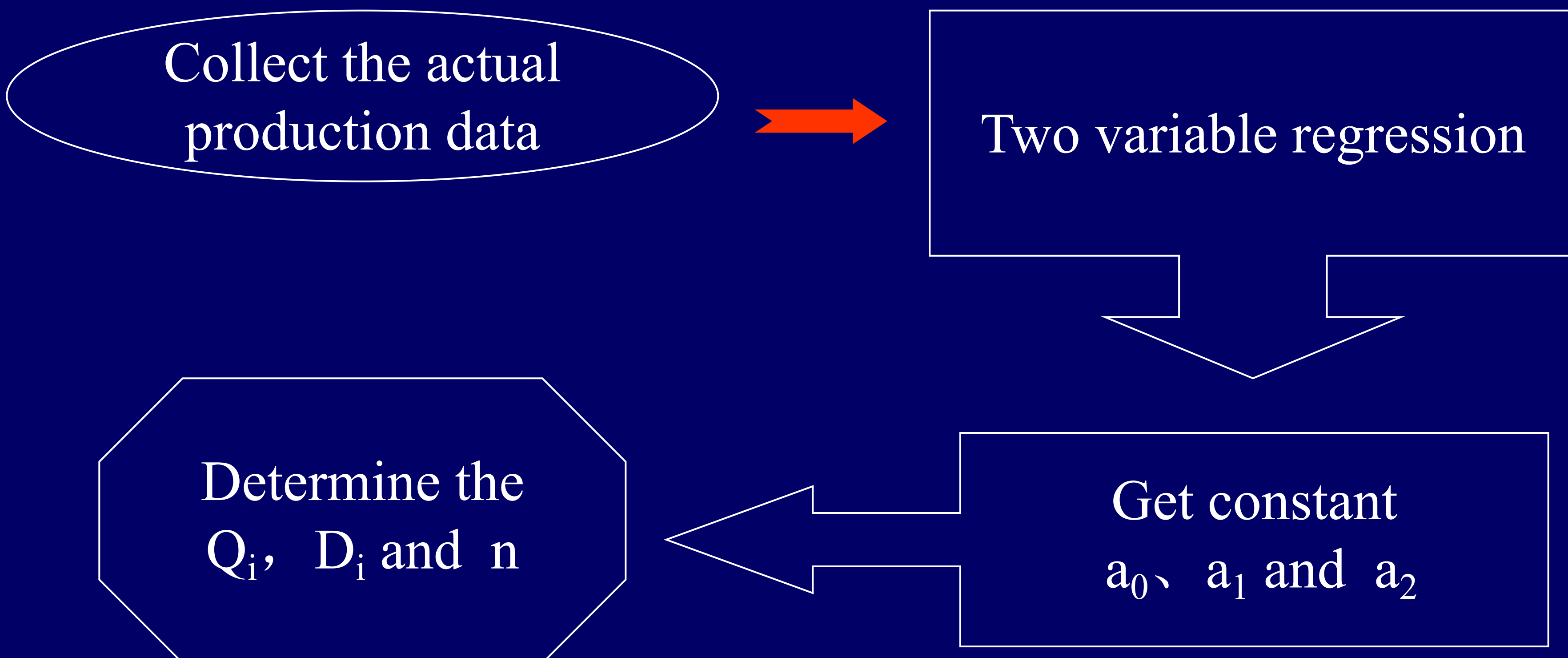
$$N_p = \boxed{\frac{EQ_i}{D_i}\left(\frac{n}{n-1}\right)} \boxed{-\frac{E}{D_i}\left(\frac{n}{n-1}\right)} Q \boxed{-\left(\frac{E}{n-1}\right)} Qt$$

transform

$$N_p = a_0 + a_1 Q + a_2 Qt$$

$$\underline{y} = a_0 + a_1 \underline{x_1} + a_2 \underline{x_2}$$

Solution procedure



Parameters are determined as follows:

$$\begin{cases} Q_{\mathrm{i}} = -a_0 / a_1 \\ \boxed{D_i} = (a_2 - 1)/a_1 \\ n = a_2 /(a_2 - 1) \end{cases}$$

