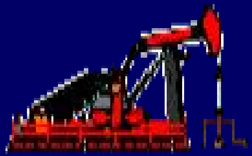
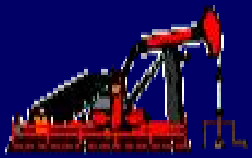
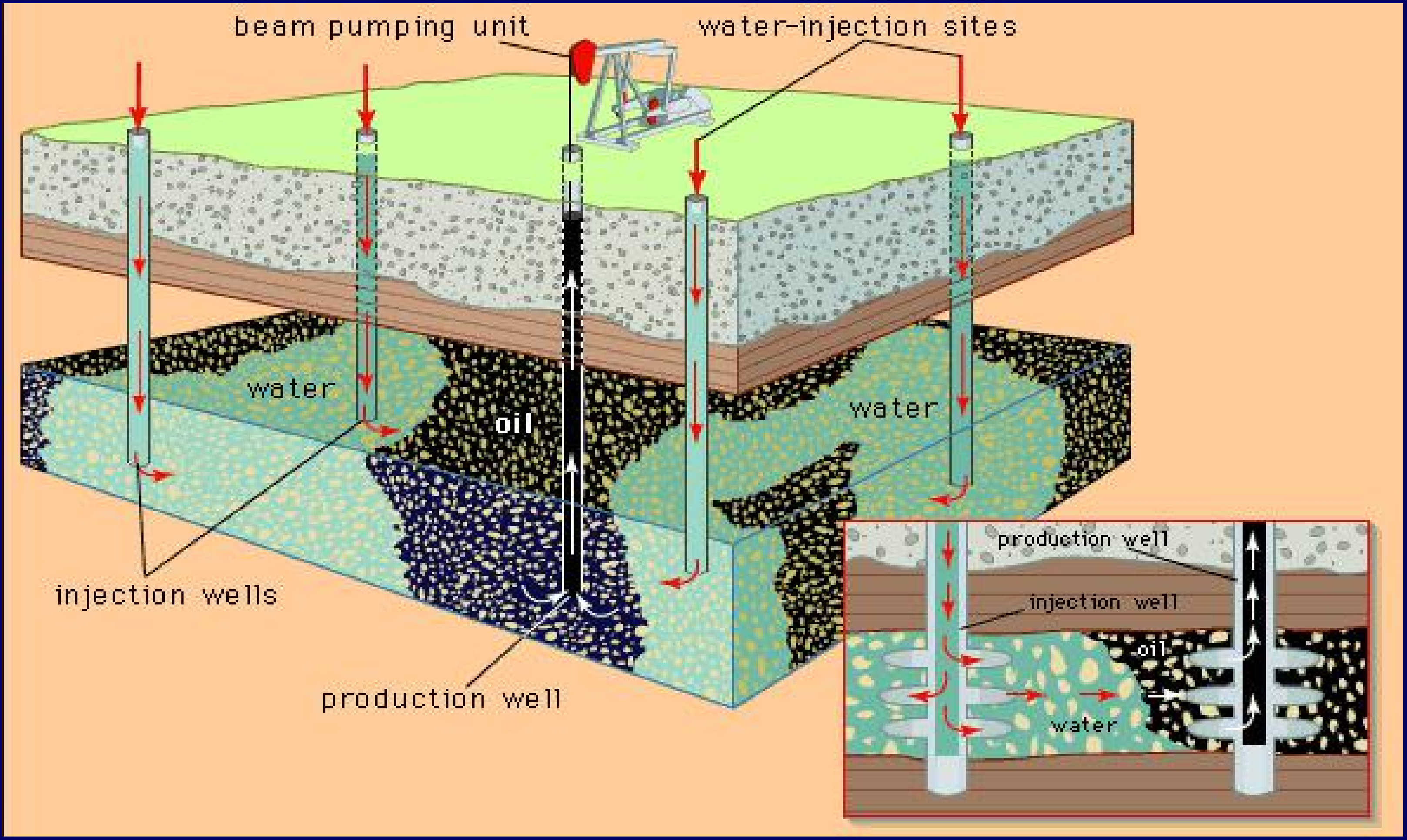
Advanced Reservoir Engineering



Teacher: Zhou Hong



Department of petroleum engineering, China University of Geosciences



The mechanism of water flooding

Chaptr 2 Basic theory of water flooding

<u>Section 1 "Leaky" piston-like displacement characteristics</u>

Section 2 Fractional-flow equation

Section 3 The frontal advance equation

Section 4 Welge's equation

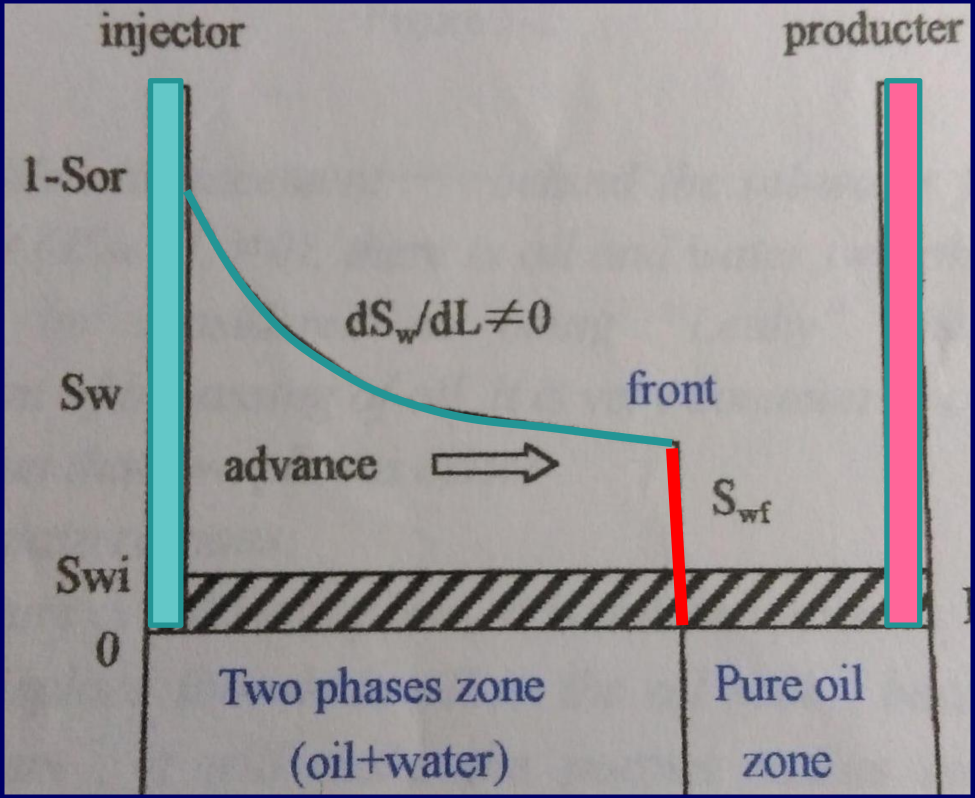
Section 1 “Leaky” piston-like displacement characteristics

I. Basic concepts

1. Water-oil front
2. ”Leaky” piston-like displacement
3. Piston-like displacement
4. Stabilized zone
5. Nonstabilized zone

II. “Leaky” piston-like displacement characteristic

Water-oil front



S_w —— water saturation

S_{wi} —— irreducible water saturation

S_{wf} —— frontal water saturation

S_{or} —— residual oil saturation

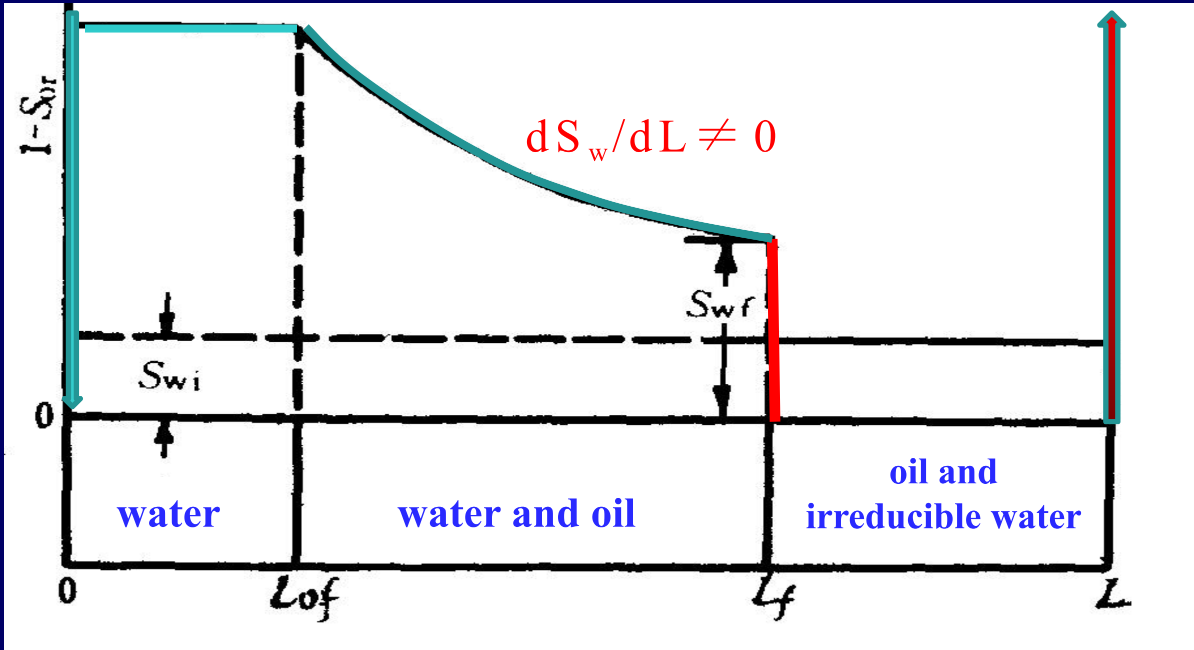
L — — distance

From water injection well to oil well, before the oil well meet water ,water saturation distribution is discontinuous, at the sudden chang of water saturation,we call it water-oil front.

In front of the water-oil front is pure oil, behind the water-oil front is water-oil two-phase zone.

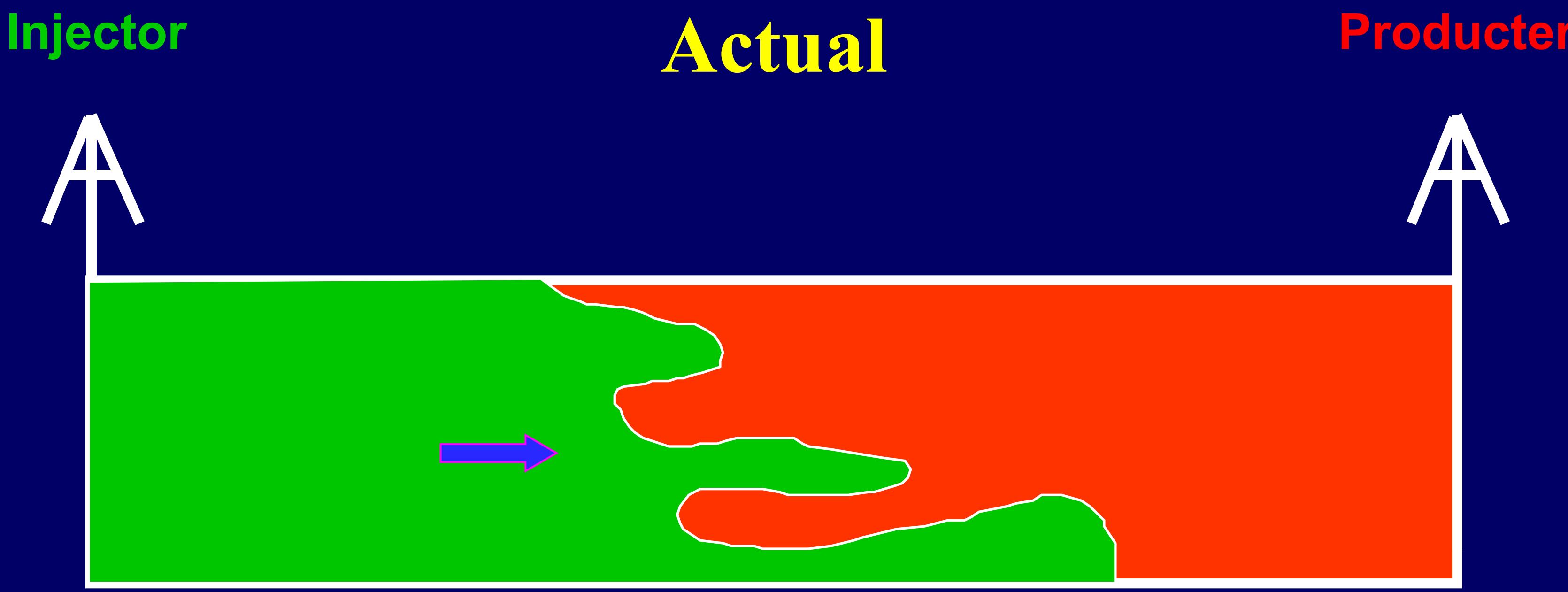
“Leaky” piston-like displacement

Behind the water-oil front , there is water saturation gradient $\frac{dS_w}{dL} \neq 0$,there is oil and water two phase ,this means the displacement can be consider as being “Leaky” piston-like , there is a considerable amount of bypassing of oil , it is very common in oilfield.



”Leaky” piston-like displacement

Oil Production



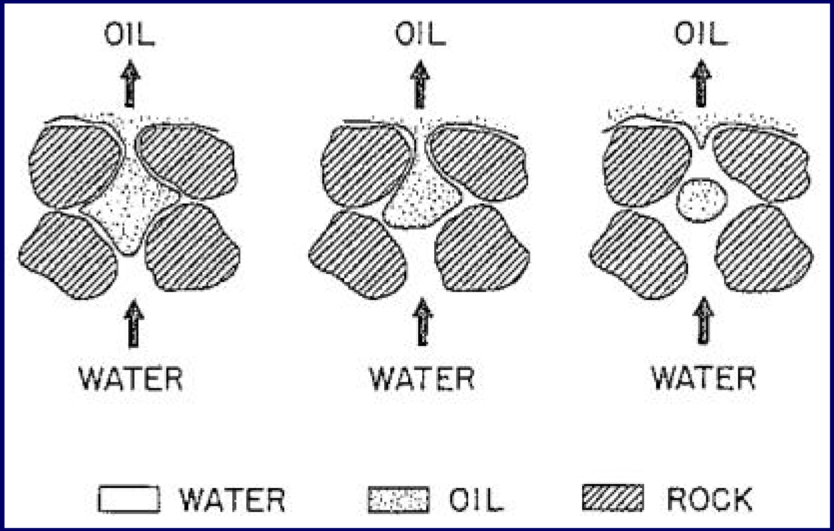
What is the reason that two phases exist?

There are three main reasons :

(1) Reservoir heterogeneity

Water can't displace the whole oil in the oil rock , because of the reservoir heterogeneity.

(2) Because of the oil-water viscosity is different ,and the rock's wetting is different.