5. The loss-ratio method

This method requires data evenly spaced in time , and smoothing of the data .

$$\left(\frac{Q}{Q_i}\right)^n = \frac{1}{1+nD_i t}$$

$$\frac{D}{D_i} = \frac{1}{1+nD_i t}$$

$$\frac{1}{D} = \frac{1}{D_i} + nt$$

derivation

$$d\left(\frac{1}{D}\right) = ndt$$

$$n = \frac{\Delta \frac{1}{D}}{\Delta t}$$

Example 1

A well has production data as shown below, please based on the loss ratio method to determine the decline type.

t (months)	Q (STB /mon)
0	9600
4	2400
8	600
12	150

solution: If we obtain the decline exponent n-value ,we can determine the decline type.

Based on the loss ratio method:

$$n = \frac{\Delta \frac{1}{D}}{\Delta t}$$

t (months)	Q (STB /mon)	Q_{av} (STB /mon)	$\frac{dQ}{dt}$	$\frac{1}{D}=\frac{Q_{av}}{\left(dQ/dt\right)}$	$\Delta\frac{1}{D}$	Δt	n
0	9600	/	/	/	/	/	/
4	2400	6000	1800	3.33			
8	600	1500	450	3.33	0	4	0
12	150	375	112.5	3.33	0	4	0

The average rate is obtained by averaging the given rates .For the first two time intervals ,Q_{av}=(9600+2400)/2=6000

The first difference is calculated by taking the incremental production , divided by the time span.

$$\frac{dQ}{dt}=(9600-2400)/4=1800$$

$$\frac{1}{D}=\frac{Q_{av}}{\left(dQ/dt\right)}=6000/1800=3.33$$

$$n=\frac{\Delta\frac{1}{D}}{\Delta t}=\frac{(3.33-3.33)}{4}=0$$

Finally , the exponent is obtained by taking the second difference divided by the time span,which in this case yields a result of zero ,because D is constant for exponential decline ,for this example : n=(3.33-3.33)/4=0

Example 2

t (months)	Q (STB /mon)	Q_{av} (STB /mon)	$\frac{dQ}{dt}$	$\frac{1}{D} = \frac{Q_{av}}{\left(dQ/dt\right)}$	$\Delta \frac{1}{D}$	Δt	n
0	29500	/	/	/	/	/	/
6	16100	22800	2233.3	10.21	/	/	/
12	9910	13005	1031.7	12.61	2.40	6	0.400
18	6820	8365	515.0	16.24	3.63	6	0.605
24	5015	5917.5	300.8	19.61	3.43	6	0.572
30	3855	4435	193.3	22.94	3.27	6	0.545
36	3050	3452.5	134.2	25.74	2.80	6	0.461
42	2475	2762.5	95.8	28.83	3.09	6	0.515
48	2050	2262.5	70.8	31.95	3.12	6	0.520

Based on the loss ratio method:

$$n = \frac{\Delta \frac{1}{D}}{\Delta t}$$

hyperbolic decline

6. Newton-iterative method

This method make use of the newton iterative formula to determine decline exponent (n)

$$\because Q = \frac{Q_i}{(1 + nD_i t)^{\frac{1}{n}}}$$

Integration of oil production, can get cumulative oil production

$$\therefore \frac{n}{1-n} \cdot \frac{1 - \left(Q_i / Q\right)^{n-1}}{\left(Q_i / Q\right)^{n} - 1} - \frac{N_p}{Q_i t} = 0$$

Construct a function

$$F(n_k) = \frac{n_k}{1-n_k} \cdot \frac{1 - \left(Q_i / Q\right)^{n_k - 1}}{\left(Q_i / Q\right)^{n_k} - 1} - \frac{N_p}{Q_i t}$$

Use Newton iteration formula

$$n_{k+1} = n_k - \frac{F(n_k)}{F'(n_k)}$$

Where : $F'(n_k) - -F(n_k)$ derivative

First , we can give a suitable initial "n_o"-value,then according to the Newton iteration formula to calculate "n_1"-value , until absolute value $|n_1 - n_o| \angle \varepsilon$, where ε is precision.

7. Correlation coefficient comparisonmethod

This method makes use of the linear relationship of Np~Q to calculate the correlation coefficient ,then compare with the correlation coefficient,maximum correlation coefficient (absolute value) corresponding decline exponent (n) is what we want .

Exponential decline: $N=0$
Hyperbolic decline: $0<N<1$
Harmonic decline: $N=1$

At last, with intercept and slope of linear relationship, it could obtain initial production (Q_i) and initial decline rate (D_i)

1. We know, exponential decline:

Oil field's Q~Np presents a linear relation

$$N_p = A_1 + B_1 Q$$

2. Harmonic decline:

Oil field's lgQ~Np presents a linear relation

$$N_p = A_3 + B_3 \ln Q$$

3. Hyperbolic decline

Oil field's Np~$Q^{(1-x)}$ presents a linear relation

$$N_p =$$

$$A_2 = \frac{Q_i}{D_i(1-X)} \qquad B_2 = -\frac{Q_i^{\,X}}{D(1-X)_i}$$

According to these , we can write computer program , if the precision $\varepsilon = 0.01$,then we can select our step to have a maximum correlation coefficient (R) , it corresponding decline exponent (n) is what we want.

Given the suitable step, so that the "n" from 0 to 1, each "n"-value always get its corresponding correlation coefficient (R). What's more, there always exists a most suitable "n"-value to make the absolute value of correlation coefficient (R) reach maximum. Among them, if n=0, it is exponential decline; if 0<n<1, it is hyperbolic decline; and if n=1, it is harmonic decline.