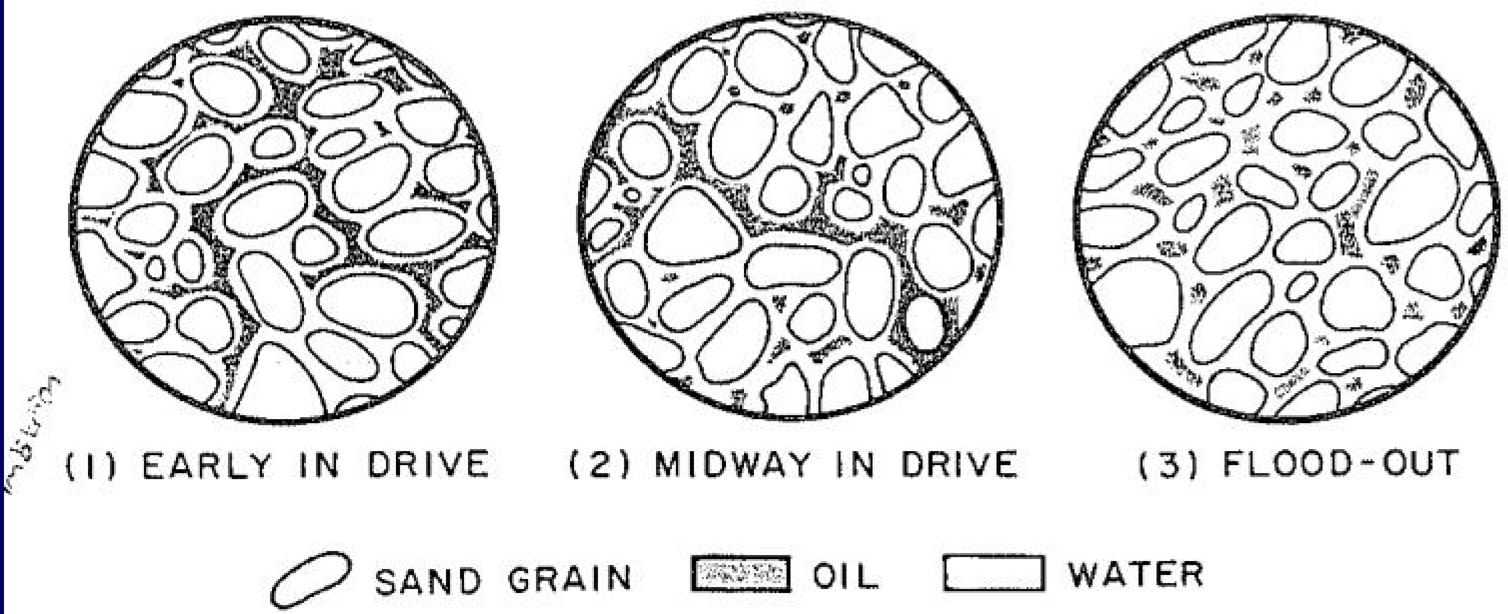
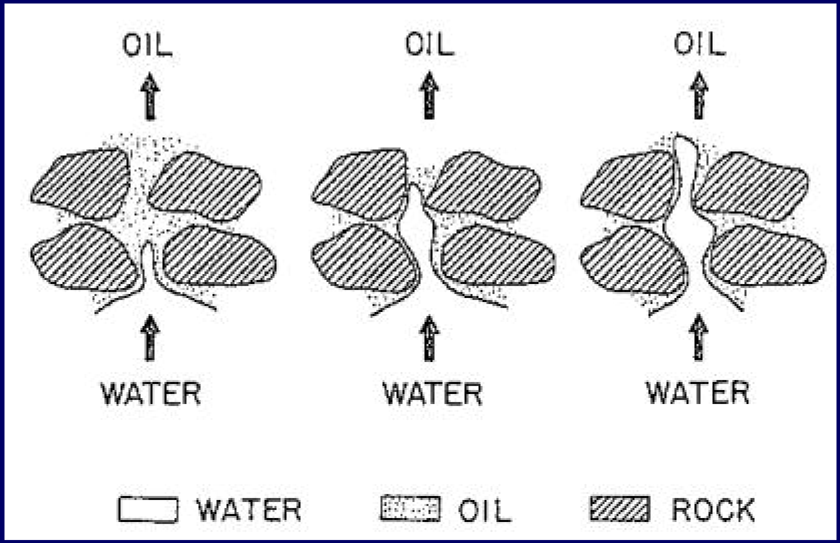


Fig. 2.5 Fluid distribution during waterflood of water-wet rock (after Ref. 24).

Water-wet rock



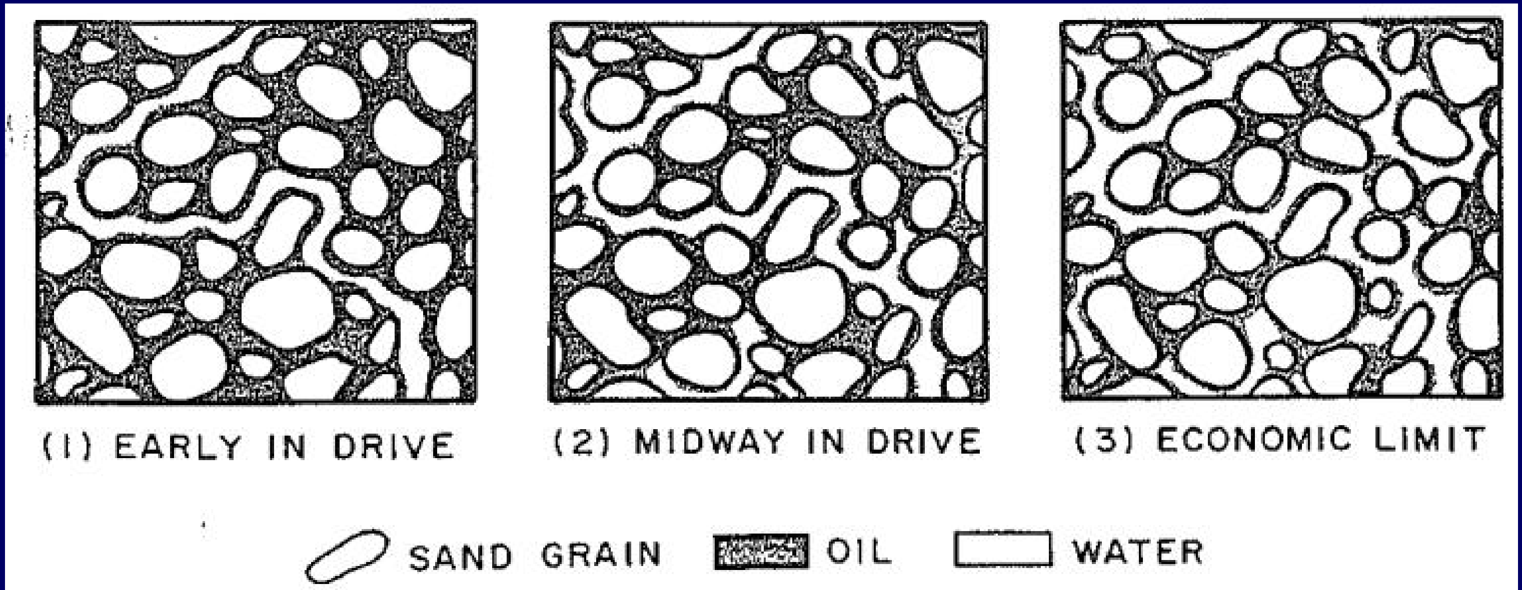
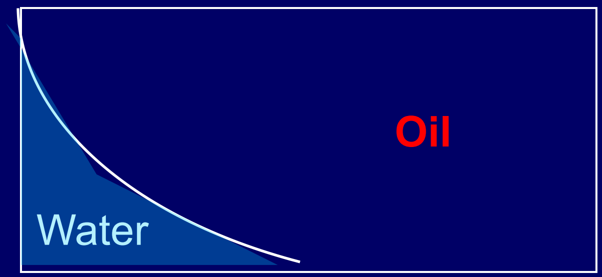


Fig. 2.6 Fluid distribution during waterflood of oil-wet rock (after Ref. 24).

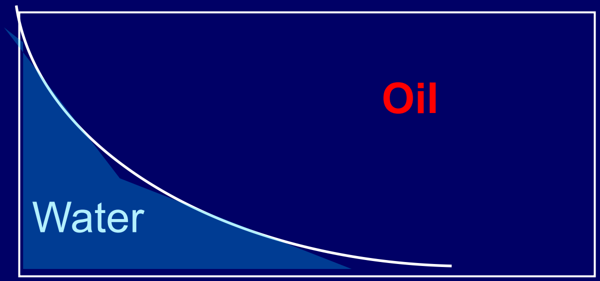
Oil-wet rock

(3)Because of the oil-water specific gravity is different ,water specific gravity is higher than oil, injected water will advance to the oil zone along the lower portion of the reservoir.

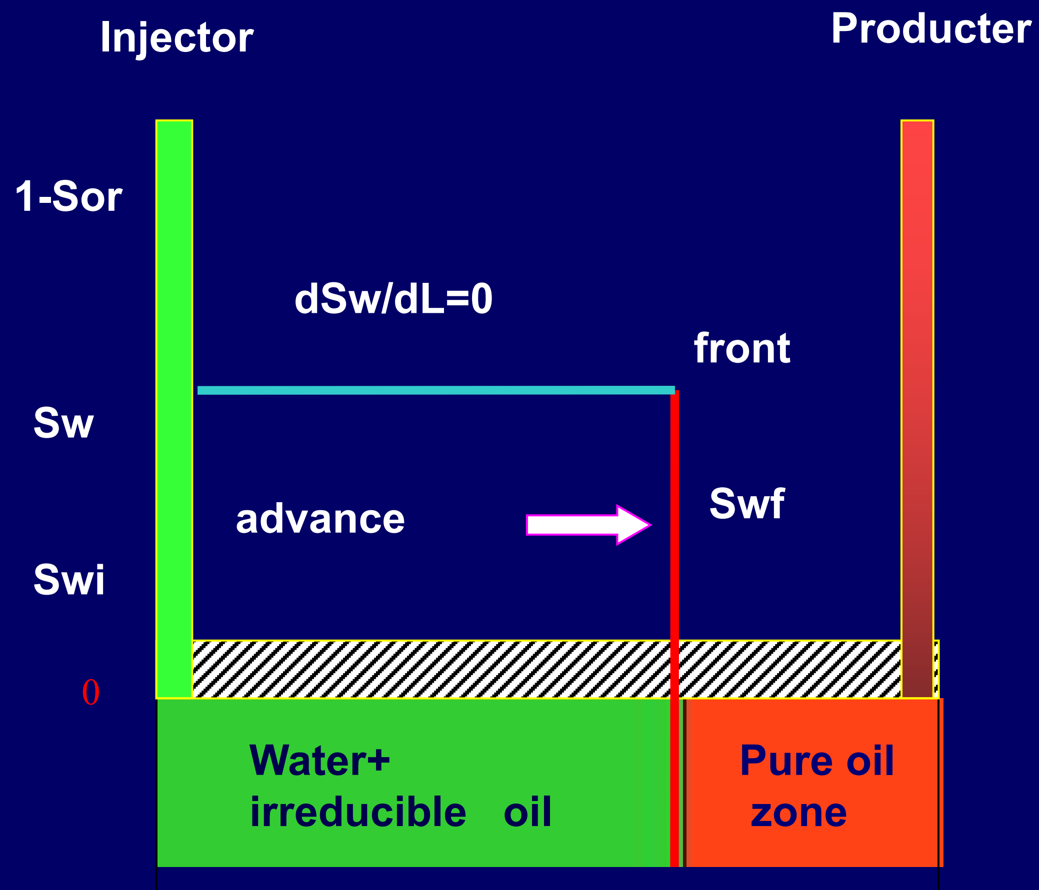
Flooding at low speed



Flooding at high speed

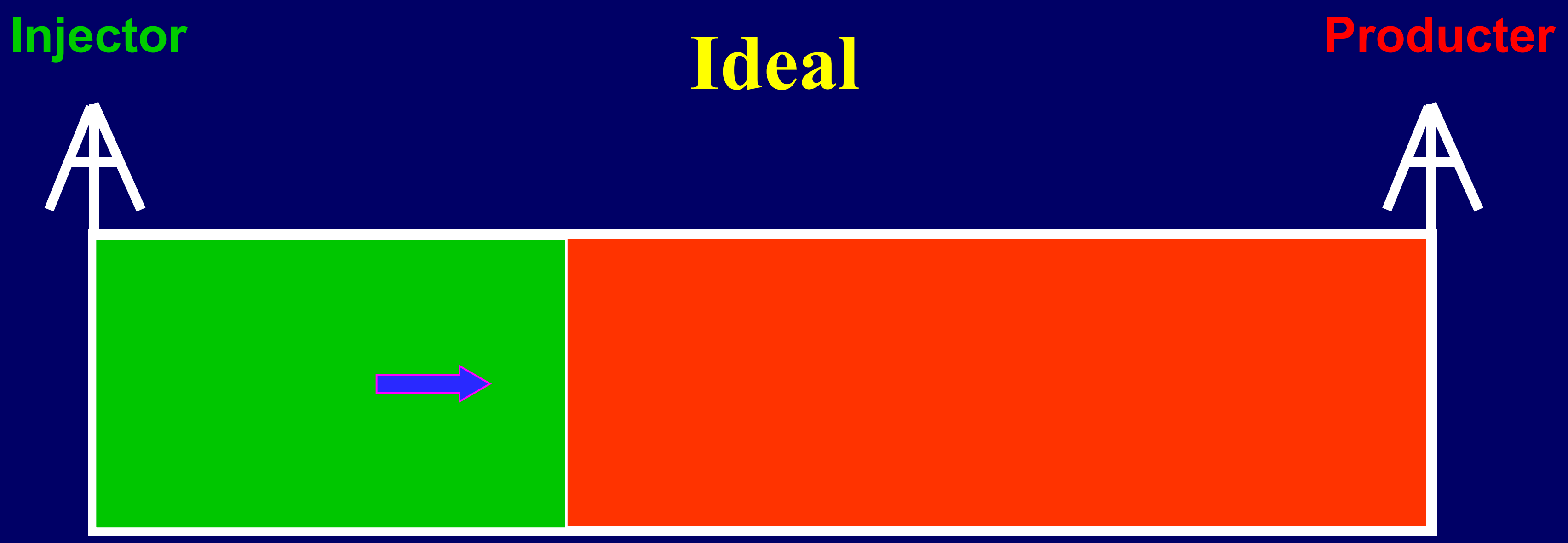


Piston-like displacement



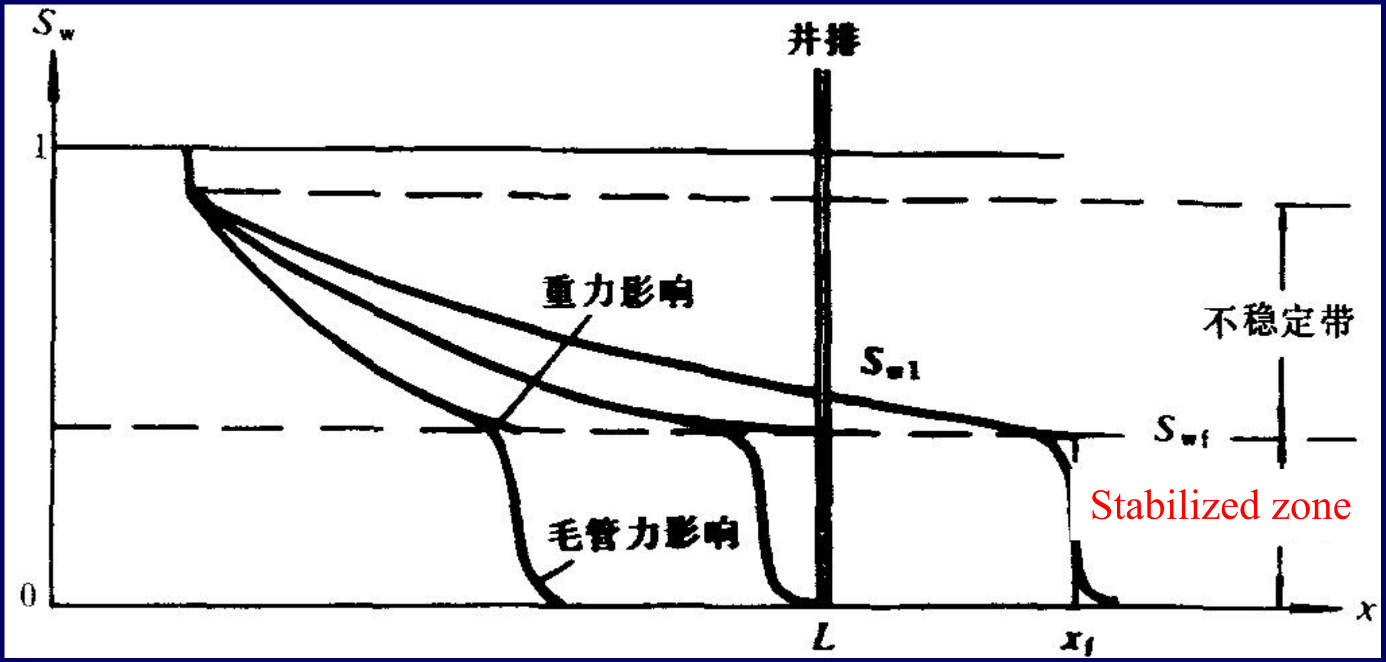
Behind the water-oil front ,there is no water saturation gradient ($dS_w/dL=0$), there is no oil flow behind the water(only residual oil). this is an ideal way of displacement. in fact , it is not common in oilfield .