8. Typical curve fitting method

Exponential decline $$Q_i / Q = e^{D_i t}$$

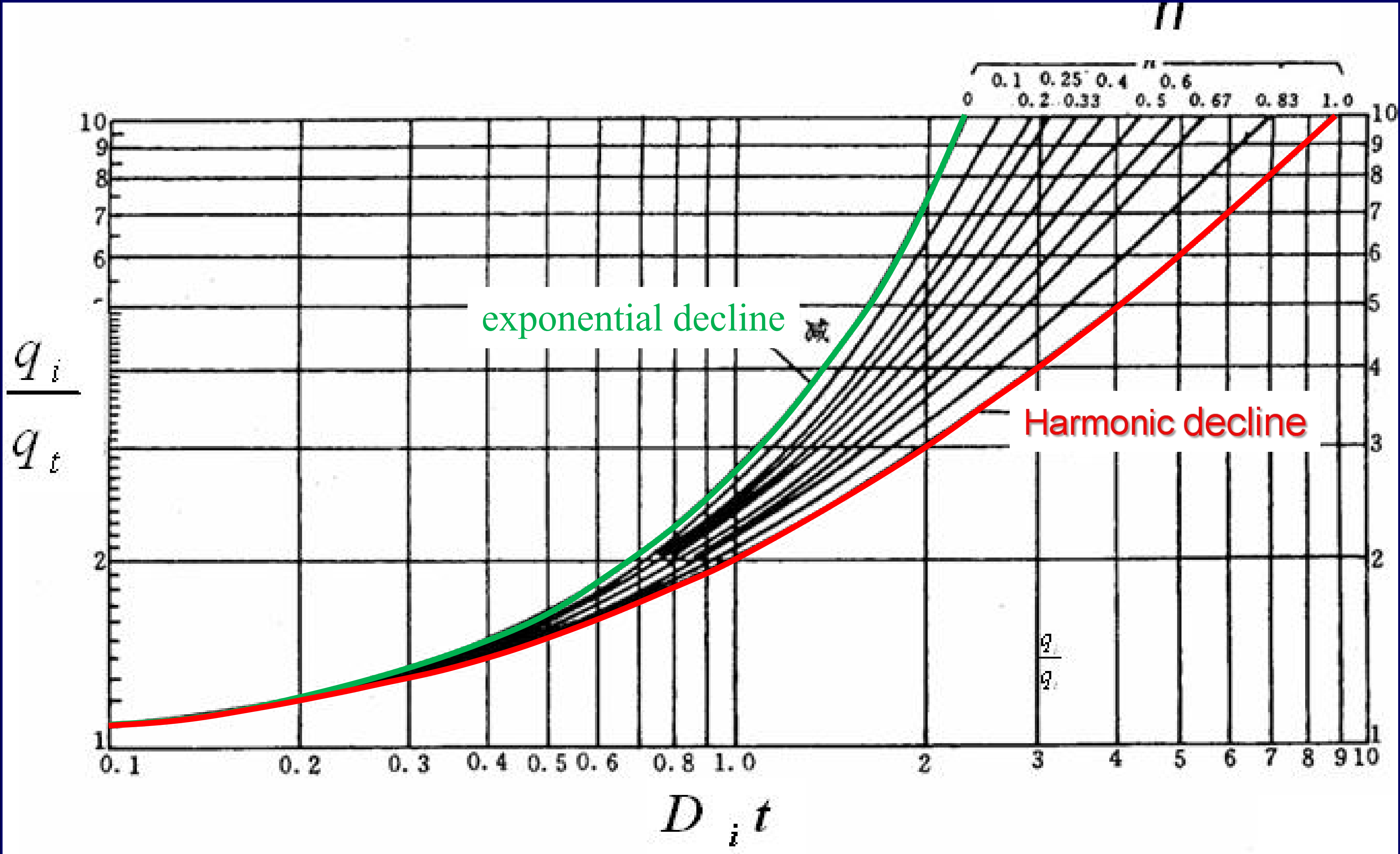
Hyperbolic decline $$Q_i / Q = \left(1 + n D_i t\right)^{1/n}$$

Harmonic decline $$Q_i / Q = 1 + D_i t$$

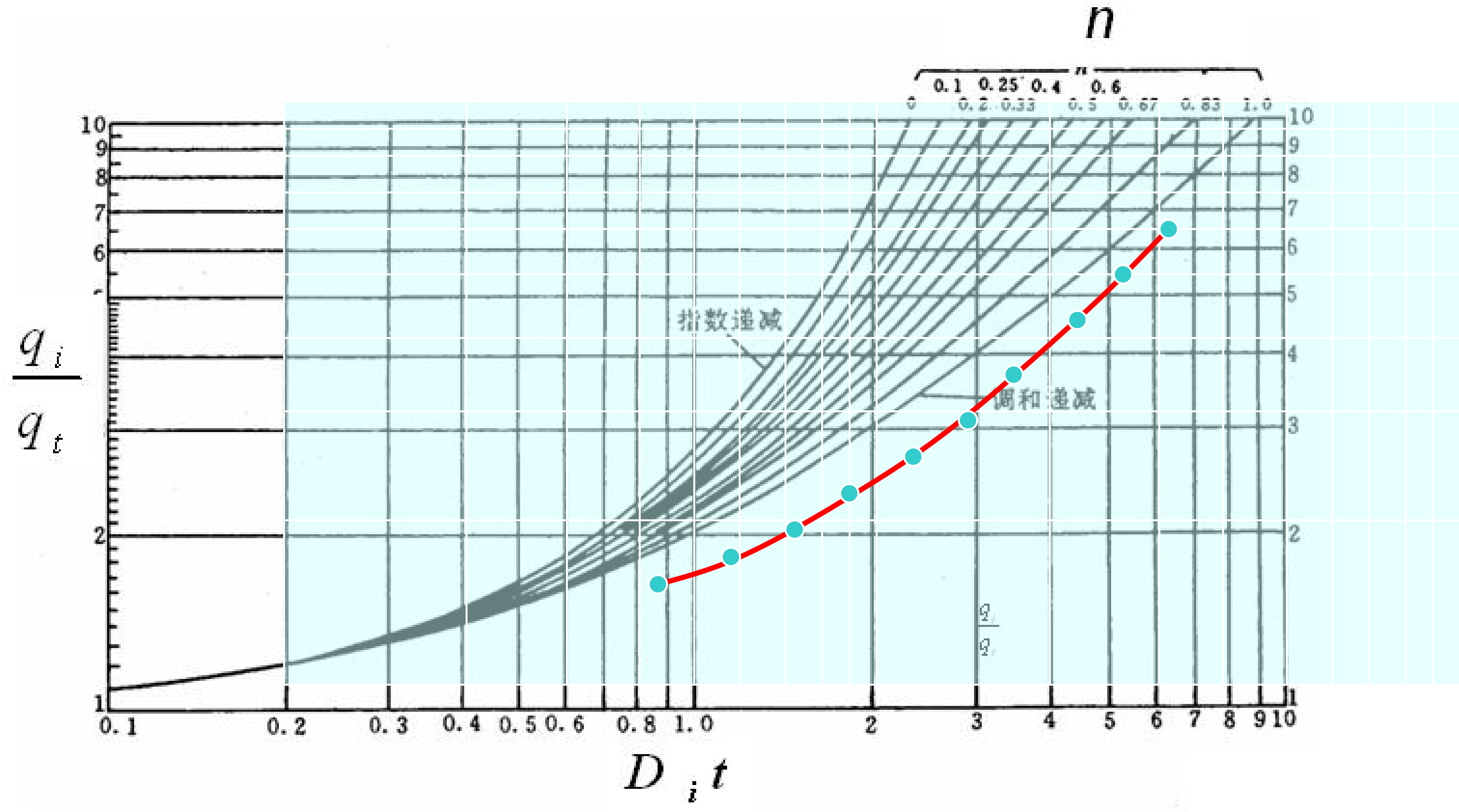
Given the different n-value and $D_i t$-value, we can calculate the different yield ratio (Q_i/Q), plotted on double logarithmic paper, can get the theoretical graph.

Hyperbolic decline

$$\lg \frac{q_i}{q_t} = \frac{1}{n} \lg(1 + nD_i t)$$



Put the transparent paper on the chart, shows the coordinate scale,the actual data for lg (qi/q) ~lgt double logarithmic curve, and chart fitting, find a theoretical curve matching with the actual curve,the corresponding n-value is obtained.



Take a point in the actual curve, read its t-value, and read the corresponding D_it-value on the theoretical curve, then: $D_i=(D_it)/t$

Brief Summary

Graphical method is the most quick-look one, while it is pettyin real practice.

Two variable regression method is very good, but it may have a problem when it is applied in non-smooth data. Because the regressions are not two independent variables, the autocorrelation property may occur . Thus, the results will incorrect . So when this method is used, it is best to smooth the data .

Newton iteration method is a quite strict and convenient way for searching answers. But one disadvantage lies in fixing the initial value. It is so hard to determine initial value that a wrong initial value may lead to failed iteration. Therefore, the calculation is completely wrong.