Oil Production



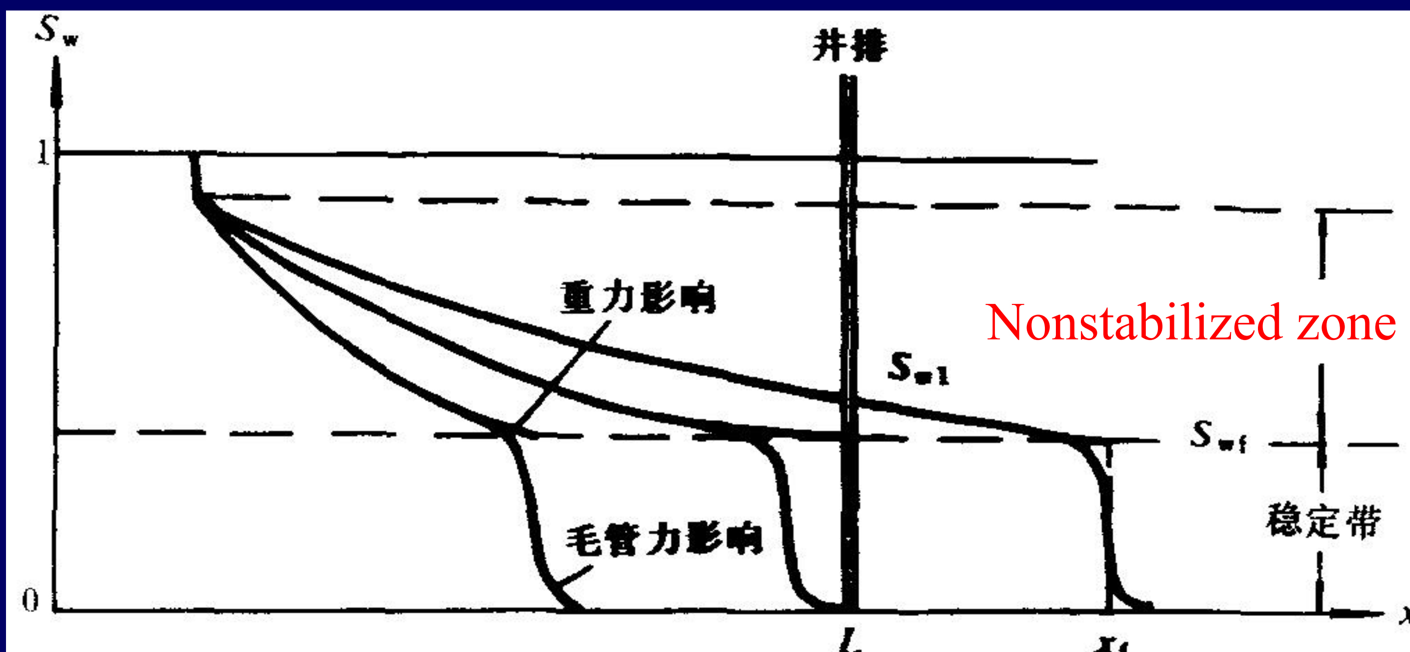
Stabilized zone

Saturation interval where all points of saturation move at the same rate



Nonstabilized zone

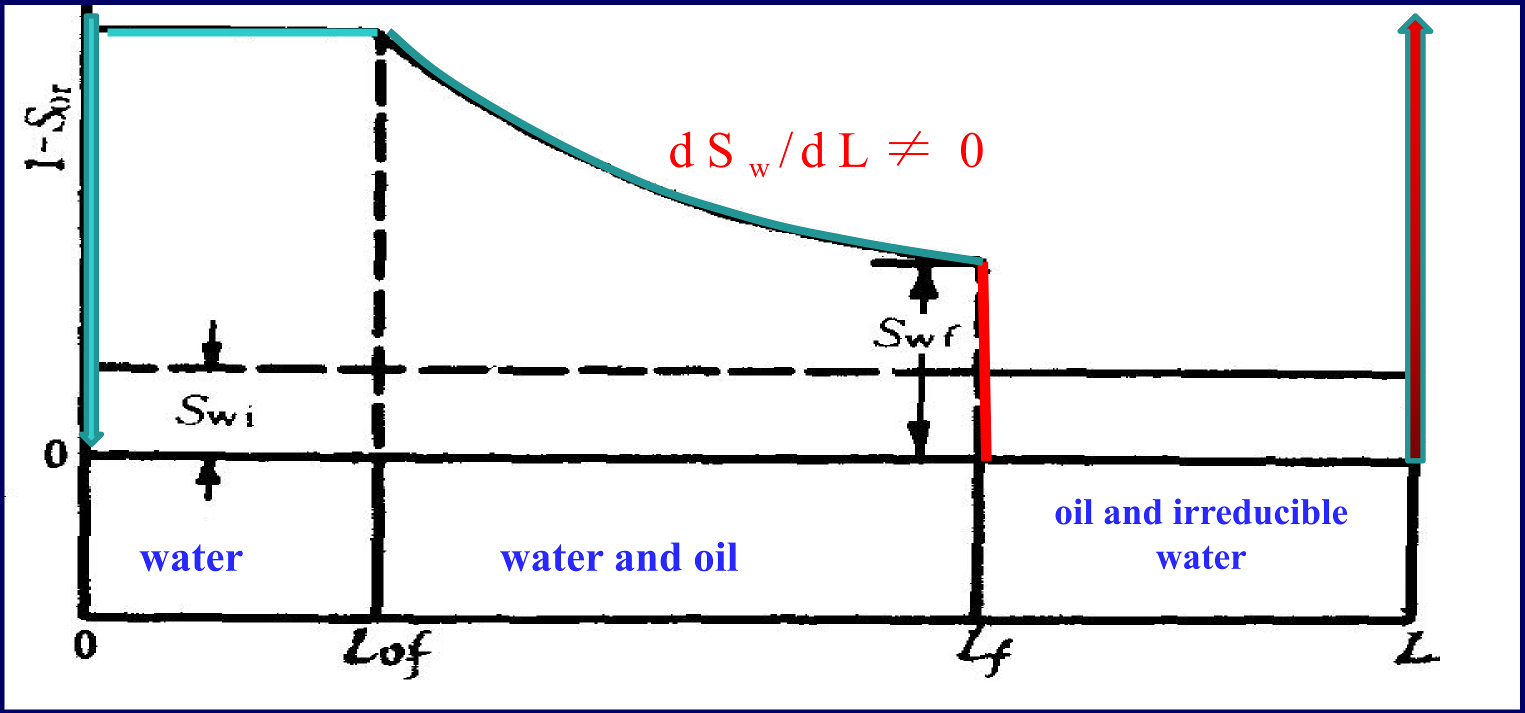
Saturation interval where all points of saturation continue to get farther apart.



II. "Leaky" piston-like displacement characteristics

In the water flooding reservoir, from water injection well to oil well, water saturation from 100% gradually reduced to the irreducible water saturation .

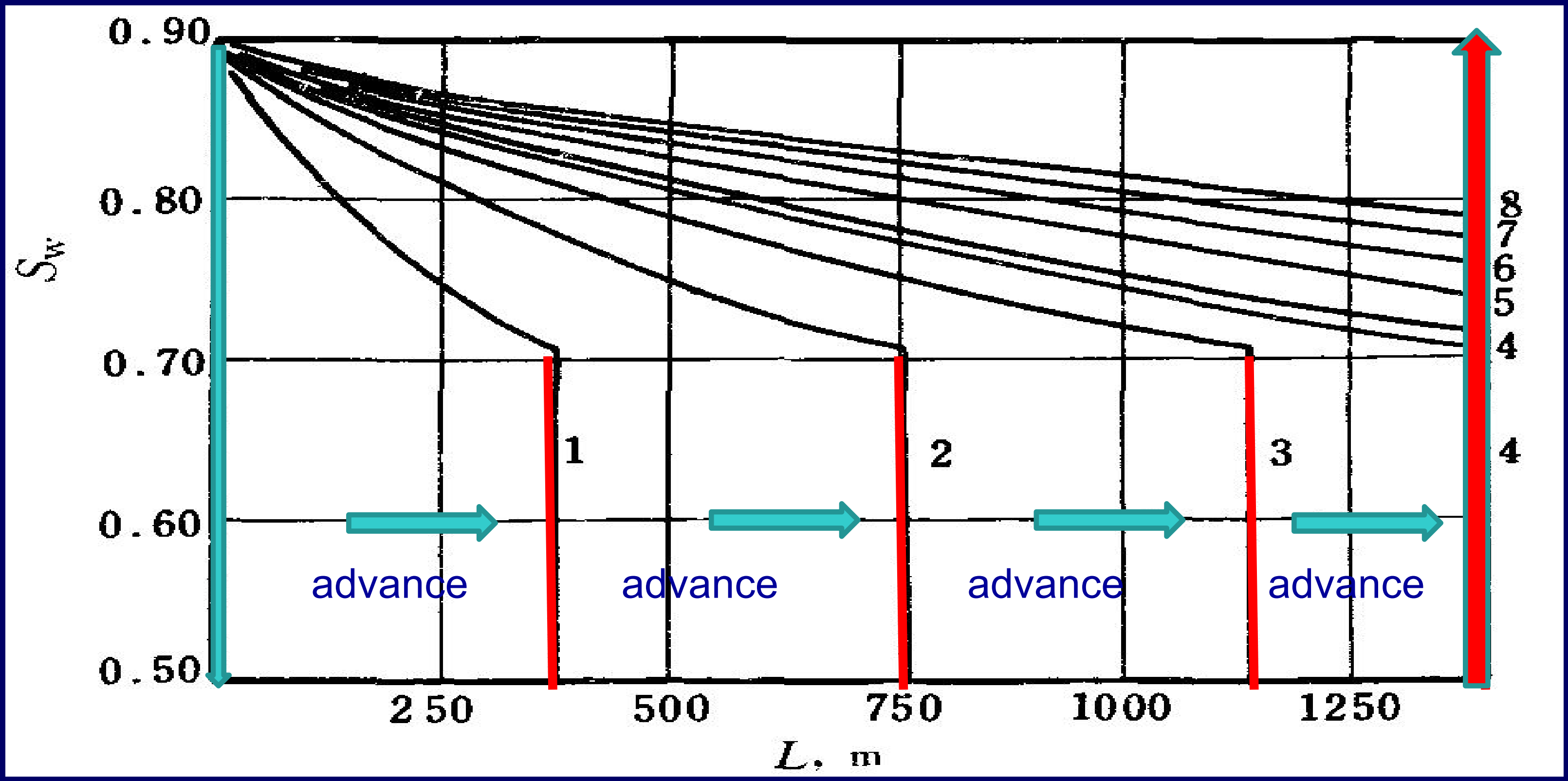
The saturation distribution is shown in Fig.



"Leaky" piston-like displacement

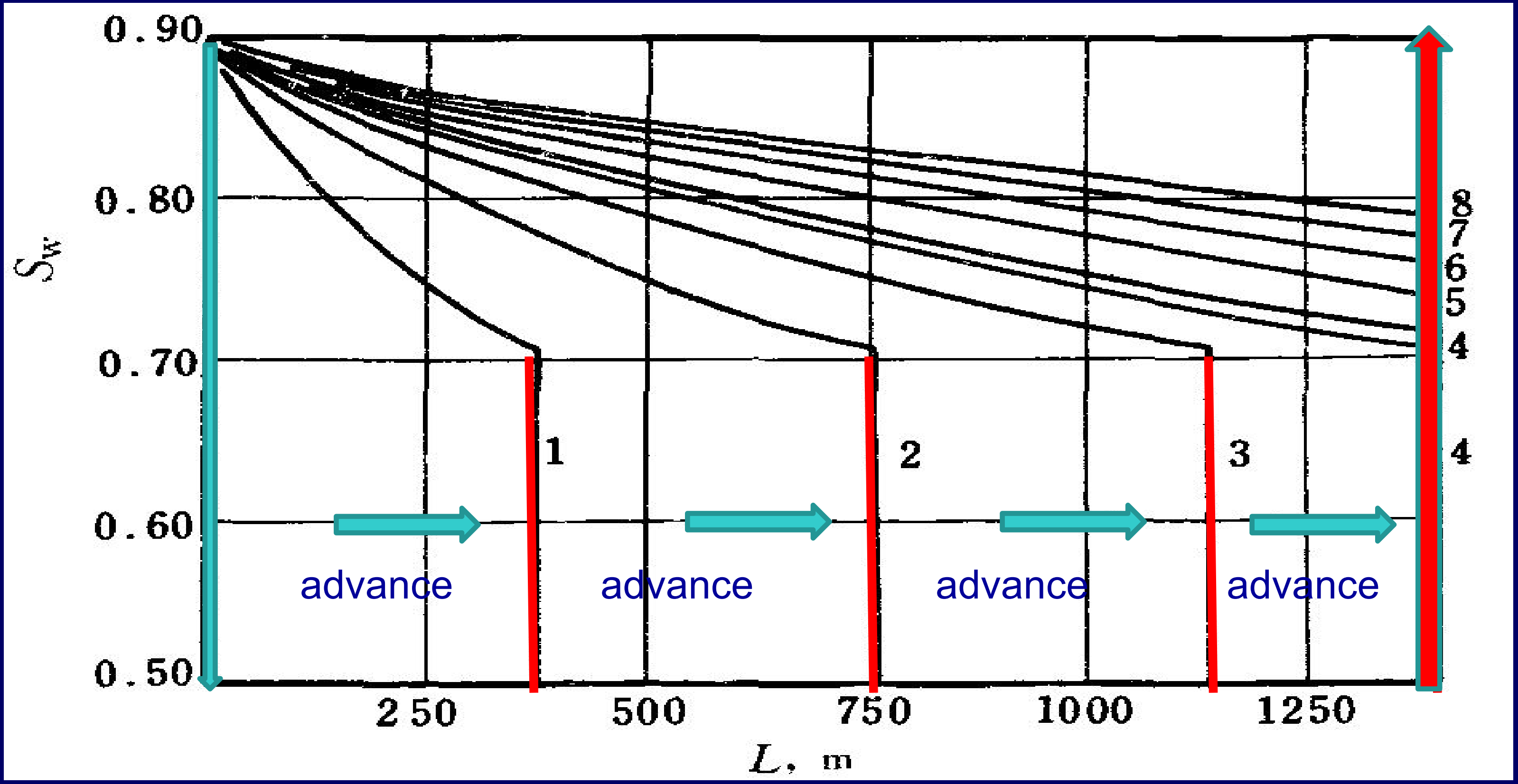
（1）Front advances from injector to producter , the two phases zone gradually enlarge , and the pure oil zone gradually reduce.

There is only two- phase flow in the oil layer when the production end meets water（breakthrough）.



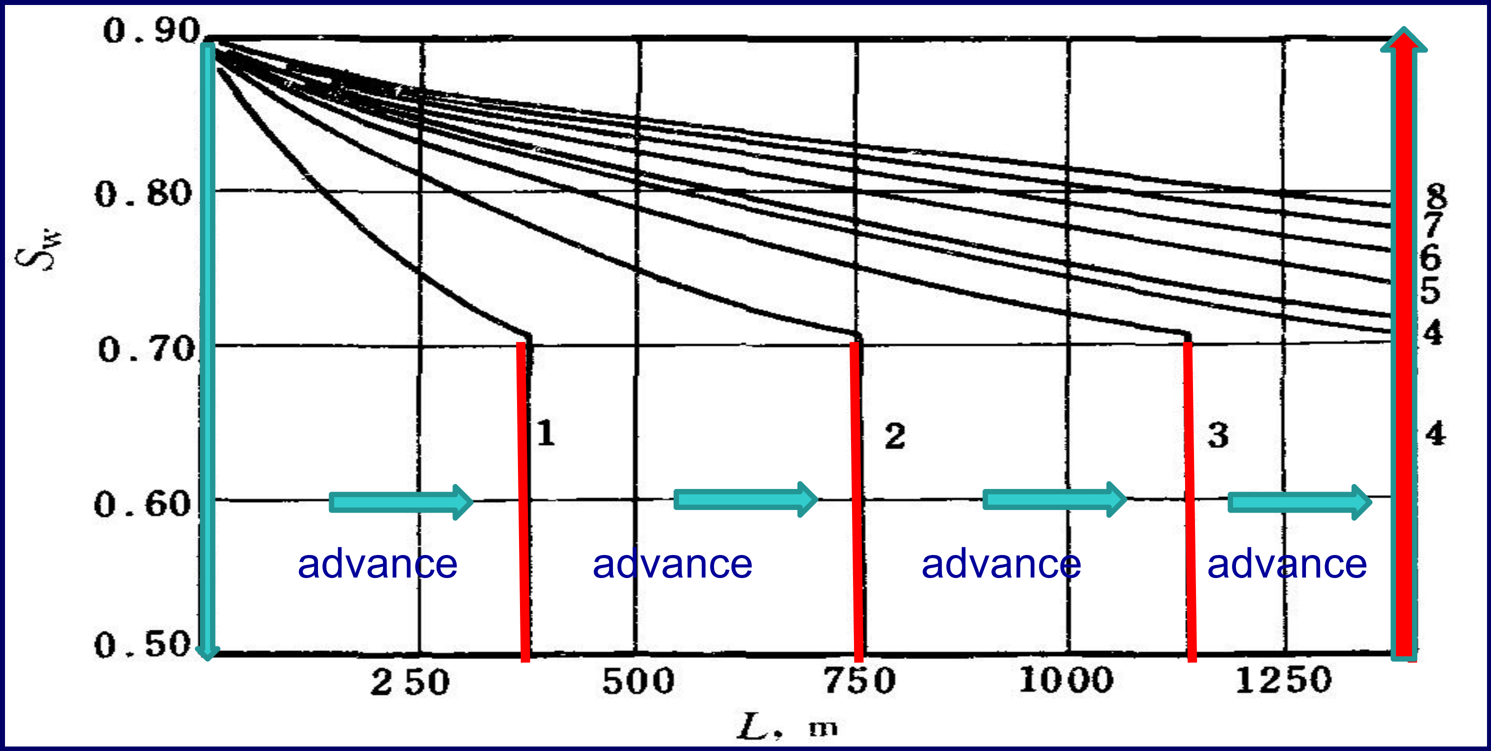
Distributions of saturation at different periods

(2) If L = constant, Sw will go up along with the time;
If S_w = constant, L will advance along with the time;
at the same time(t = constant), water saturation of different reservoir is different;



Distributions of saturation at different periods

(3) Although the front advance , but the frontal saturaton is constant.



(4) When the reservoir enters into ultimate flooding stage, the saturation is S_{or}

S_{or}---- the residual oil saturation

Chaptr 2 Basic theory of water flooding

Section 1 "Leaky" piston-like displacement characteristics

Section 2 Fractional-flow equation

Section 3 The frontal advance equation

Section 4 Welge's equation

Section 2 Fractional-flow equation

1.Fractional-flow equation

2.Fractional-flow equation derivation

3.Comprehension of Fractional-flow equation

4.Fractional-flow curve and its application

(1) Fractional-flow curve

(2) The application of fractional- flow curve

1.Fractional-flow equation

In 1941, Leverett deduced the fractional flow equation:

$$f_w = \frac{1 + \frac{K_o}{q_t \mu_o}\left(\frac{\partial P_c}{\partial L} - g\Delta\rho \sin\theta\right)}{1 + \frac{K_{ro}\mu_w}{K_{rw}\mu_o}}$$

K_o —— effective permeability to oil

K_{rw} —— relative permeability to water

K_{ro} —— relative permeability to oil

f_w —— water cut

q_t —— total flow rate

$\Delta\rho$ —— watere-oil density differences

g —— acceleration due to gravity

μ_w —— water viscosity

μ_o —— oil viscosity

$\frac{\partial P_c}{\partial L}$ —— capillary pressure gradient

θ —— angle of the formation dip to the horizontal

All the factors necessary to calculate the value of f_w are readily available except one, the capillary pressure gradient. This gradient can be expressed as

$$\frac{\partial P_c}{\partial L} = \frac{\partial P_c}{\partial S_{10}} \frac{\partial S_{10}}{\partial L}$$

Although the value of $\frac{\partial P_c}{\partial S_{10}}$ can be determined from the appropriate water-oil capillary pressure curve , the value of the saturation gradient , $\frac{\partial S_{10}}{\partial L}$, is not available; so in practical use the capillary pressure term in Eq. is neglected (but not forgotten). Then Eq. simplifies to

$$f_w = \frac{1 - \frac{K_o}{q_t \mu_o}\left(g \Delta \rho \sin \theta\right)}{1 + \frac{K_{ro} \mu_w}{K_{rw} \mu_o}}$$

For the further simplification where displacement occurs in a horizontal system, this equation reduces to

$$f_w = \frac{1}{1 + \frac{K_{ro}}{K_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$

2. Fractional-flow equation derivation

(1) Darcy's Law:

Darcy's Law is the starting point for many reservoir engineering calculations. The law may be written in the form:

$$V = -\frac{K}{\mu}\frac{dp}{dx}$$

Where: V——Velocity,(v=q/A)

q——flow rate

P——Pressure

A——cross-sectional area

K——Permeability

x——distance

(2) Fractional-flow equation's derivation

Fractional flow equation reflects the percentage of water flows of the total fluid amount, so:

$$f_w = \frac{q_w}{q_o + q_w} \times 100\% = \frac{q_w}{q_t} \times 100\% \qquad (1)$$

If using seepage velocity to express fractional flow, then:

$$f_w = \frac{V_w}{V_o + V_w} = \frac{V_w}{V_t} \qquad (2)$$

According to Darcy's law, the seepage velocity of oil water two-phase can be written respectively as:

$$V_o = -\frac{K_o}{\mu_o}\left(\frac{\partial P_o}{\partial L} + g\rho_o \sin\theta\right) \quad (3)$$

negative

$$V_w = -\frac{K_w}{\mu_w}\left(\frac{\partial P_w}{\partial L} + g\rho_w \sin\theta\right) \quad (4)$$

Deformation of formula (3) and (4):

$$V_o \cdot \frac{\mu_o}{K_o} = -\frac{\partial P_o}{\partial L} - g\rho_o \sin\theta \qquad (5)$$

$$V_w \cdot \frac{\mu_w}{K_w} = -\frac{\partial P_w}{\partial L} - g\rho_w \sin\theta \qquad (6)$$

Formula (6) minus formula (5),

$$V_w \cdot \frac{\mu_w}{K_w} - V_o \cdot \frac{\mu_o}{K_o} = \left(\frac{\partial P_o}{\partial L} - \frac{\partial P_w}{\partial L} \right) - g(\rho_w - \rho_o)\sin\theta \qquad (7)$$

$$\frac{\partial P_c}{\partial L}$$

Let

$$\Delta\rho = \rho_w - \rho_o$$

$$P_c = P_o - P_w$$

Get:

$$V_w \cdot \frac{\mu_w}{K_w} - V_o \cdot \frac{\mu_o}{K_o} = \frac{\partial P_c}{\partial L} - g\Delta\rho\sin\theta \qquad (8)$$

$$V_w \cdot \frac{\mu_w}{K_w} - V_o \cdot \frac{\mu_o}{K_o} = \frac{\partial P_c}{\partial L} - g\Delta\rho \sin\theta \qquad (8)$$

Put $V_o = V_t - V_w$ into the above formula to obtain:

$$\frac{V_w}{V_t}\left(\frac{\mu_w}{K_w} + \frac{\mu_o}{K_o}\right) = \frac{\mu_o}{K_o} + \frac{1}{V_t}\left(\frac{\partial P_c}{\partial L} - g\Delta\rho \sin\theta\right) \qquad (9)$$

Change this to the right side of the formula, get the formula 10

$$f_w = \frac{V_w}{V_t} = \frac{1 + \frac{K}{V_t} \cdot \frac{K_{ro}}{\mu_o}\left(\frac{\partial P_c}{\partial L} - g\Delta\rho \sin\theta\right)}{1 + \frac{K_{ro}}{K_{rw}} \cdot \frac{\mu_w}{\mu_o}} \qquad (10)$$

Regardless of capillary pressure, if the flooding carries in a horizontal systems, then the formation dip θ=0, the equation can be further simplified as:

$$f_w = \frac{1}{1 + \frac{K_{ro}}{K_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$

3.Comprehension of fractional-flow equation

$$f_w = \frac{1 + \frac{K}{V_t} \cdot \frac{K_{ro}}{\mu_o}\left(\frac{\partial P_c}{\partial L} - g\Delta\rho \sin\theta\right)}{1 + \frac{K_{ro}}{K_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$

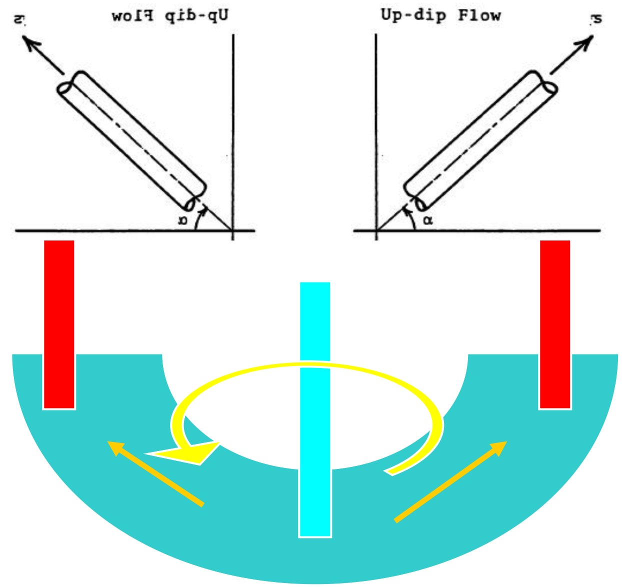
(1) Fractional flow equation shows the percentage of water flows of the total fluid amount. and the water-cut is related to viscosity , the oil and water relative permeability， total flow rate, capillary pressure gradient， formation dip, gravity and other factors.

(2) Water displacing oil updip will result in a lower value of f_w at any water saturation than will water displacing oil downdip

Because when

Updip displacement

$$\pi > \theta > 0 \quad \sin\theta > 0$$

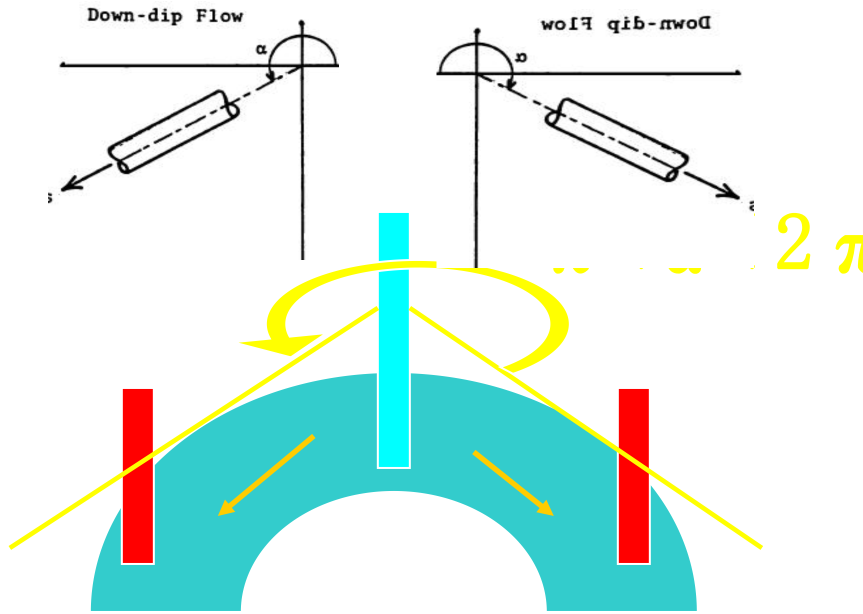


$0 < \alpha < \pi$

$$f_{w(updip)} = \frac{1 + \dfrac{K}{V_t} \cdot \dfrac{K_{ro}}{\mu_o}\left(\dfrac{\partial P_c}{\partial L} - g\Delta\rho \sin\theta\right)}{1 + \dfrac{K_{ro}}{K_{rw}} \cdot \dfrac{\mu_w}{\mu_o}}$$

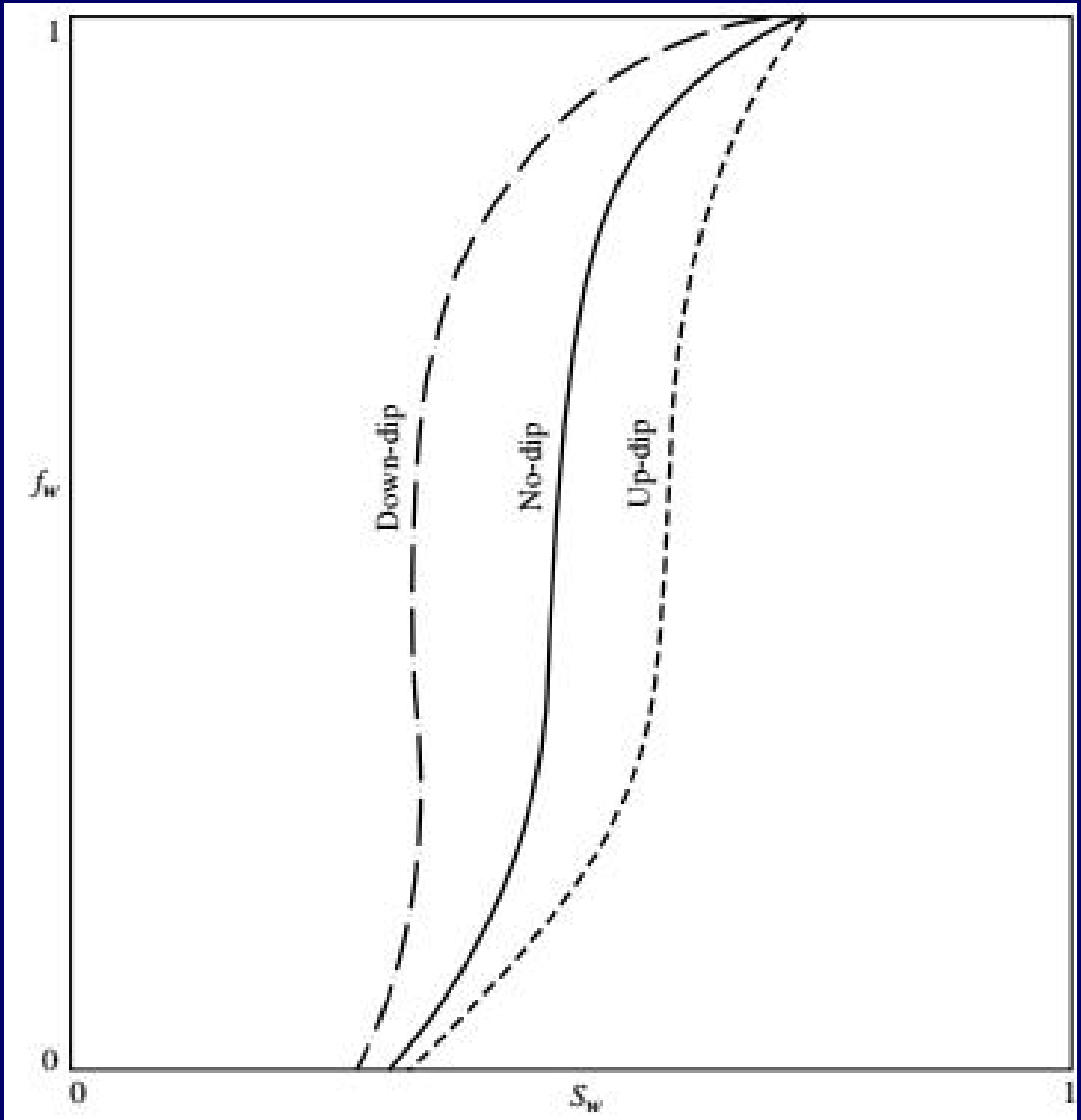
When $2\pi > \theta > \pi$ $\sin\theta < 0$

Downdip displacement



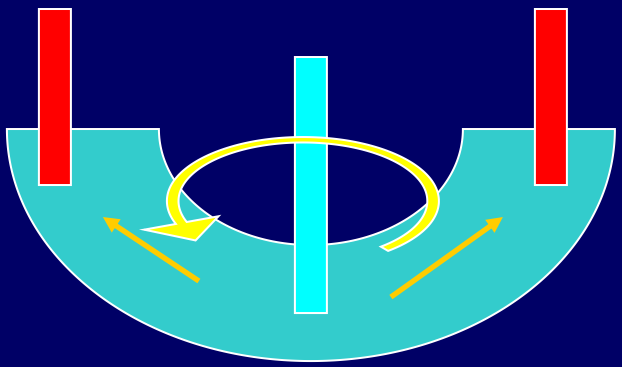
$$f_{w(downdip)} = \frac{V_w}{V_t} = \frac{1 + \dfrac{K}{V_t} \cdot \dfrac{K_{ro}}{\mu_o}\left(\dfrac{\partial P_c}{\partial L} + g\Delta\rho\left|\sin\theta\right|\right)}{1 + \dfrac{K_{ro}}{K_{rw}} \cdot \dfrac{\mu_w}{\mu_o}}$$

So : $f_{w(downdip)} > f_{w(updip)}$



Effect of dip angle on fw

The best location to inject water is located at the lower areas of the structure. In this way, it displaces oil from the bottom to up and is effective.