Chapter 5 Decline Curve Analysis

Section 1 Basic conception

Section 2 Decline type

Section 3 Comparison of decline types

Section 4 Determine the decline type

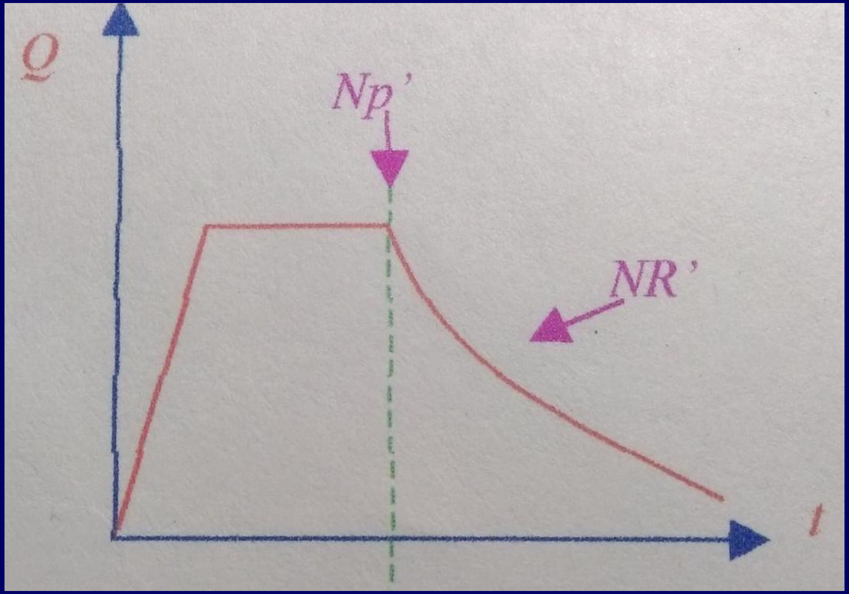
Section 5 Application of decline curve analysis

Section 5 Application of production decline analysis

(1) Forecast the oil production rate in the future

According to the empirical formula , Q~t ,Np~t , then we can forecast the oil production rate in the future.

(2)Determine oil recoverable reserves (N_R)



$$N_R = N_R' + N_p'$$

Where : N_R——*oilfield 's oil recoverable reserves;*

N_R' —— *decline period's oil recoverable reserves;*

N_p' —— *cumulative oil production before decline.*

Use the following formula , we can determine NR.

Hyperbolic decline : $$N_R = N_p' + \frac{Q_i^n}{D_i(1-n)}\left[Q_i^{1-n} - Q_{\min}^{1-n}\right]$$

Exponential decline : $$N_R = N_p' + \frac{\left(Q_i - Q_{\min}\right)}{D_i}$$

Harmonic decline : $$N_R = N_p' + \frac{Q_i}{D_i}\ln\frac{Q_i}{Q_{\min}}$$

Where : Q_{min} —— *economic limit oil production rate.*

(3)Determine oil recovery factor (E_R)

$$E_R = \frac{N_R}{N_o}$$

If we have the N_o-value ,then we can use this formula to calculate E_R

Where: N_o——Original oil in place , gelogical reserves

(4) Calculating oilfield's life time

Use the following formula , we can determine the productive life.

Hyperbolic decline : $$t = \frac{1}{nD_i}\left[\left(\frac{Q_i}{Q_{\min}}\right)^n - 1\right]$$

Exponential decline : $$t = \frac{1}{D_i}\ln\frac{Q_i}{Q_{\min}}$$

Harmonic decline: $$t = \frac{Q_i - Q_{\min}}{D_i Q_{\min}}$$

Where: Q_{min} —— economic limit oil production rate.

Example-3

An oilfield,cumulative oil production before decline $Np'=118.44\times10^4m^3$, the data at decline period are shown in the table below.