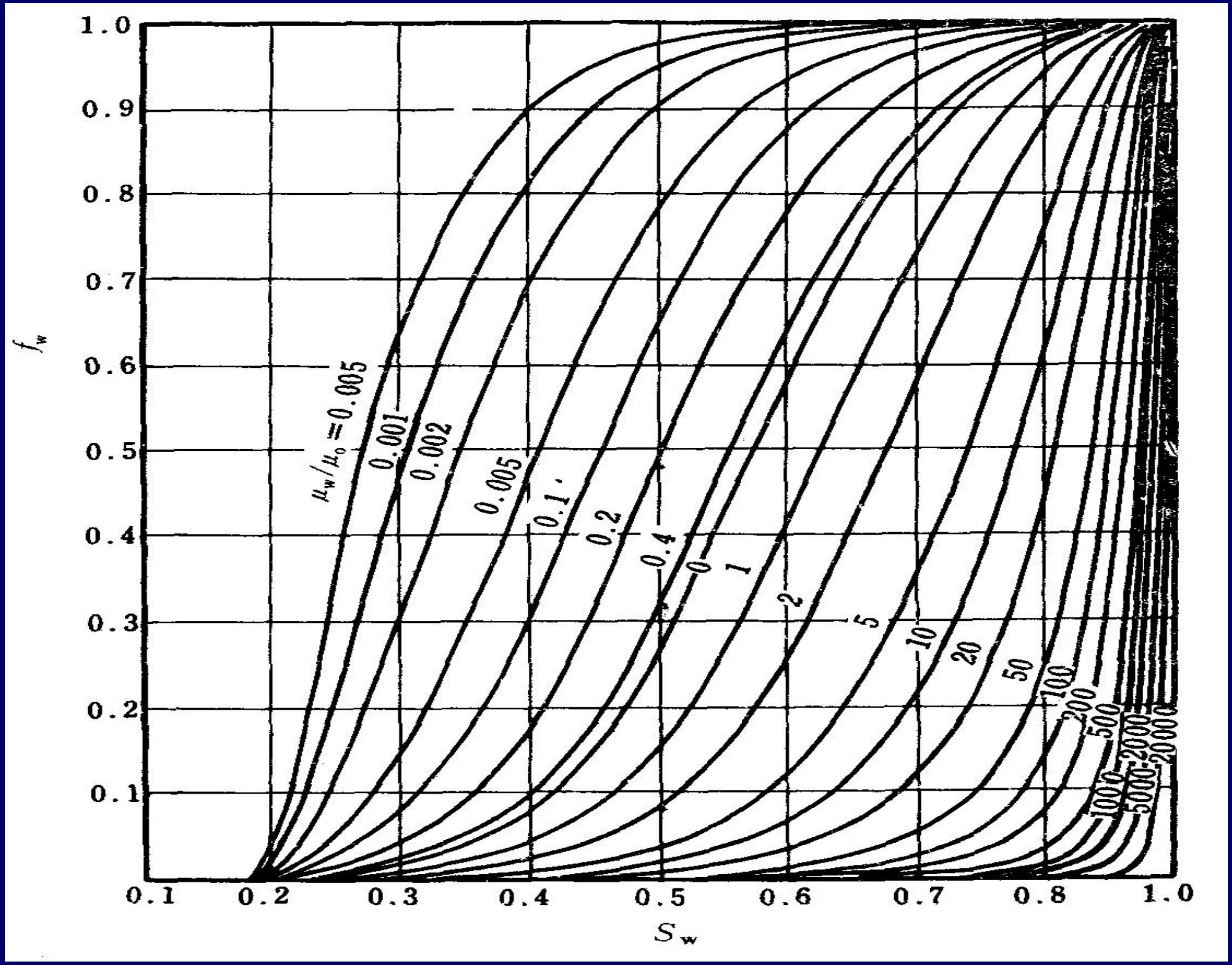
$0 < \alpha < \pi$

(3) Influence of water saturation on water-cut

Water-cut changes with water saturation. The water saturation increases, water-cut increased; In addition, the closer the water area, the greater the degree of water saturation。

Therefore, water-cut can be regarded as a function of distance X, that is:

$$f_w = f_w\left[S_w\left(X\right)\right]$$



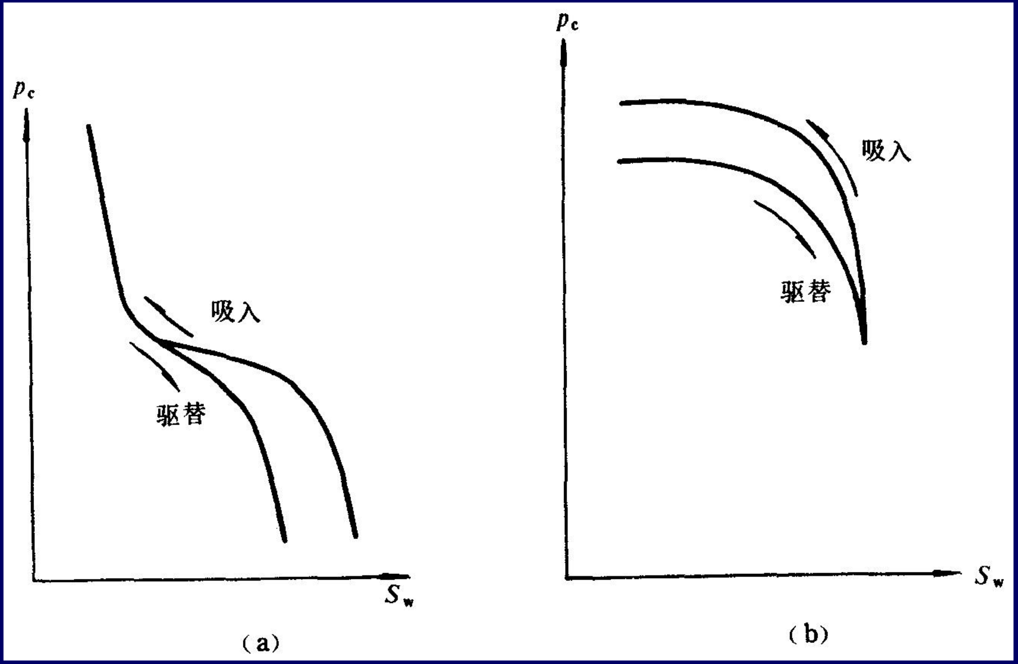
Fw---Sw curves of different oil and water viscosity ratio

Water-cut changes with water saturation. The water saturation increases, water-cut increased

(4) Capillary pressure will influence the f_w

No matter oil-wet rock or water-wet rock, capillary pressure is declining with the increase of water saturation in the core, thus the slope of capillary pressure curve is negative.

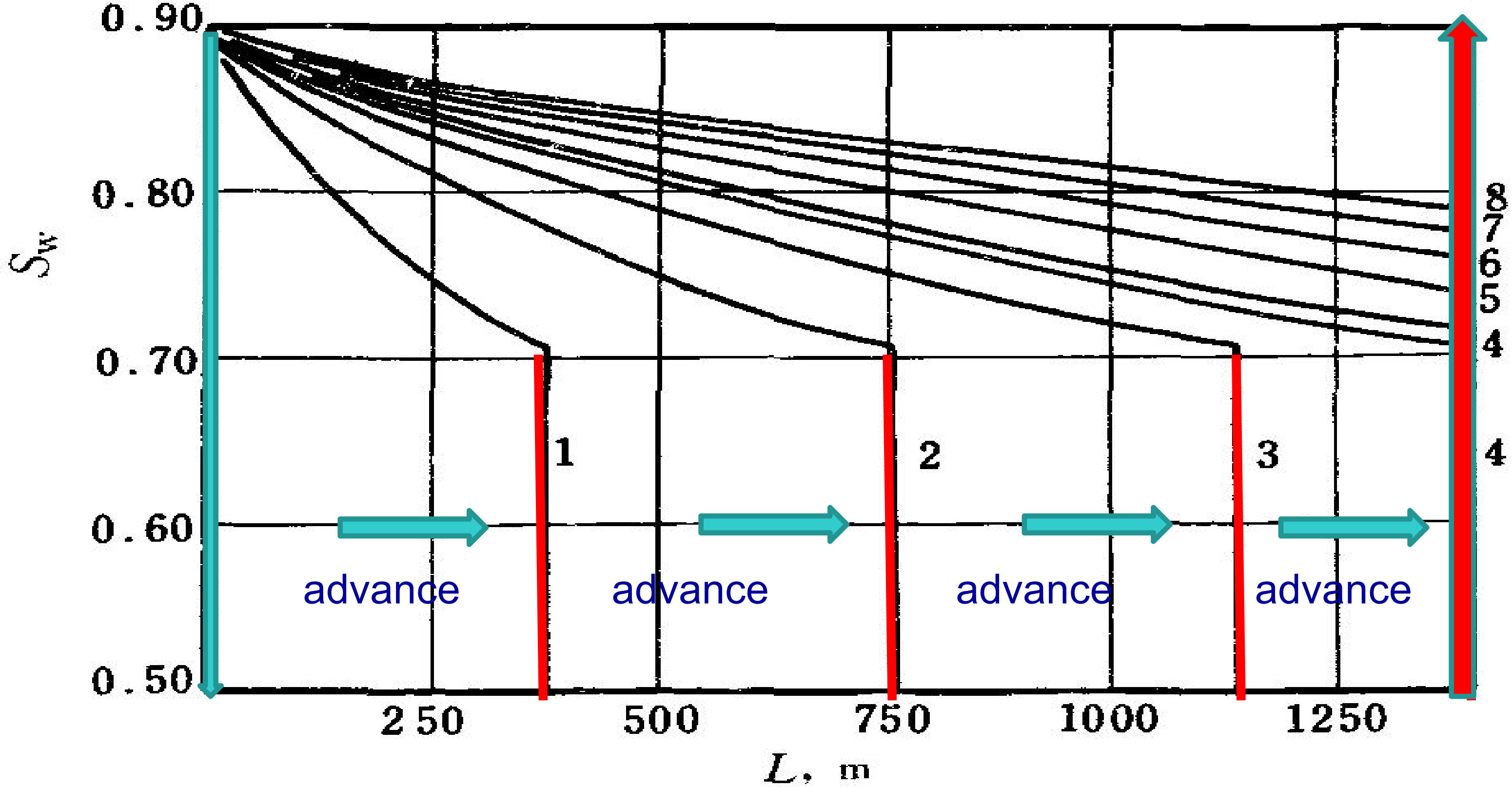
$$\frac{\partial P_c}{\partial S_w} < 0$$



Water wetting and non wetting phase when the capillary pressure curve

With the increase of distance from the water area, the water saturation is gradually reduced.

So $$\frac{\partial S_w}{\partial L} < 0$$



$$\because \frac{\partial P_c}{\partial S_w} < 0, \quad \frac{\partial S_w}{\partial L} < 0$$

$$\therefore \frac{\partial P_c}{\partial L} = \frac{\partial P_c}{\partial S_w} \cdot \frac{\partial S_w}{\partial L} > 0$$

$$\because f_w = \frac{1 + \frac{K_o}{q_t \mu_o}\left(\frac{\partial P_c}{\partial L} - g\Delta\rho \sin\theta\right)}{1 + \frac{K_{ro}}{K_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$

If we consider the capillay pressure gradient term ,then it will increase the water-cut (f_w).

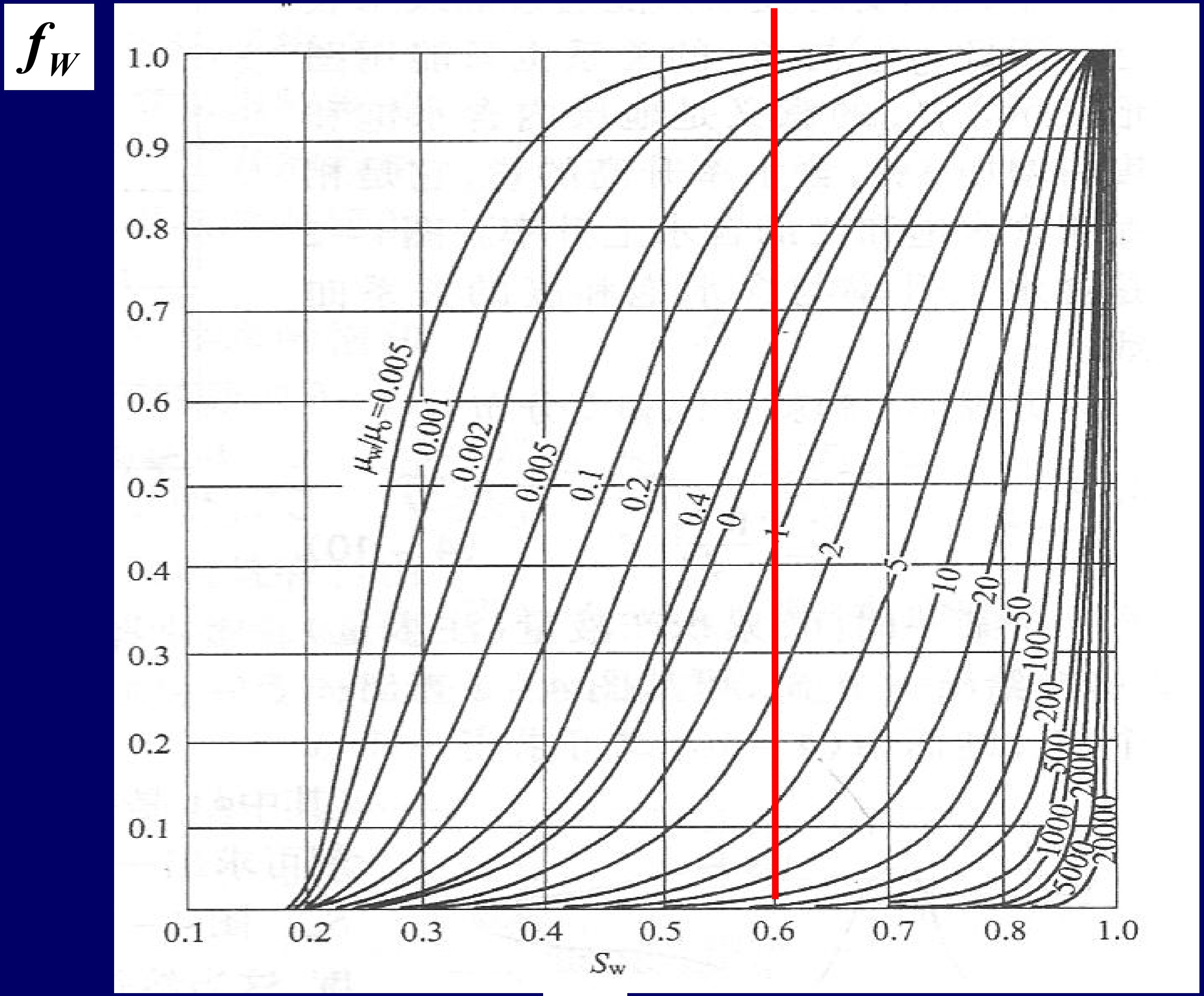
(5) Regardless of capillary pressure, If the flooding carries in level systems, then the formation dip θ=0, the formula can be further simplified as:

$$f_w = \frac{1}{1 + \frac{K_{ro}}{K_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$

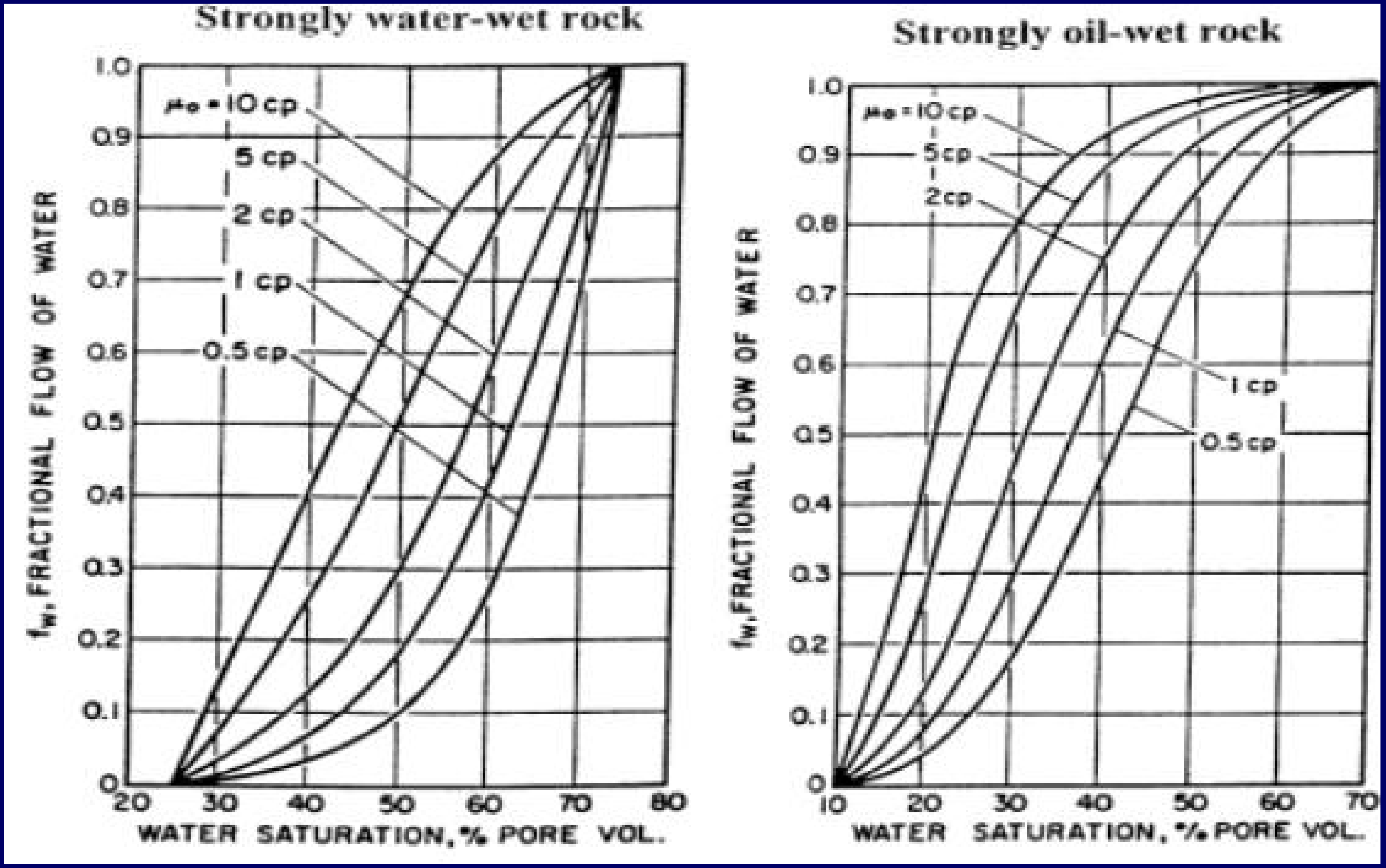
From this formula , we know:

If $\frac{\mu_w}{\mu_o}$ =constant , $\frac{k_{ro}}{k_{rw}}$ will be the main factor to influence the fw . $\frac{k_{ro}}{k_{rw}}$ go up , then fw will go down.

If $\frac{k_{ro}}{k_{rw}}$ =constant , $\frac{\mu_w}{\mu_o}$ will be the main factor to influence the fw . $\frac{\mu_w}{\mu_o}$ go up , then fw will go down.



S_w =constant , If $\frac{\mu_w}{\mu_o}$ go up , then fw will go down.



Effect of oil viscosity on fw

(6) Mobility ratio (M) will influence the f_w

Mobility refers to a fluid permeability of rocks divided by the viscosity of the fluid. Therefore, water mobility is K_w/μ_w , and oil mobility is K_o/μ_o

Mobility ratio is the ratio between driving phase mobility and the oil mobility, namely:

$$M = \frac{k_d / \mu_d}{k_o / \mu_o}$$

In the oil field, the drive phase is water, so the mobility ratio of water displacing oil is as follow:

$$M = \frac{K_w / \mu_w}{K_o / \mu_o}$$

If M=1,it means water and oil's flowability is the same;

If M<1,it will be favorable
oil mobility is greater than the displacing phase mobility. Therefore, less oil will go around, which is beneficial to mobility.

If M>1, it will be unfavorable.
In this case, the displacing phase is easier to flow compared to the oil. the displacing phase which is easier to flow will bypass many of the oil, while the oil will be left behind.

$$\because \ f_w = \frac{1}{1+\frac{k_{ro}\mu_w}{k_{rw}\mu_o}} = \frac{1}{1+\frac{k_{ro}/\mu_o}{k_{rw}/\mu_w}} = \frac{1}{1+\frac{1}{M}}$$

As can be seen from the above equation

∴ If M go up , then fw will go up ;
If M go down , then fw will go down.

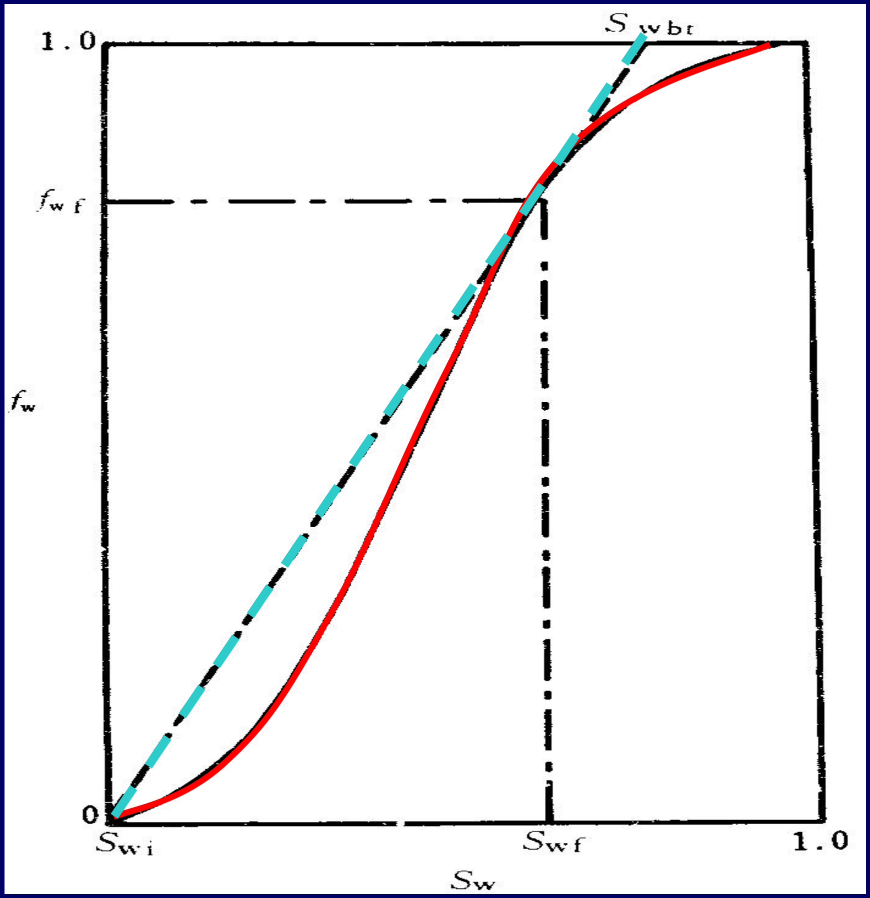
(7) Effect of water injection intensity on water-cut

The requirement of water injection intensity is not high for hydrophilic reservoir layer. Because strong water injection intensity will increase water-cut , it is unfavorable for oil recovery.

So taking strong water injection intensity will get good effect for oil reservoir layer.

四.Fractional-flow curve and its application

(1) Fractional-flow curve



(2) The application of fractional flow curve

①Calculate water saturation of water-oil front S_{wf}

②Determination of average water saturation before breakthrough $\overline{S}_{wf}$

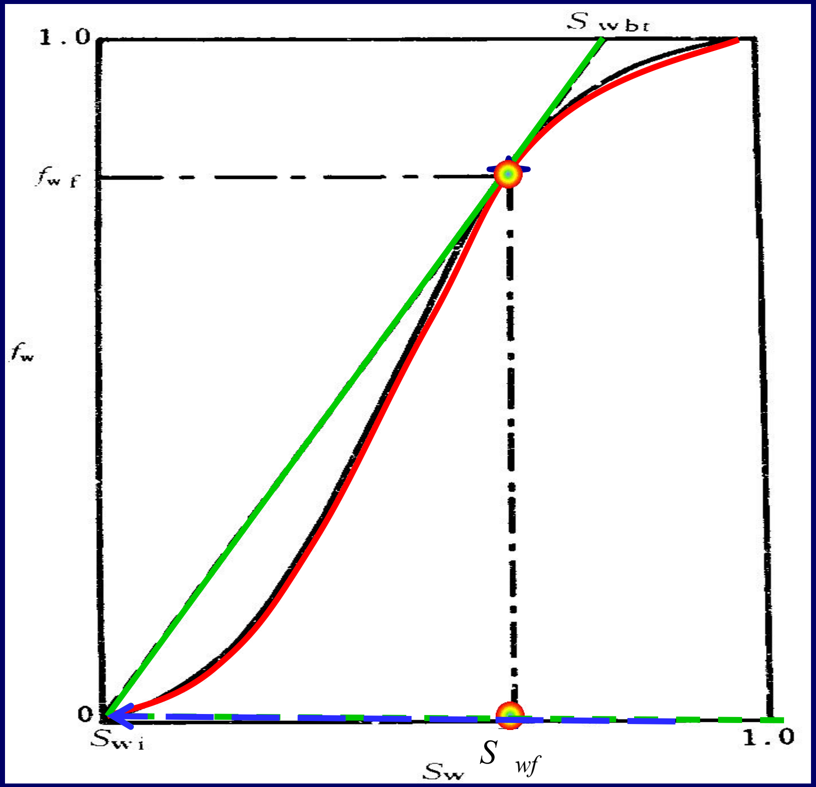
③Determination of average water saturation after breakthrough $\overline{S}_{w}$

①Calculate water saturation of oil-water front S_{wf}

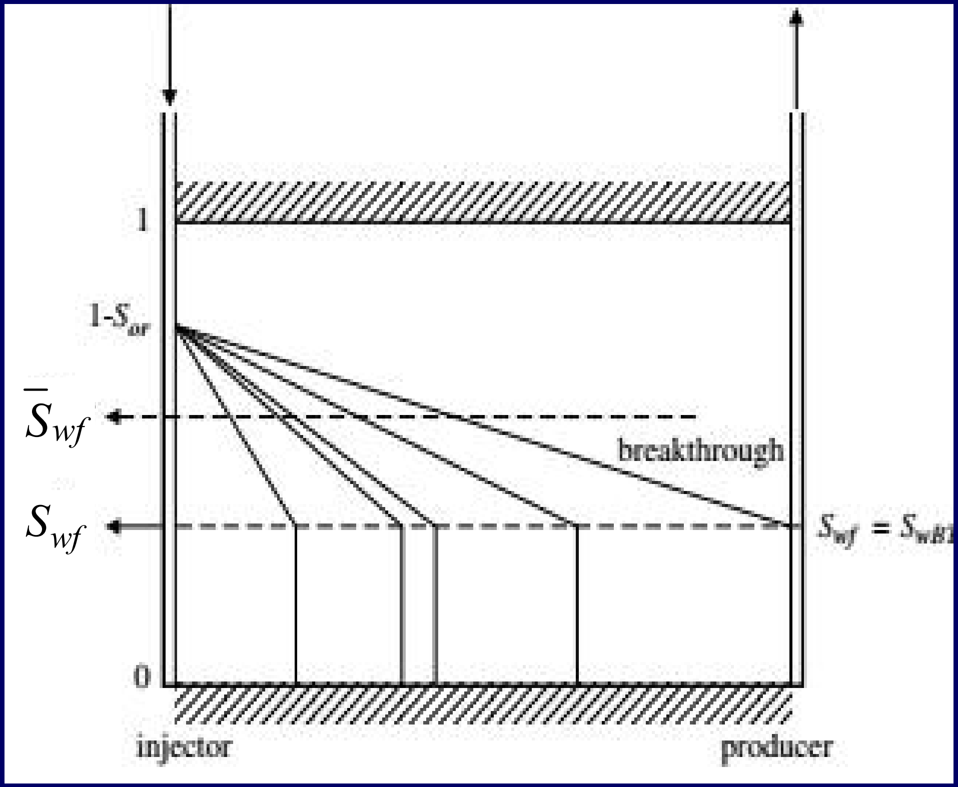
Ignoring the capillary pressure term, construct the fractional flow curve, i.e., f_w vs. S_w

Draw a straight line tangent from S_{wi} to the curve.

Identify the point of tangency and read off the values of S_{wf} and f_{wf}



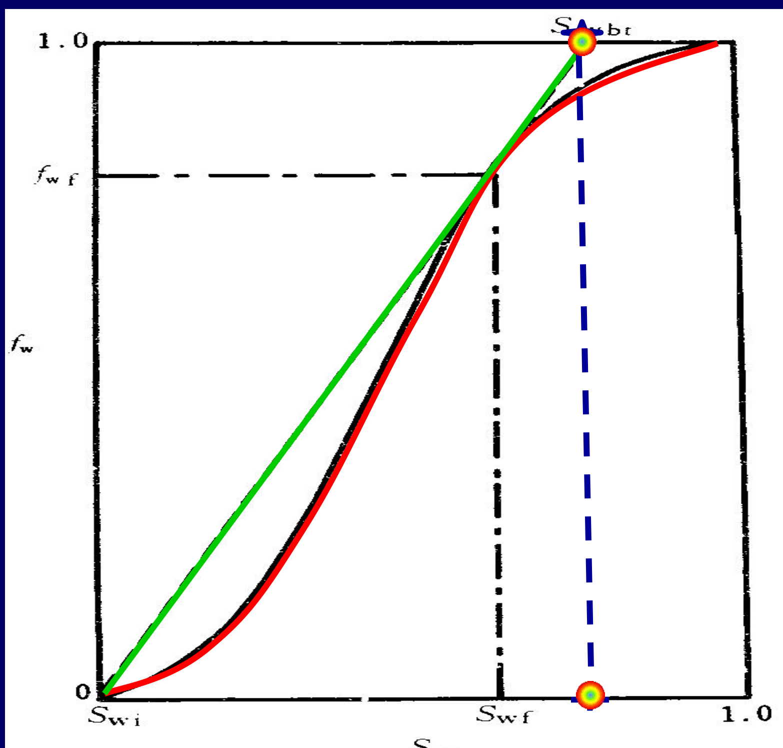
S_{wbt}
1.0
f_{wf}
f_w
0
S_{wi}
S_{wf}
1.0
S_w



1
1-S_{or}
$\overline{S}_{wf}$
breakthrough
S_{wf}
S_{wf} = S_{wBT}
0
injector
producer

②Determination of average water saturation before breakthrough $\overline{S}_{wf}$

Extending to tangent of fractional flow curve intersect with the water-cut equal to 100% of the level line. The saturation of the intersection point is equal to $\overline{S}_{wf}$



$\overline{S}_{wf}$

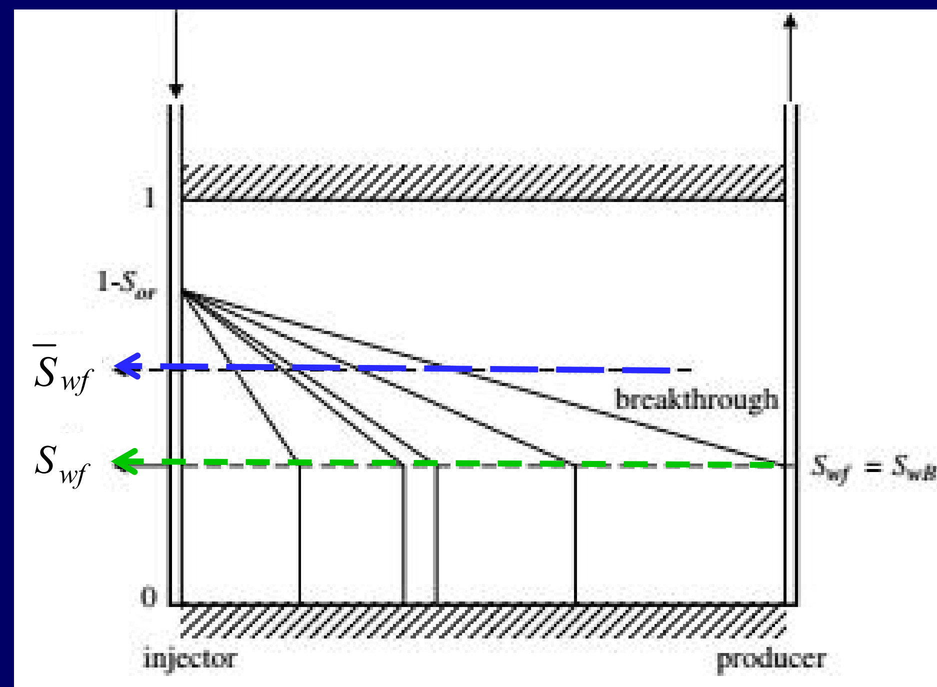
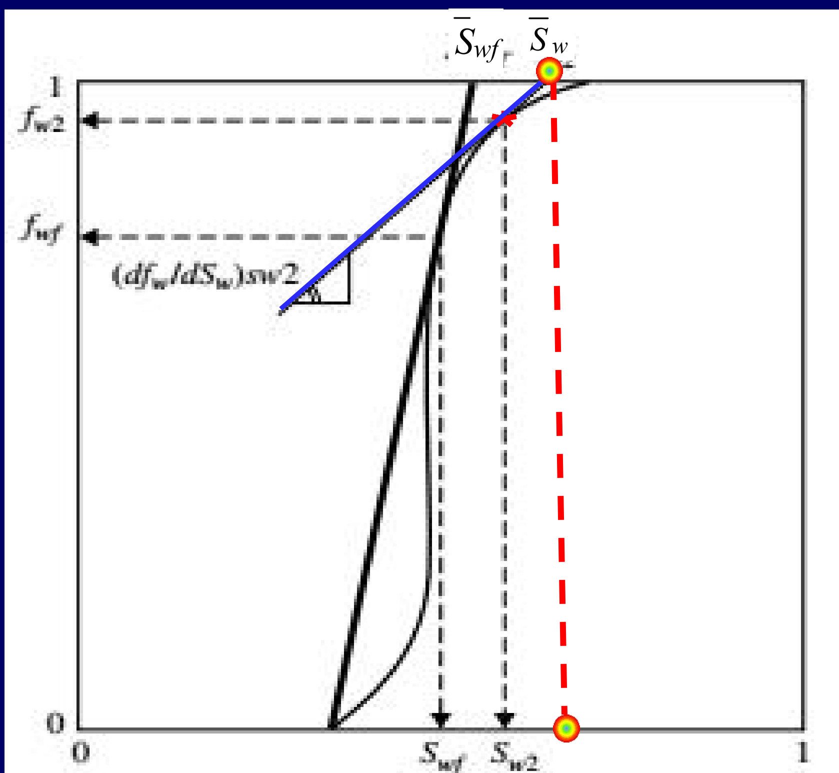


Figure 14-27. Average water saturation before breakthrough

③Determination of average water saturation after breakthrough $\overline{S}_w$

Do a tangent corresponding to the point (S_w,f_w), thus, the tangent intersect with the water-cut equal to 100% of the level line, the saturation of the intersection point is equal to $\overline{S}_w$



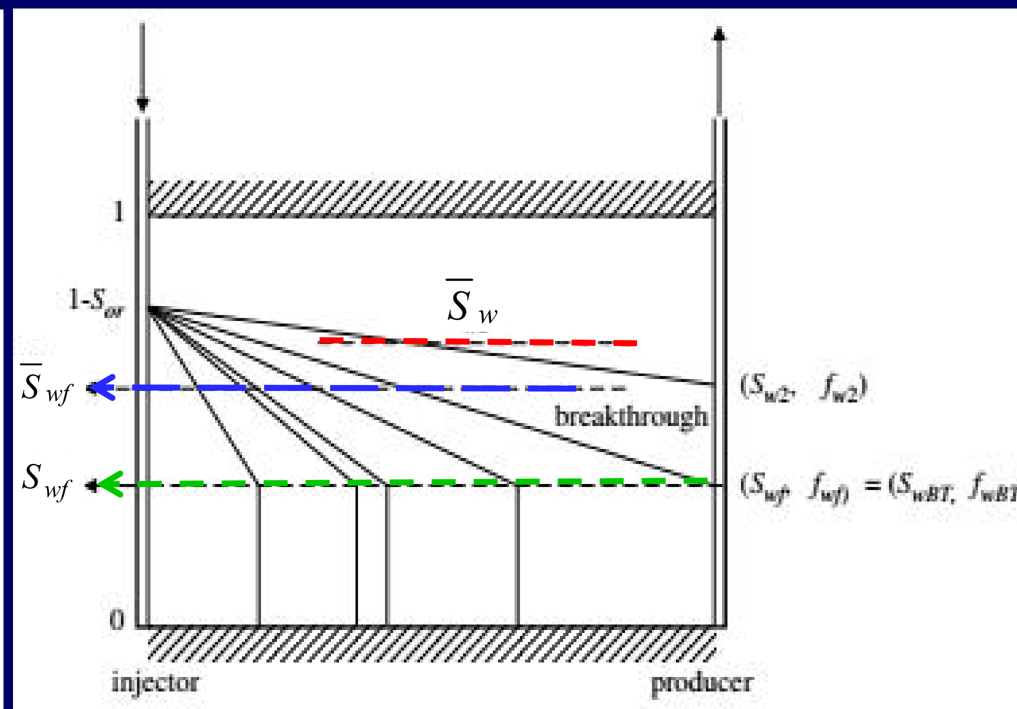


Figure 14-28. Average water saturation after breakthrough.