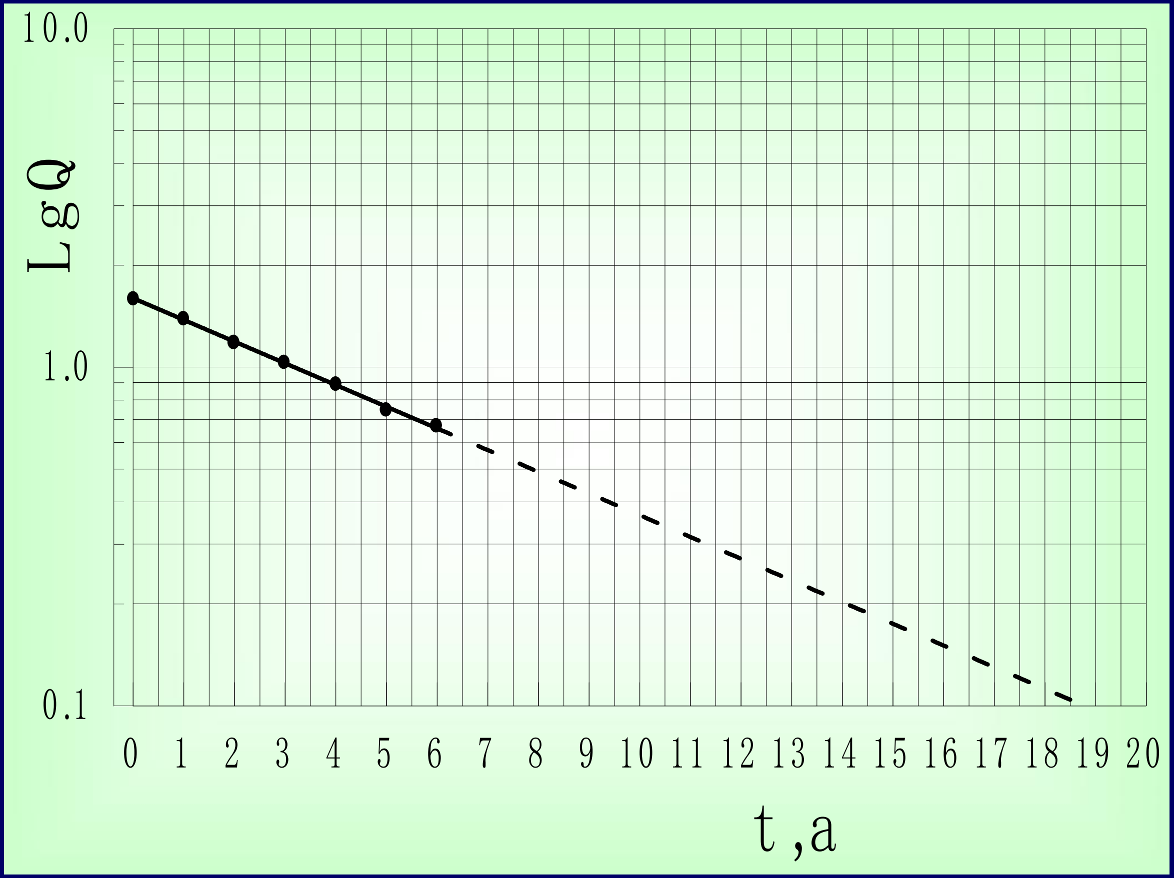
(1) What type of decline undergoing ?

(2)If the economic limit production rate (the abanndonment rate) $Q_{min}=0.1(10^4m^3/year)$, please calculate the oil recoverable reserves $N_R=?$

(3) Determine the productive life ?

Decline time t (year)	Production oil Q ($10^4 m^3$/year)	Cumulative production oil N_p ($10^4 m^3$)
0	1.5937	0.0000
1	1.3866	1.3866
2	1.1820	2.5686
3	1.0329	3.6015
4	0.8879	4.4894
5	0.7472	5.2366
6	0.6677	5.9043

Solution:



lgQ—t

(1) Plot lgQ—t to get the regression formula

from the plot lgQ vs t we found:

$lgQ=A_1+B_1t=0.2042+0.06429t$

Correlation coefficient $R=0.9990$

lgQ vs t is a straight-line relationship

The decline type is exponential decline

(2)

$$LgQ = A_1 - B_1 t$$

$$A_1 = LgQ_i \qquad B_1 = D/2.303$$

$$Q_i = 10^{0.2042} = 1.5937\left(10^4 m^3 / a\right)$$

$$D = 2.303 \times 0.06429 = 0.148 a^{-1}$$

$$N_R = N_i + \frac{E\left(Q_i - Q_a\right)}{D_i}$$

The oil recoverable reserves :

$$N_R = 118.44 + \frac{1\left(1.5937 - 0.1\right)}{0.148} = 128.53\left(10^4 m^3\right)$$

(3) Determine the productive life ?

$$t = \frac{1}{D}\ln\frac{Q_i}{Q_a}$$

$$t = \frac{\ln(1.5937/0.1)}{0.148} = 18.7(a)$$

In a conclusion, the development time in decline period is 18.7 years. As it has been producing for 6 years, it has 12.7 years in residue.

Example-4

According to the data in the following table, using Two variable regression method to determine the decline type. And forecast the decline rate（D) , production(Q) , cumulative oil production (Np）.

The Production Data in a Gas Well

日期	t (a)	$Q=X_1$ $(10^4 m^3/d)$	$Qt=X_2$ $(10^4 m^3)$	$G_p=Y$ $(10^4 m^3)$	$\frac{Q_i}{Q}$
1979.1.1	0.0	28.47	0	0	1
1979.7.1	0.5	23.91	4363.79	4753.77	1.19
1980.1.1	1.0	20.27	7398.24	8767.44	1.40
1980.7.1	1.5	17.53	9601.48	12240.25	1.62
1981.1.1	2.0	15.26	11138.63	15229.15	1.86
1981.7.1	2.5	13.44	12260.18	17847.99	2.12
1982.1.1	3.0	11.90	13028.75	20153.71	2.39
1982.7.1	3.5	10.59	13526.90	22146.31	2.69
1983.1.1	4.0	9.56	13965.27	24025.05	2.98

Solution: Input the data groups of X_1、X_2 and Y in time sequence into the two variable regression computer program, we could get constant coefficients. that is $a_0 = 58817.9418$ $a_1 = -2066.7747$ $a_2 = -1.08207$

To get the decline parameter is:

$$Q_i = -\frac{a_o}{a_1} = 28.4588\left(10^4 m^3 / d\right)$$

$$D_i = \frac{a_2 - 1}{a_1} = 0.3677 \ a^{-1}$$

$$n = \frac{a_2 - 1}{a_2} = 1.9241$$

$n = 1.9241$ → Hyperbolic decline

Forecast the decline rate (D) , production(Q) , cumulative oil production (Np)

$$Q = \frac{Q_i}{\left(1 + \frac{D_i}{n} t\right)^n} = \frac{28.4588}{(1 + 0.1911 t)^{1.9241}}$$

$$D = D_i \left(1 + \frac{D_i}{n} t\right)^{-1} = 0.3677 (1 + 0.1911 t)^{-1}$$

$$G_p = 5.8820 \times 10^8 \left[1 - \left(\frac{28.4588}{Q}\right)^{-0.4803}\right]$$

time	actual Q	Forecast Q	decline rate	actual Np	Forecast Np
$t(a)$	$10^4 m^3/d$	$10^4 m^3/d$	(a^{-1})	$10^8\ m^3$	$10^8\ m^3$
0.0	28.47	28.46	0.3677	0	0
0.5	23.91	23.86	0.3356	0.4754	0.4774
1.0	20.27	20.33	0.3087	0.8767	0.8774
1.5	17.53	17.52	0.2858	1.2240	1.2225
2.0	15.26	15.27	0.2660	1.5229	1.5202
2.5	13.44	13.42	0.2488	1.7848	1.7825
3.0	11.90	11.90	0.2337	2.0154	2.0125
3.5	10.59	10.62	0.2203	2.2146	2.2183
4.0	9.56	9.54	0.2084	2.4025	2.4022
5.0		7.83	0.1880		2.7171
6.0		6.54	0.1713		2.9793
7.0		5.55	0.1573		3.1993
8.0		4.77	0.1454		3.3875
9.0		4.15	0.1352		3.5489
10.0		3.64	0.1263		3.6913
11.0		3.22	0.1185		3.8166
12.0		2.89	0.1116		3.9276
13.0		2.58	0.1056		4.0251
14.0		2.33	0.1000		4.1137
15.0		2.11	0.0951		4.1960
15.6		2.00	0.0924		4.2888