Chaptr 2   Basic theory of water flooding

Section 1   "Leaky" piston-like displacement characteristics

Section 2 Fractional-flow equation

Section 3 The frontal advance equation

Section 4 Welge's equation

Section 3 The frontal advance equation

1.The frontal advance equation

The frontal advance equation was proposed by Buckly and Leverett in 1942.

Advance rate

$$\left(\frac{\partial X}{\partial t}\right)_{S_w} = \frac{q_t}{A\Phi}\left(\frac{\partial f_w}{\partial S_w}\right)_t = \frac{V_t}{\Phi}\left(\frac{\partial f_w}{\partial S_w}\right)_t$$

The formula of integral transform is:

$$X = \frac{W_i}{A\Phi}\left(\frac{df_w}{dS_w}\right) = \frac{W_i}{A\Phi} f_w'$$

Where:

X——advance distance

S_w——water saturation

W_i——cumulative water injection

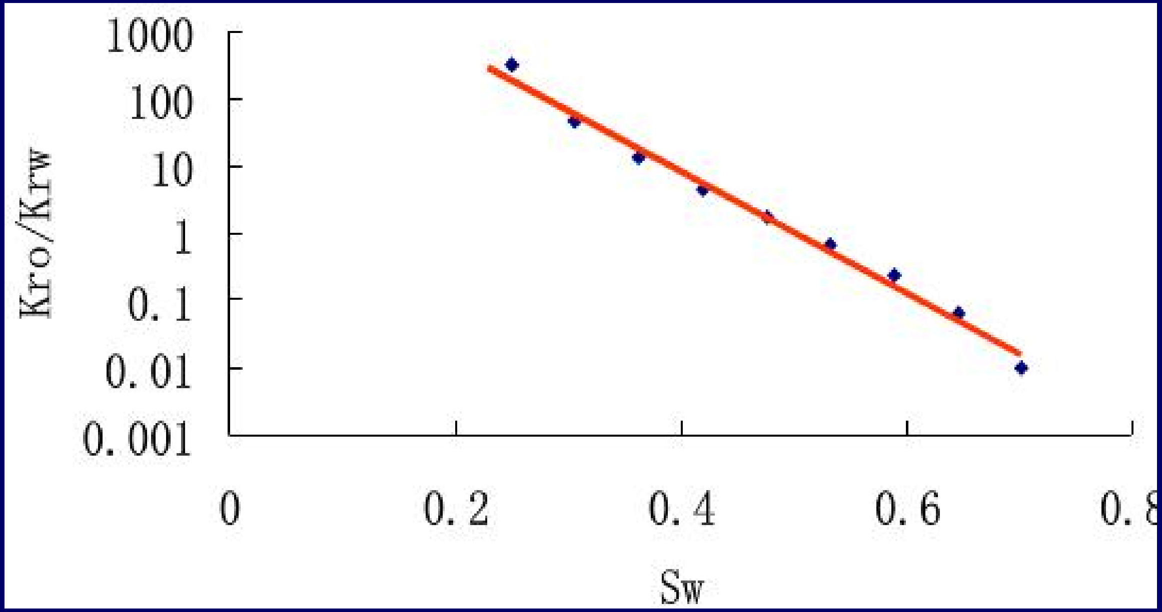
A——area

q_t——total flow rate

Φ ——porosity

$\left(\frac{\partial X}{\partial t}\right)_{Sw}$ ——advance rate

$f_w' = \frac{df_w}{dS_w} --$ It may be obtained by plotting fw vs. S_w and graphically take the slopes at value of S_w . A mathematical method also may be used.



$$K_{ro}/K_{rw} = ae^{-bSw}$$

Take logarithm on both sides of the formula:

$$\ln K_{ro}/K_{rw} = \ln a - bS_w$$

The constants a and b may be determined from the graph

$$\because \; f_w = \frac{1}{1+\frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}}$$

$$K_{ro}/K_{rw} = ae^{-bSw}$$

According to the above two formulas , with ae^{-bSw} instead of K_{ro}/K_{rw} ,then we can get :

$$\therefore \; f_w = \frac{1}{1 + ae^{-bS_w}\left(\frac{\mu_w}{\mu_o}\right)}$$

Derivative

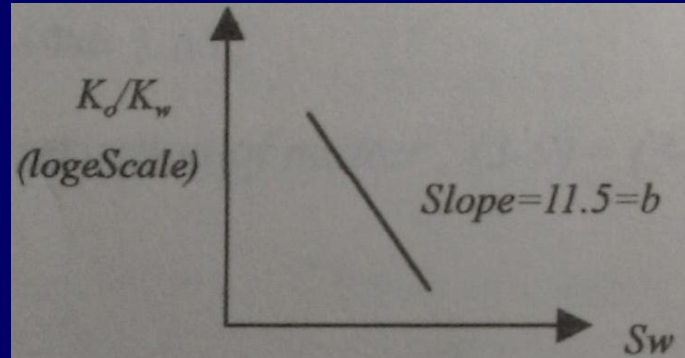
$$\therefore \; \frac{df_w}{dS_w} = \frac{\left(\frac{\mu_w}{\mu_o}\right) bae^{-bS_w}}{\left[1+\left(\frac{\mu_w}{\mu_o}\right)ae^{-bS_w}\right]^2} = \frac{\left(\frac{\mu_w}{\mu_o}\right) b\left(\frac{K_{ro}}{K_{rw}}\right)}{\left[1+\left(\frac{\mu_w}{\mu_o}\right)\left(\frac{K_{ro}}{K_{rw}}\right)\right]^2}$$

This equation does not apply to high or low value of S_w

If we have $\frac{\mu_w}{\mu_o}, \frac{K_o}{K_w}, b$,then we can use equation to solve the value of $\frac{df_w}{dS_w}$

$$\frac{df_w}{dS_w} = \frac{\left(\frac{\mu_w}{\mu_o}\right) bae^{-bS_w}}{\left[1 + \left(\frac{\mu_w}{\mu_o}\right) ae^{-bS_w}\right]^2} = \frac{\left(\frac{\mu_w}{\mu_o}\right) b \left(\frac{K_{ro}}{K_{rw}}\right)}{\left[1 + \left(\frac{\mu_w}{\mu_o}\right)\left(\frac{K_{ro}}{K_{rw}}\right)\right]^2}$$

For example:



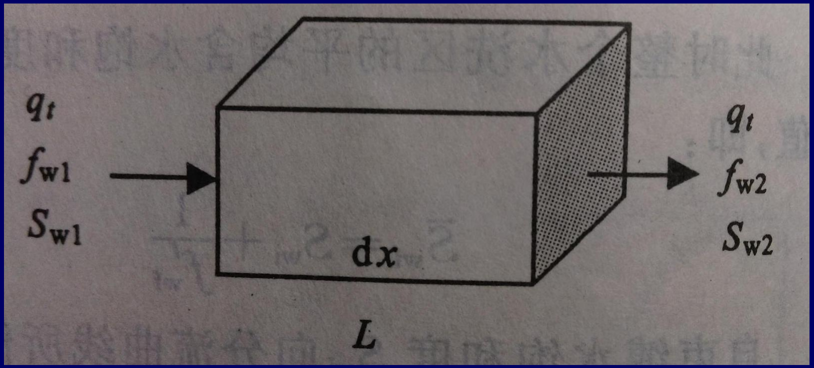
$\frac{\mu_w}{\mu_o} = 0.5$

S_w	$\frac{K_{ro}}{K_{rw}}$	$\frac{df_w}{dS_w}$
0.5	1.70	?
0.6	0.55	?

$$\left(\frac{df_w}{dS_w}\right)_{S_w=0.5} = \frac{\left(\frac{\mu_w}{\mu_o}\right) bae^{-bS_w}}{\left[1+\left(\frac{\mu_w}{\mu_o}\right) ae^{-bS_w}\right]^2} = \frac{\left(\frac{\mu_w}{\mu_o}\right) b \left(\frac{K_{ro}}{K_{rw}}\right)}{\left[1+\left(\frac{\mu_w}{\mu_o}\right)\left(\frac{K_{ro}}{K_{rw}}\right)\right]^2} = \frac{\frac{1}{2}\times 11.5 \times 1.70}{(1+\frac{1}{2}\times 1.70)^2} = 2.86$$

$$\left(\frac{df_w}{dS_w}\right)_{S_w=0.6} = \frac{\left(\frac{\mu_w}{\mu_o}\right) bae^{-bS_w}}{\left[1+\left(\frac{\mu_w}{\mu_o}\right) ae^{-bS_w}\right]^2} = \frac{\left(\frac{\mu_w}{\mu_o}\right) b \left(\frac{K_{ro}}{K_{rw}}\right)}{\left[1+\left(\frac{\mu_w}{\mu_o}\right)\left(\frac{K_{ro}}{K_{rw}}\right)\right]^2} = \frac{\frac{1}{2}\times 11.5 \times 0.55}{(1+\frac{1}{2}\times 0.55)^2} = 1.95$$

2.The rate-of-frontal-advance equation derivation



Consider an elemental volume from a linear porous media , as shown in above diagram, containing two fluids , the displaced fluid , oil , and the displacing fluid , water.

The element has a cross-sectional area , A and a porosity Φ

The entering water quantity $W_1 = q_t f_{w1} dt$

The exiting water quantity $W_2 = q_t f_{w2} dt$

Amount of water left in the unit

$$\Delta W = q_t (f_{w1} - f_{w2}) dt = q_t \Delta f_w dt \qquad (2\text{-}14)$$

The change in the water saturation , ΔS_W ,in the pore volume $A\Phi dx$ of the element of the porous media= $(A\Phi dx)\Delta S_W$ (2-15)

According to the law of conservation of mass , (2-14)=(2-15) , so :

$$(A\Phi dx)\Delta S_W = q_t \Delta f_w dt$$

$$\therefore\ dx = \frac{q_t}{A\Phi}\left(\frac{\partial f_w}{\partial S_w}\right)_t dt$$

$$\because\ W_i = q_t t$$

$$\therefore\ X = \frac{W_i}{A\Phi}\left(\frac{\partial f_w}{\partial S_w}\right)$$

3.The significance of frontal advance equation

$$\left(\frac{\partial X}{\partial t}\right)_{S_w} = \frac{q_t}{A\Phi}\left(\frac{\partial f_w}{\partial S_w}\right)_t$$

The equation indicates that water saturation with fixed constant and the surface are equivalent to the speed of $\frac{q_t}{A\Phi}\left(\frac{\partial f_w}{\partial S_w}\right)_t$ moving along the flow path. When the total flow rate (q_t) increases, the speed of a certain saturation surface also increases ; when the total flow rate (q_t) is reduced, the speed of water saturation is reduced.

4.The application of frontal advance equation

From the frontal advance equation

$$\left(\frac{\partial X}{\partial t}\right)_{S_w} = \frac{q_t}{A\Phi}\left(\frac{\partial f_w}{\partial S_w}\right)_{Sw}$$

$$\Delta X = \frac{q_t}{A\Phi}\left(\frac{\partial f_w}{\partial S_w}\right)_{Sw} \Delta t$$

By using the above equation, water saturation distribution at any moment can be calculated. At last, the water saturation position of water-oil front can be determined.

Chaptr 2 Basic theory of water flooding

Section 1 “Leaky” piston-like displacement characteristics

Section 2 Fractional-flow equation

Section 3 The frontal advance equation

Section 4 Welge's equation

Section 4 Welge's equation

1.Welge's equation

2 .The application of welge's equation

1.Welge's equation

In 1952, Welge expanded Barclays Leverett's early studies and proposed Welge's equation:

$$\overline{S}_w - S_{w2} = Q_i f_{o2}$$

Where: $\overline{S}_w$ —— Average water saturation

S_{w2} —— Water saturation at the producing end of the system；

Q_i —— Pore volumes of cumulative injected fluid

f_{o2} —— Oil-cut $f_{o2} = 1 - f_{w2}$

The value of Q_i after breakthrough is found by:

$$Q_i = \frac{1}{\left(\frac{df_w}{dS_w}\right)_{Sw2}}$$

Where:

Q_i —— Volumes of cumulative injected fluid

In this equation, the difference between the average and producing end saturation is related to the cumulative injected fluid and the producing oil cut . In practice, the values of the terms S_w, Q_i, and f_{o2} can be determined at any time from the production history of a flow experiment .Then the saturation at the outflow face, S_{w2}, can be calculated from Welge's equation.

2 .The application of welge's equation

Example： it is known outlet end of the water saturation, the outlet end of the water-cut and the fractional flow curve slope . Please calculate $\overline{S}_w$

Solution: Because of

$$Q_i = \frac{1}{\left(\frac{df_w}{dS_w}\right)_{Sw2}}$$

When : $S_{w2} = 0.469$

$$Q_i = \frac{1}{2.16} = 0.463$$

$$\overline{S}_w = S_{w2} + Q_i f_{o2} = 0.469 + 0.463 \times (1 - 0.798) = 0.563$$

The outlet end of different water saturation of $\overline{S}_w$ can be calculated successively.

The value of Q_i after breakthrough is found by:

$$Q_i = \frac{1}{\left(\frac{df_w}{dS_w}\right)_{Sw2}}$$

Where：

Q_i —— Pore volumes of cumulative injected fluid

Examples of dynamic water flooding

Already known data | **Seek data**

S_{w2} 出口端含水饱和度（占孔隙体积的分数）	f_{w2} 出口端液流中含水率（分数）	df_w/dS_w 分流量曲线的斜率	Q 累积注入水量的孔隙体积数	$\bar{S}_w$ 平均含水饱和度（孔隙体积的分数）
0.469	0.798	2.16	0.463	0.563
0.495	0.848	1.75	0.572	0.582
0.520	0.888	1.41	0.711	0.600
0.546	0.920	1.13	0.887	0.617
0.572	0.946	0.851	1.176	0.636
0.597	0.965	0.649	1.540	0.652
0.622	0.980	0.477	2.100	0.666
0.649	0.990	0.317	3.157	0.681
0.674	0.996	0.195	5.13	0.694
0.700	1.000	0.102	9.80	0.700

Example-1

The following data are available for a linear-reservoir system:

S_w	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
k_{ro}/k_{rw}	30.23	17.00	9.56	5.38	3.02	1.70	0.96	0.54	0.30	0.17	0.10

B_o=1.25 bbl/STB B_w=1.02 bbl/STB   h = 20 ft   area A = 26,400 ft

Porosity = 25%   Injection rate i_w = 900 bbl/day

Distance between producer and injector   L=600 ft

u_o = 2.0 cp u_w = 1.0 cp   Dip angle = 0° S_{wi} = 20% S_{or} = 20%

Seek for:

(1)Calculate and plot the water saturation profile after 60, 120, and 240 days.

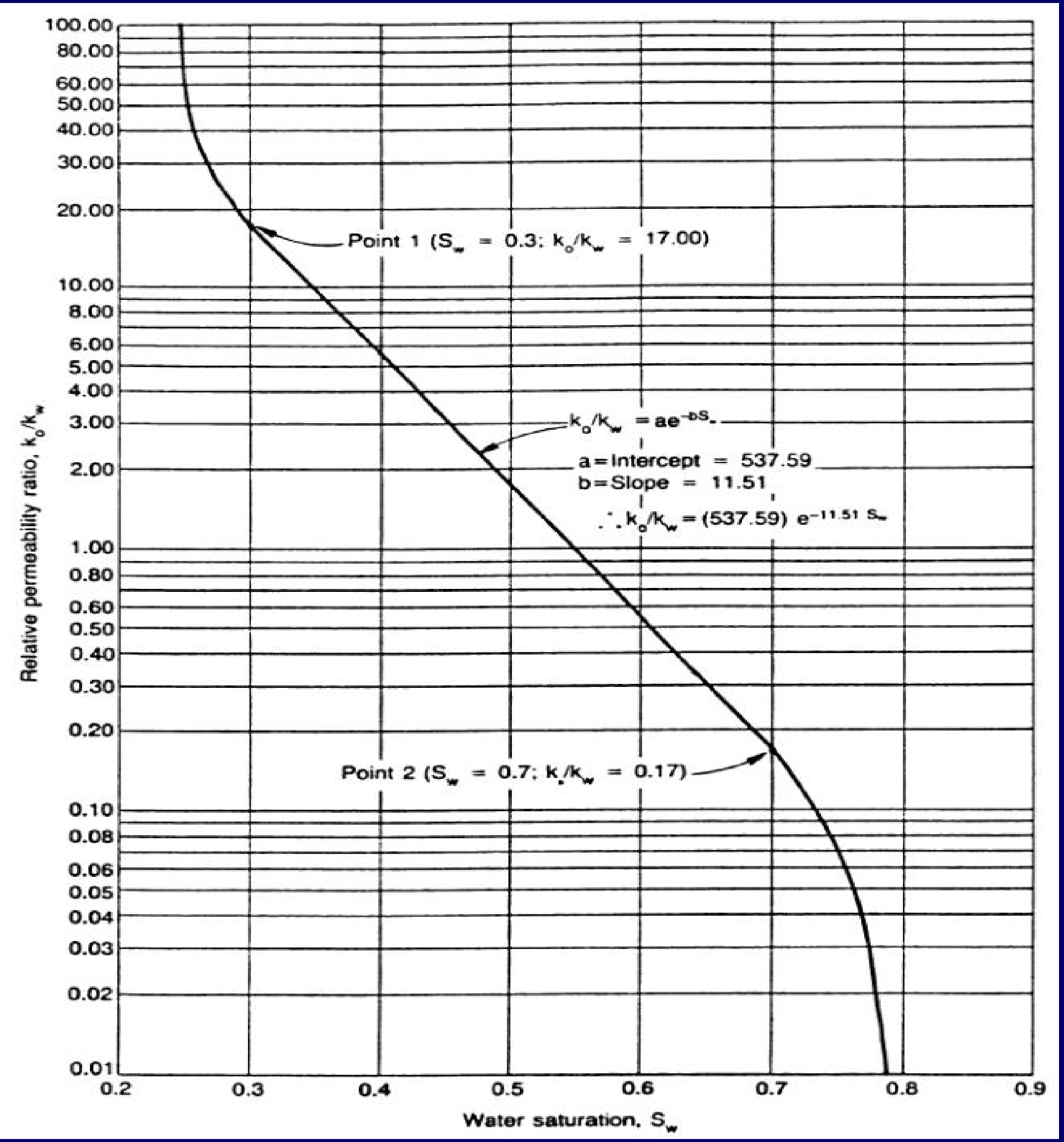
(2)Time to breakthrough

(3)Cumulative water injected at breakthrough

(4)Total pore volumes of water injected at breakthrough

Solution

Step 1. Plot **k_{ro}/k_{rw} vs. S_w** on a semilog paper and determine the coefficients a and b of Equation



Therefore,

$$\frac{k_{ro}}{k_{rw}} = 537.59e^{-11.51S_w}$$

Step 2. Assume several values of water saturation and calculate the fractional flow curve at its derivatives by applying Equations

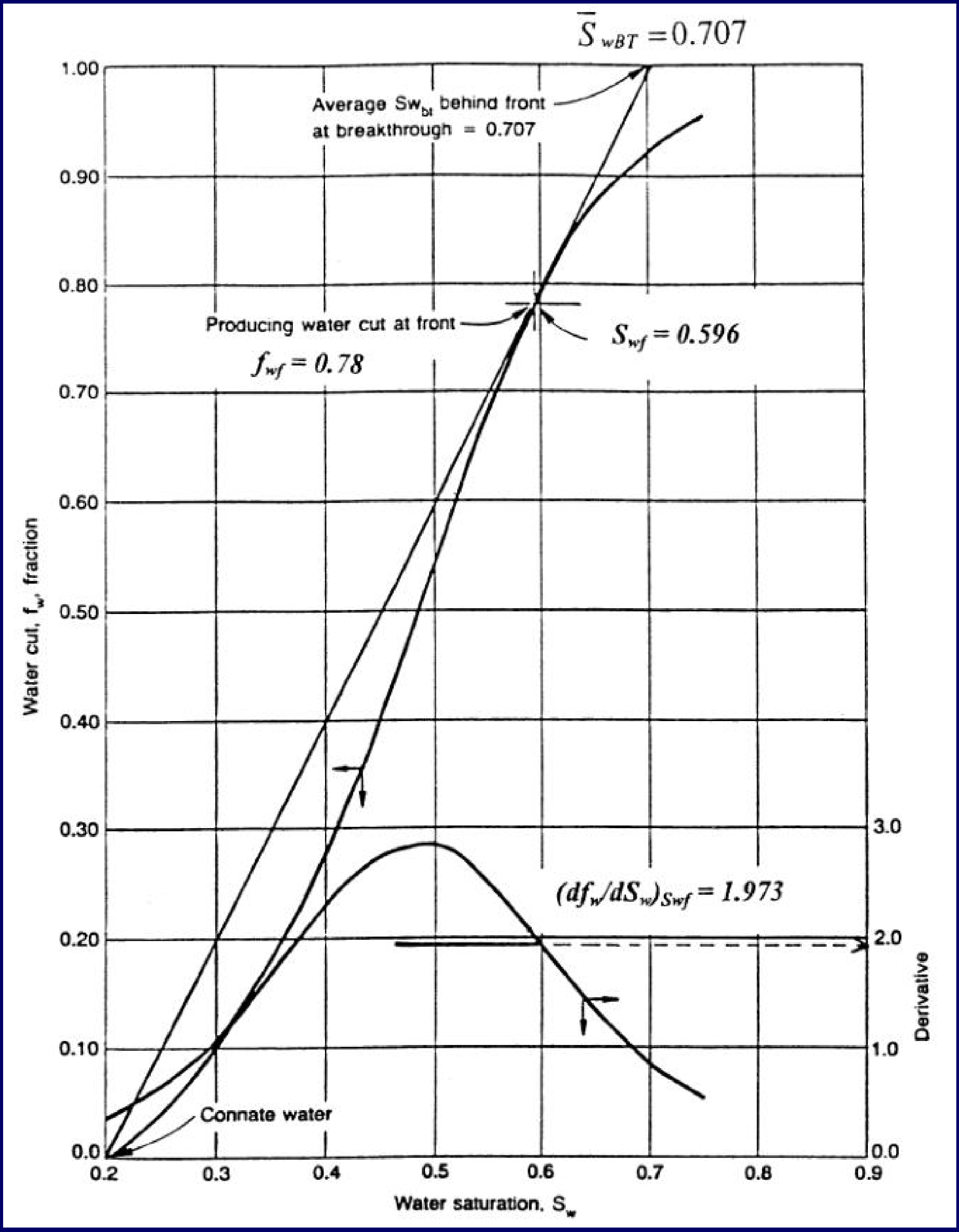
$$\frac{df_w}{dS_w} = \frac{\left(\frac{\mu_w}{\mu_o}\right) bae^{-bS_w}}{\left[1 + \left(\frac{\mu_w}{\mu_o}\right) ae^{-bS_w}\right]^2} = \frac{\left(\frac{\mu_w}{\mu_o}\right) b \left(\frac{K_{ro}}{K_{rw}}\right)}{\left[1 + \left(\frac{\mu_w}{\mu_o}\right)\left(\frac{K_{ro}}{K_{rw}}\right)\right]^2}$$

S_w	k_{ro}/k_{rw}	f_w, Equation 14-37	(df_w/dS_w), Equation
0.25	30.23	0.062	0.670
0.30	17.00	0.105	1.084
0.35	9.56	0.173	1.647
0.40	5.38	0.271	2.275
0.45	3.02	0.398	2.759
0.50	1.70	0.541	2.859
0.55	0.96	0.677	2.519
0.60	0.54	0.788	1.922
0.65	0.30	0.869	1.313
0.70	0.17	0.922	0.831
0.75	0.10	0.956	0.501

Step 3. Plot f_w and (df_w/S_w) vs. S_w on a Cartesian scale as shown in Figure . Draw a straight line from S_{wi} and tangent to the f_w curve. Determine the coordinates of point of tangency and the slope of the tangent $(df_w/dS_w)_{Swf}$, to give:

$$(S_{wf}, f_{wf}) = (0.596, 0.78) \quad \text{and} \quad \left(\frac{df_w}{dS_w}\right)_{S_{wf}} = 1.973$$

This means that the leading edge of the water front (stabilized zone) has a constant saturation of 0.596 and water cut of 78%.



Step 4. When constructing the water saturation profile, it should be noted that no water saturation with a value less than S_{wf}, i.e., 59.6%, exists behind the leading edge of the water bank. Assume water saturation values in the range of S_{wf} to $(1–S_{or})$, i.e., 59.6% to 75%, and calculate the water saturation profile as a function of time by using Equation

The frontal advance equation:

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{q_t}{\phi A}\left(\frac{\partial f_w}{\partial S_w}\right)_{Sw}$$

$$(x)_{S_w} = \frac{q_t t}{\phi A}\left(\frac{\partial f_w}{\partial S_w}\right)_{Sw}$$

In field units, the above equation can be expressed as:

$$(x)_{S_w} = \left(\frac{5.615 i_w t}{\phi A}\right)\left(\frac{df_w}{dS_w}\right)_{S_w}$$

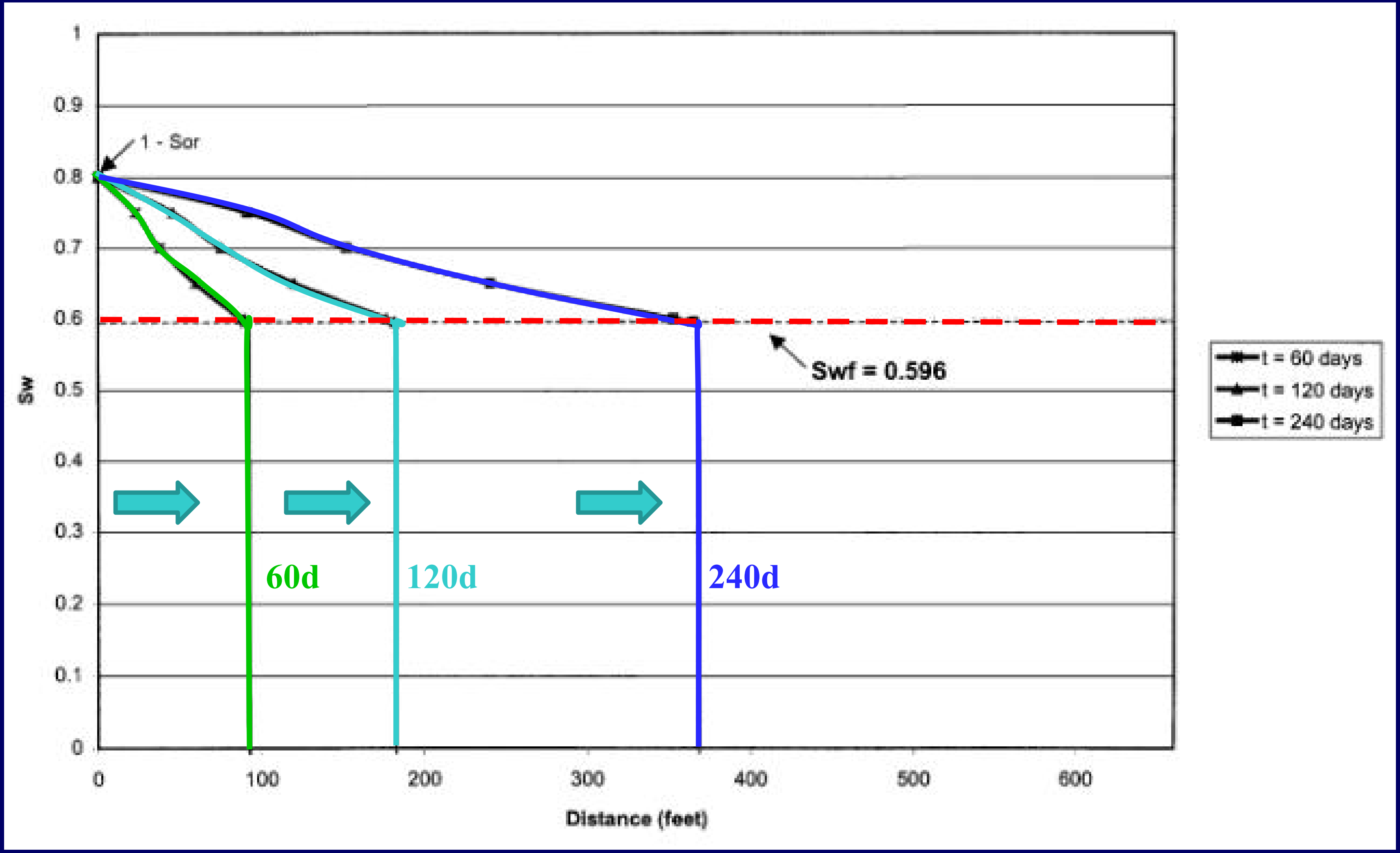
i_w = water-injection rate, bbl/day

$$(x)_{S_w} = \left(\frac{(5.615)(900)t}{(0.25)(26,400)}\right)\left(\frac{df_w}{dS_w}\right)_{S_w}$$

$$(x)_{S_w} = (0.77t)\left(\frac{df_w}{dS_w}\right)_{S_w}$$

Assumed S_w	(df_w/dS_w)	$x = 0.77t(df/dS_w)$ t = 60 days	$x = 0.77t(df/dS_w)$ t = 120 days	$x = 0.77t(df/dS_w)$ t = 240days
0.596	1.973	91	182	365
0.60	1.922	88	177	353
0.65	1.313	60	121	241
0.70	0.831	38	76	153
0.75	0.501	23	46	92

Step 5. Plot the water saturation profile as a function of distance and time, as shown in Figure



Water saturation profile

The above example shows that after 240 days of water-injection, the leading edge of the water front has moved 365 feet from the injection well (235 feet from the producer). The water front (leading edge) will eventually reach the production well and water reakthrough occurs.

The example also indicates that at water breakthrough, the leading edge of the water front would have traveled exactly the entire distance between the two wells, i.e., 600 feet. Therefore, to determine **the time to breakthrough, t_{BT}**, simply set $(x)_{Swf}$ equal to the distance between the injector and producer L in Equation and solve for the time:

$$L = \left(\frac{5.615 i_w t_{BT}}{\phi A} \right) \left(\frac{df_w}{dS_w} \right)_{S_{wf}}$$

$$L = \left(\frac{5.615 i_w t_{BT}}{\phi A} \right) \left(\frac{df_w}{dS_w} \right)_{S_{wf}}$$

Note that the pore volume (PV) is given by:

$$(PV) = \frac{\phi A L}{5.615}$$

Combining the above two expressions and solving for the time to breakthrough t_{BT} gives:

$$t_{BT} = \left[\frac{(PV)}{i_w} \right] \frac{1}{\left(\frac{df_w}{dS_w} \right)_{S_{wf}}}$$

Where t_{BT} = time to breakthrough, day

PV = total flood pattern pore volume, bbl

L = distance between the injector and producer, ft

Assuming a constant water-injection rate, the **cumulative water injected at breakthrough** is calculated

$$W_{iBT} = i_w t_{BT} = \frac{(PV)}{\left(\frac{df_w}{dS_w}\right)_{S_{wf}}}$$

Where: Wi_{BT} = cumulative water injected at breakthrough, bbl

Step 6. Calculate the reservoir pore volume:

$$(PV) = \frac{\phi A L}{5.615}$$

$$(PV) = \frac{(0.25)(26{,}400)(660)}{5.615} = 775{,}779 \text{ bbl}$$

Step 7. Calculate the time to breakthrough

$$t_{BT} = \frac{(PV)}{i_w} \frac{1}{\left(\frac{df_w}{dS_w}\right)_{S_{wf}}}$$

$$t_{BT} = \left(\frac{775{,}779}{900}\right)\left(\frac{1}{1.973}\right) = 436.88 \text{ days}$$

Step8. Determine cumulative water injected at breakthrough:

$$W_{iBT} = i_w t_{BT}$$

$$W_{iBT} = (900)(436.88) = 393{,}198 \text{ bbl}$$

Step 9. Calculate pore volumes of water injected at breakthrough:

$$Q_{iBT} = \frac{1}{\left(\frac{df_w}{dS_w}\right)_{S_{wf}}}$$

$$Q_{iBT} = \frac{1}{1.973} = 0.507 \text{ pore volumes}$$

That's all for today

Class is over

See you next time